«Оптика атмосферы и океана», **23**, № 4 (2010)

ОПТИКА КЛАСТЕРОВ, АЭРОЗОЛЕЙ И ГИДРОЗОЛЕЙ

УДК 519.216.24, 519.6

Monte Carlo simulation of angular characteristics for polarized radiation in water-drop and crystal clouds

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Поступила в редакцию 20.12.2009 г.

In the paper we present the results of computational experiments aimed to define the angular distributions for the polarized radiation scattered in a cloudy layer. The angular distributions for Stokes parameters were computed by Monte Carlo method for different optical models of water-drop and crystal clouds. The ulterior objective of the research is to develop effective techniques to study the particles shape and size by measuring angular characteristics of the scattered radiation emanating from clouds.

Key words: polarized radiation transfer, water-drop and crystal clouds, Monte Carlo simulation, angular distributions, particle shape and size.

Introduction

The role of clouds in the global climate system is important but not well studied. Cloud feedbacks are the largest source of uncertainty in estimates of radiation balance and climate sensitivity, therefore a better understanding and representation of radiation transfer processes in clouds is of paramount importance for climate science. On the other hand, the adequate optical models of clouds are necessary to investigate properties of cloudiness by active and passive optical remote sensing. Our paper deals with numerical modeling of the solar radiation transfer in the atmosphere clouds taking into account specific features caused by polarization of light. By computational experiments we studied angular distributions for the polarized radiation scattered upward and downward by water-drop and crystal clouds. The angular distributions were computed for the Stokes parameters, degree of polarization, and direction of preferable polarization. For computations we used Monte Carlo method and several optical models of clouds. The ultimate aim of the research is to develop effective techniques to study phase structure of clouds, shape and size of particles by measuring characteristics of the scattered radiation.

1. Mathematical model and Monte Carlo algorithms

Assume that an optically isotropic scattering medium consists of particles randomly oriented in space, extinction coefficient and single scattering albedo in the medium does not depend on light polarization, and a field of reference-vectors $\rho(\omega)$ is fixed, i.e. for every direction $\omega \in \Omega = \{(a, b, c\} \in R^3: a^2 + b^2 + c^2 = 1\}$ there is defined a unit vector $\rho(\omega)$ orthogonal to ω. Then the process of stationary polarized radiation transfer in the scattering medium may be described by integral equations of the second kind with the generalized kernel

$$S[\rho](r, \omega) = \int_\Omega \int_{R^3} \frac{e^{-\tau(r',r)}}{\|r-r'\|^2} q(r')\sigma(r') M[\rho',\rho](\omega',\omega,r') \times$$

$$\times S'[\rho'](r',\omega')\delta\left(\omega - \frac{r-r'}{\|r-r'\|}\right) dr' d\omega' + S_0[\rho](r,\omega),$$

$$r', r \in R^3, \omega', \omega \in \Omega, \rho = \rho(\omega), \rho' = \rho(\omega'). \quad (1)$$

Here $S[\rho](r, \omega)$ is the Stokes vector (we shall consider Stokes vectors of the type (I,Q,U,V)) with respect to the reference vector $\rho = \rho(\omega)$ for the radiation at the point r spreading in the direction ω, $q(r')$ is the single scattering albedo at the point r', $\sigma(r')$ is the extinction coefficient at the point r', δ is the delta-function, $S_0[\rho](r,\omega)$ is the Stokes vector of the source at the point r in the direction ω, $\tau(r', r) = \int_0^l \sigma(r(s),\omega) ds$ is the optical length of the segment $[r', r]$, $r(s) = r' + s(r - r')/l$, $l = \|r-r'\|$, $M[\rho', \rho](\omega', \omega, r')$ is the 4×4-phase matrix of the medium at the point r' (ω' is the direction before scattering, and ω is the direction after scattering):

$$M[\rho',\rho](\omega',\omega,r') = L[\rho,\rho^*]^{-1} M(\omega',\omega,r')\, L[\rho',\rho^*],$$

where $M(\omega', \omega, r')$ is the Mueller matrix with normalization for the first element $\int_\Omega m_{11}(\omega',\omega,r')d\omega = 1$, $L[\rho, \rho^*]$ is the rotation matrix corresponding to exchange of reference vectors: $S[\rho^*] = L[\rho, \rho^*]\ S[\rho]$, and ρ^* is the reference vector orthogonal to the plane of scattering (ω', ω). Assume that ρ^* can be obtained by rotation of reference vector $\rho = \rho(\omega)$ at angle φ around ω (in the right-handed coordinate system). The rotation matrix has the form

$$L[\rho(\omega),\ \rho^*] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & -b & a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ a = \cos 2\varphi,\ b = \sin 2\varphi;$$

$$a = \cos^2\varphi - \sin^2\varphi = 1 - 2\sin^2\varphi,\ b = 2\cos\varphi\sin\varphi;$$

$$\rho^* = \frac{\omega' \times \omega}{\|\omega' \times \omega\|},\ \cos\varphi = <\rho, \rho^*>,\ \sin\varphi = |\rho\rho^*\omega|.$$

Here symbol $\times$ denotes the vector product, $<.,.>$ is the scalar product, and $|\,abc\,| = <a,(b\times c)>$ is the mixed product of vectors a, b, and c.

Remark. About the polarized radiation transfer equation see, for example, [1–7]. The integral form (1) of the radiation transfer equation is presented in [2] (usually reference-vectors are assumed to be orthogonal to the vertical planes, which contain the direction vectors). In papers [6, 7] the radiation transfer equation is derived as the Kolmogorov backward equation for a specific time continuous stationary Markov jump process.

Under made assumptions about the optical medium, the elements of the Mueller matrix depend only on cosine of the angle between ω' and ω:

$$M(\omega', \omega, r')d\omega = M(\mu, \psi, r')d\mu d\psi,\ \mu = <\omega', \omega>, \quad (2)$$

where μ is the cosine of the angle between directions before and after scattering, and ψ is the azimuthal scattering angle, i.e. an angle between the scattering plane and a fixed plane containing ω'. (Note, that in (2) for notation simplicity we use the same letter M for different mathematical objects.) Moreover, for crystal clouds the Mueller matrix has the form (see, for example, [8, 9])

$$M(\mu, \psi, r') = \frac{1}{2\pi}\begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{12} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & -m_{34} & m_{44} \end{bmatrix}, \quad (3)$$

where $m_{ij} = m_{ij}(\mu, r')$ and the first element m_{11} can be interpreted as the phase function in case the beam before scattering is unpolarized,

$$\int_{-1}^{1} m_{11}(\mu, r')d\mu = 1.$$

For water-drop clouds with spherical particles the Mueller matrix (3) satisfies $m_{22} = m_{11}$, $m_{44} = m_{33}$.

One of the most effective methods to simulate processes of the polarized radiation transfer in a scattering medium is the Monte Carlo method. The main idea of the Monte Carlo method is to estimate characteristics of radiation fields by computer simulation of a huge number of random photons trajectories in the medium. Several realizations of Monte Carlo approach were developed to solve the polarized radiation transfer equation (1) (see, for example, [2, 6, 7, 10–15]). Below we present a brief description of stochastic algorithms that we used for numerical experiments.

By a “photon” we shall call a quasi-monochromatic wave that will be described by the Stokes vector $S[\rho](r, \omega)$ with respect to the reference-vector ρ, where r is the vector of the photon’s coordinates in space and ω is the unit vector of its movement direction (the reference-vector ρ is always orthogonal to ω). A trajectory of a photon in scattering medium was simulated in following steps.

Step 1. Initial point $r_0 = (x_0, y_0, z_0)$, initial direction $\omega_0 = (a_0, b_0, c_0)$, $|\omega_0| = 1$, and Stokes vector $S[\rho_0](r_0, \omega_0)$ of a photon are simulated according to the density of sources; $n = 0$.

Step 2. The photon’s free-path length l is simulated according to the probability density

$$p(l) = \sigma(x_n + l\omega_n)\exp\left[-\int_0^l \sigma(x_n + t\omega_n)dt\right],\ l > 0,$$

where σ is the extinction coefficient.

Step 3. We set $n = n + 1$ and calculate the coordinates of the next collision point in the medium:

$$x_n = x_{n-1} + a_{n-1}l,\ y_n = y_{n-1} + b_{n-1}l,$$
$$z_n = z_{n-1} + c_n l,\ r_n = (x_n, y_n, z_n).$$

Step 4. The scattering of the photon is simulated: a new direction of the photon ω_n is simulated according to a scattering function $g(\omega_{n-1}, \omega_n, r_n)$ and a new value of the Stokes vector is calculated according to the formula

$$S[\rho_n](r_n, \omega_n) = q(r_n)g^{-1}(\omega_{n-1}, \omega_n, r_n)M(\omega_{n-1}, \omega_n, r_n) \times$$
$$\times L[\rho_{n-1}, \rho_n]S[\rho_{n-1}](r_{n-1}, \omega_{n-1}), \quad (4)$$

where new reference-vector ρ_n is orthogonal to the scattering plane,

$$\rho_n = \frac{\omega_{n-1} \times \omega_n}{\|\omega_{n-1} \times \omega_n\|},$$

and the scattering function g can be chosen in different ways as follows. An algorithmically simpler variant that is often used in stochastic simulation of the radiation transfer is to take the first element of the Mueller matrix [2, 10–12]:

$$g(\omega_{n-1}, \omega_n, r_n) = m_{11}(\omega_{n-1}, \omega_n, r_n). \quad (5)$$

Another possibility is to use a physically adequate scattering function for which (4) should give

$$I(r_n, \omega_n) = q(r_n) I(r_{n-1}, \omega_{n-1}),$$

where $I = S_1$ is the first element of the Stokes vector corresponding to the intensity of the photon. Such scattering function $g(\omega_{n-1}, \omega_n, r_n)$ coincides with the first element of the vector

$$M(\omega_{n-1}, \omega_n, r_n) L[\rho_{n-1}, \rho_n] S[\rho_{n-1}](r_{n-1}, \omega_{n-1}) / S_1(r_{n-1}, \omega_{n-1})$$

see [6, 7, 13–15]. The simulation algorithm in this case is more complicated in comparison with (5), but the Monte Carlo estimators are more stable because there is no artificial distortion of the photon's weight. In our algorithms we realized both methods of scattering and numerical results for the two methods were in good agreement with each other.

Step 5. Go over to the step 2.

The trajectory is terminated if the photon leaves the scattering medium or the 'intensity' of the photon (the first element of the Stokes vector) becomes negligible.

2. Numerical experiments and simulation results

The calculations were made for the plane parallel source of unpolarized radiation and visible wavelength range (in the region near 532 nm) with the refractive index of water being equal to 1.337 and refractive index of ice being equal to 1.311. The scattering and the absorption in the cloudless atmosphere as well as the absorption in the cloud were neglected. We assumed that the cloud is a homogeneous plane layer with the scattering coefficient 20 km^{-1}. Three models of water-drop clouds and tree models of ice clouds were considered. For water-drop clouds we used Mueller matrices corresponding to models C1, C2 and C3 from [16]. For simplified models K1, K2 and K3 of crystal clouds we assumed that the ice particles are randomly oriented in space and they are of the same size and shape: for model K1 the particles are oblate cylinders with diameter-to-length ratio equal to 2, for model K2 the particles are hexagonal columns with length 50 μm and edge 10 μm, and for model K3 the particles are dendrites (stellar crystals) of diameter 200 μm with 'arms' of length 100 μm and thickness 50 μm. Prof. L. Xu provided us with the Mueller matrix for model K1 [17] and Prof. D. Winker provided us with the Mueller matrices for models K2 and K3.

In Figs. 1–7 we present the results of Monte Carlo simulation of angular distributions for different radiation characteristics, models of clouds and zenith angles of the source. The circles correspond to the dome of the sky using a projection of the hemisphere on the plane: the point in the middle of the circle corresponds to the zenith and the points on the outer circumference correspond to the horizon.

Fig. 1 represents the shapes of angular distributions of intensity of radiation reflected upward and the values of integral albedo for the six cloud models with the same optical thickness equal to 1 when the source is in zenith.

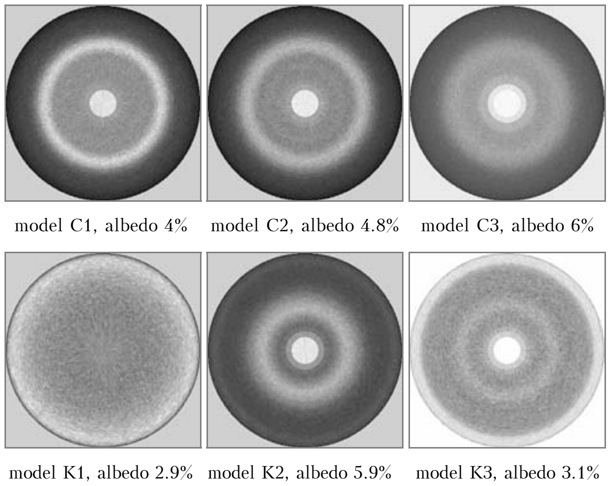


Fig. 1. Angular distributions for the upward radiation for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 1, source zenith angle = 0°

For the same input parameters Fig. 2 represents the angular distributions of the degree of polarization

$$\sqrt{Q^2 + U^2 + V^2} / I$$

for the upward radiation. The images reflect the shapes of the distributions and, in addition, the values of the corresponding maximal values of the degree of polarization are given in brackets. In the similar way Figs. 3 and 4 represent the angular distributions of intensity and degree of polarization of the upward radiation for the source zenith angle of 30°.

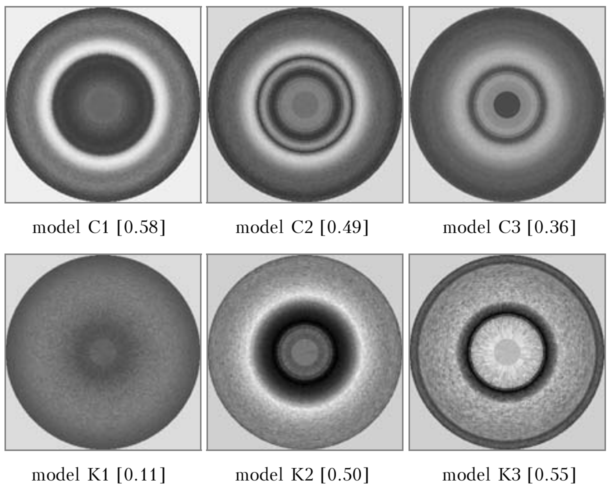
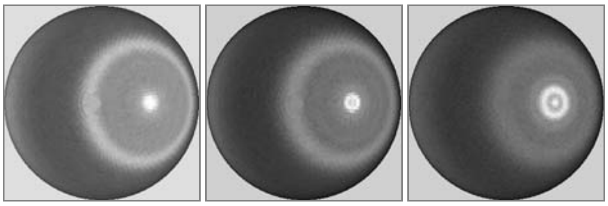
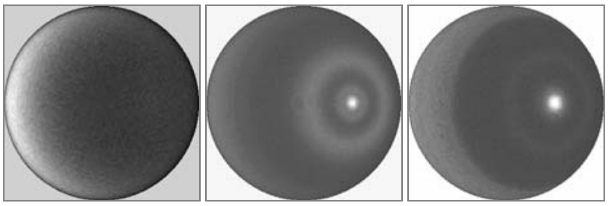


Fig. 2. Degree of polarization for the upward radiation distributed on the dome of the sky for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 1, source zenith angle = 0°. Maximum values of degree of polarization for every model are given in brackets

The bright spots in the center of the images in Fig. 1 and on the right side of the images in Fig. 3 correspond to radiation reflected by clouds in the direction backward to the source (the different size of spots in Fig. 1 and Fig. 3 is due to the grid resolution and specific projection of the hemisphere of directions on the plane).

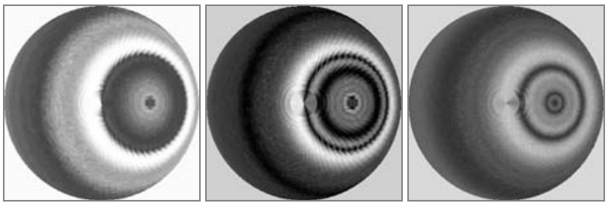


model C1, albedo 5.5% model C2, albedo 6.4% model C3, albedo 8%

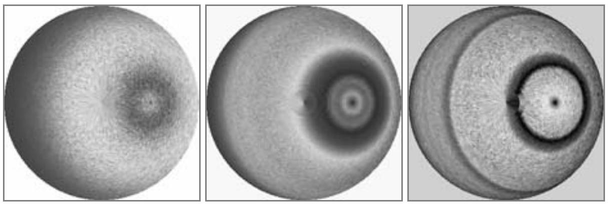


model K1, albedo 4.5% model K2, albedo 7.7% model K3, albedo 4%

Fig. 3. Angular distributions for the upward radiation for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 1, source zenith angle = 30°



model C1 [0.58] model C2 [0.48] model C3 [0.37]

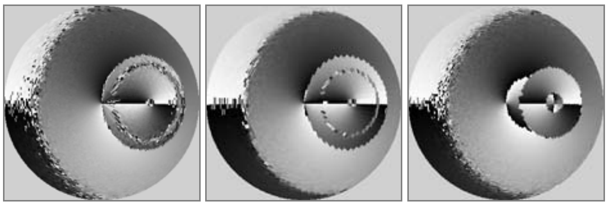


model K1 [0.16] model K2 [0.52] model K3 [0.57]

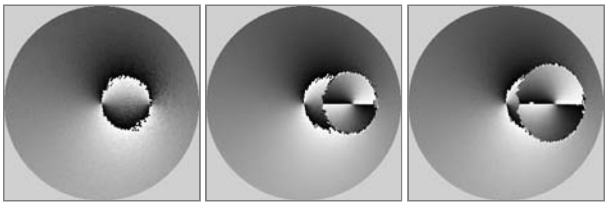
Fig. 4. Degree of polarization for the upward radiation distributed on the dome of the sky for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 1, source zenith angle = 30°. Maximum values of degree of polarization for every model are given in brackets

The distributions for directions of preferable polarization for the six cloud models are presented in Fig. 5. The direction of preferable polarization was computed as the angle in the range from −90 to +90 degrees between the main axis of the polarization ellipse and vertical plane containing the direction vector for a photon outgoing from the cloud. The sharp borders between white and black colors correspond to the directions of preferable polarization perpendicular to the vertical axes (i.e., borders of the imaginary jumps of the directions from +90 to −90 degrees).

For the optically thick clouds the distributions for the upward radiation become smoother, but they are still informative enough to distinguish between cloud models: see Fig. 6, where the angular distributions for the degree of polarization are presented for the optical thickness of clouds equal to 10.

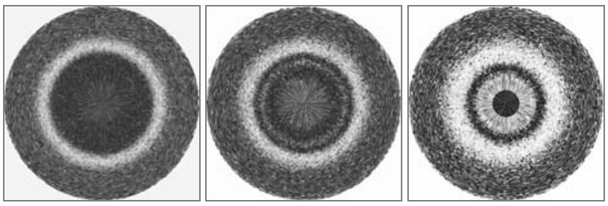


model C1 model C2 model C3

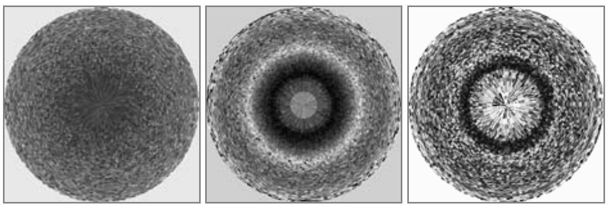


model K1 model K2 model K3

Fig. 5. Angular distributions for the preferable polarization of the upward radiation for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 1, source zenith angle = 30°. The direction of preferable polarization was computed as the angle in the range from − 90 to + 90 degrees between the main axis of the polarization ellipse and vertical plane containing the direction vector for a photon outgoing from the cloud. The sharp borders between white and black colors correspond to the directions of preferable polarization perpendicular to the vertical axes



model C1 [0.23] model C2 [0.22] model C3 [0.17]



model K1 [0.05] model K2 [0.15] model K3 [0.13]

Fig. 6. Degree of polarization for the upward radiation distributed on the dome of the sky for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 10, source zenith angle = 0°. Maximum values of degree of polarization for every model are given in brackets

The transmitted solar radiation for optically thick clouds is practically unpolarized, but for the thin clouds the degree of polarization can reach significant values, see Fig. 7. Basically, the angular distributions for the reflected solar radiation are more informative than the distributions for the transmittered radiation.

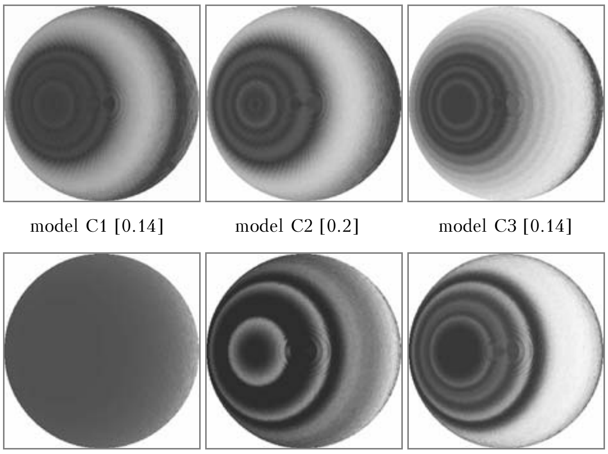


Fig. 7. Degree of polarization for the downward radiation distributed on the dome of the sky for models of water-drop (upper row) and ice (lower row) clouds, optical thickness = 1, source zenith angle = 30°. Maximum values of degree of polarization for every model are given in brackets

3. Conclusion

The results of Monte Carlo simulation clearly indicated that information about angular distributions of radiation scattered by clouds could be appreciably useful in studying the phase, the size and the shape of cloud particles. Angular distributions for different radiation characteristics (intensity, polarization degree, preferable direction of polarization) supplement each other and enable us to define the microphysical structure of clouds more precisely.

Acknowledgements

We are grateful to Prof. D.M. Winker and Prof. L. Xu who provided us with Mueller matrices for ice particles. In addition, we would like to thank A.S. Prigarin for the help in development of the auxiliary software, Dr. M. Wengenmayer and G. Czerwinski for fruitful discussions. The research was partially supported by RFBR (09-05-00963).

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