ON THERMAL DESTRUCTION OF ATMOSPHERIC ICE PARTICLES UNDER THE ACTION OF RADIATION AT λ = 10.6 μm

A.P. Prishivalko, V.A. Babenko, I.R. Katseva, and S.T. Leiko

B.I. Stepanov Institute of Physics,
National Academy of Sciences, Minsk, Belarus
Received January 30, 1998

We discuss here modeling of thermal destruction of the atmospheric ice particles under the action of high-intensity radiation at λ = 10.6 μm. *Solution of the problem is divided into the following stages: (1) choosing and justification of a geometrical model of the ice particles; (2) electrodynamics calculation of the heat release distribution inside a particle; (3) numerical solution of the heat transfer equation with the relevant initial and boundary conditions; (4) approximate solution of the elasticity equation. The results obtained for sufficiently large spherical and cylindrical ice particles are compared. The exposure time, during which a particle is destroyed, is estimated as a function of the radiation intensity. The specific, volume-averaged, energy sufficient for the destruction of particles is shown to slightly depend on the radiation intensity and particle shape while being mainly dependent on the particle radius.*

1. INTRODUCTION

The problem on nonlinear interactions of radiation with an individual particle of atmospheric aerosol is a key problem in the investigations of high-power laser beam propagation in clouds and fogs.[1] One of the most complicated task is the study of radiation interaction with crystal particles.[2,3] Since experiments on interaction of high-power radiation with ice clouds are very complicated, the mathematical simulation of some aspects of this problem based on some physically justified assumptions seems to be useful.

The aim of this study was simulation of thermal destruction (cracking) of atmospheric ice particles under the action of high-power radiation at λ = 10.6 μm. Solution of the problem involves the following stages: (1) choosing and justifying of the geometrical model of ice particles; (2) electrodynamics calculation of the optical field distribution and heat release inside particles with the allowance for all the main factors determining the distribution; (3) numerical solution of the heat transfer equation with the corresponding initial and boundary conditions, which allows one to follow up the dynamics of the temperature field inside a particle; (4) solution of the thermal elasticity equation in order to determine the conditions under which thermoelastic stresses arising inside a particle exceed the critical value for ice and lead to thermal destruction of a particle.

At present, thus complicated problem can hardly be solved in its most general formulation. So, we have made some physical assumptions, which allow an approximate solution of the problem to be obtained.

2. MODELING OF A PARTICLE

Ice particles are characterized by various shapes and size varying in a very wide range. The above problem can now be solved only for bodies of some very simple geometry. We have chosen an infinite circular cylinder as a model of ice particles. Regardless of the idealization, the model enables one to describe semi-quantitatively some aspects of thermal destruction of strongly elongated cirular and columnar ice particles, whose length significantly exceeds their cross size and the wavelength of incident radiation. For comparison, similar model calculations have been performed for spherical ice particles, following our earlier papers.[4,5]

3. DISTRIBUTION OF THE OPTICAL FIELD AND HEAT RELEASE INSIDE PARTICLES

As known,[6] the heat release (the power of a heat source Q per unit volume) at a point within a cylindrical particle described by cylindrical coordinates r, z, φ or a spherical particle with the spherical coordinates r, θ, φ is determined by the following relation:

$$Q = 4\pi\, n\, \varkappa I B / \lambda\, n_0,$$

where n and $\varkappa$ are the real and imaginary parts of the complex refractive index of ice (in our calculations, we assumed $n = 1.1013$ and $\varkappa = 0.134$ (Ref. 7)). It was also assumed that the medium around the particle did not absorb radiation and its refractive index is $n_0 = 1$. In the above expression, λ is the wavelength of incident radiation; I is the radiation intensity; $B = (E_r^* E_r + E_\varphi^* E_\varphi + E_z^* E_z)/E_0^2$ (for a spherical particle, the subscript z must be substituted by θ); E_0 is the strength of the electric field of incident radiation. The values of the electric field components E_r, E_φ, and E_z inside the cylinder have been calculated according to the exact theory of diffraction on an infinite cylinder.[8,9] However, in contrast to Ref. 9, the cylindrical functions were computed using the continued fraction method[10] and direct calculation of Bessel functions J_0 by the method proposed in Ref. 11. The components E_r, E_φ, and E_θ in the case of a sphere have been calculated by the Mie theory.[6,9]

As was noted in Ref. 2, elongated crystals in clouds may have a preferred orientation, i.e., the largest deviation from the horizontal orientation with reference to the Earth's surface does not exceed several degrees. If a high-power radiation source is on the ground, it is reasonable to suppose that cylindrical particles are illuminated along the normal ($\alpha = 0°$). To compare the results obtained for spherical and cylindrical particles and to simplify the calculations, we have assumed the incident wave to be unpolarized. The heat release has been computed for large ice particles with the cross-section radius $R = 5–70$ μm irradiated with radiation at $\lambda = 10.6$ μm. The calculated results are not presented here, as they are preliminary for the problem to be solved.

4. EQUATION OF THE HEAT TRANSFER IN A PARTICLE

The problem on particle warming up by radiation in an unbounded medium can be reduced to solving the equation of heat transfer with the corresponding boundary and initial conditions. In the case of cylindrical particles, we have

$$c_P(T)\,\rho(T)\,\frac{\partial T(r,\varphi,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_1(T)\,r\,\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda_1(T)\,\frac{\partial T}{\partial \varphi}\right) + Q(r,\varphi,R). \qquad (1)$$

For spherical particles (with no dependence on the angle φ, what is valid if the incident wave is unpolarized)

$$c_P(T)\,\rho(T)\,\frac{\partial T(r,\theta,t)}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_1(T)\,r^2\,\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\lambda_1(T)\sin\theta\,\frac{\partial T}{\partial \theta}\right) + Q(r,\theta,R), \qquad (1a)$$

where $0 \le r < R$; $0 \le \theta \le \pi$ (sphere) or $0 \le \varphi \le \pi$ (cylinder); T is the temperature; $\rho(T)$, $c_P(T)$, and $\lambda_1(T)$ are the specific density, specific heat, and heat conductivity. The following conditions must be satisfied for the cylinder:

$$-\lambda_1(\mathrm{T})\left.\frac{\partial T}{\partial t}\right|_{r=R} = a\,[T(R,\varphi,t) - T_{\mathrm{amb}}],$$

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=0} = \left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=\pi} = 0, \quad |T(0,\varphi,t)| < \infty,$$

$$T(r,\varphi,0) = T_0, \qquad (2)$$

where a is the heat exchange coefficient at the interface with the ambient gaseous medium; T_{amb} is the ambient temperature; T_0 is initial temperature of a particle (the conditions for a sphere can be obtained from the conditions (2) when replacing the coordinate φ by θ).

The algorithm for numerical solution of the systems of equations (1) and (2) is similar to that presented in Ref. 6. An absolutely stable locally one-dimensional iteration scheme is being constructed on a space-time grid. The initial problem is put into the correspondence with the difference problem. Thus obtained system of equations is then solved by the sweep method.

The thermal characteristics of the fresh-water ice and their temperature dependence have been taken from Ref. 12. Thus, within the temperature T range from 210 to 273 K the relations $\rho = 0.951 - 0.0012\,T$; $c_P = 0.0078T - 0.0094$, $\lambda_1 = 0.004685 + 4.8819T^{-1}$ (where ρ is in g/cm^3; c_P is in J/g · K; and λ_1 is in W/(cm^2 · K)) provide for accuracy of 1, 0.16, and 15%, respectively. The coefficient a can be found from the known relation[13]

$$a = \beta\,\frac{p}{\sqrt{2\pi\,MkT}}\left(c_V + \frac{1}{2}\,k\right), \qquad (3)$$

where β is the accommodation coefficient ($\beta \approx 0.7 - 1.0$); p is the gas pressure near the particle surface; M is the mass of a gas molecule; T is the gas temperature; c_V is the heat capacity at constant volume (for nitrogen and air $c_V = 5k/2$); k is the Boltzmann constant. For nitrogen, $a = 0.0209/\sqrt{T}$, in W/(cm^2·K), in the temperature range from 210 to 273 K.

5. THE HEATING MODE AND APPROXIMATIONS USED

Suppose that intensity of laser radiation is sufficiently high for quick heating of a particles. In such a case, considerable temperature differences and, consequently, considerable thermoelastic stresses should arise inside a particle. In the most general case, the problem of calculation of these stresses requires joint solution of the heat transfer equation (1) and the equilibrium equation[14] for a solid body:

$$\frac{1-\gamma}{1+\gamma}\,\text{grad div}\,\mathbf{U} - \frac{1-2\gamma}{2\,(1+\gamma)}\,\text{rot rot}\,\mathbf{U} = \alpha_T\,\nabla T, \qquad (4)$$

where **U** is the deformation vector; γ is the Poisson coefficient; α_T is the linear coefficient of heat expansion for ice. Since solving of the problem in its general form is too difficult, we restrict ourselves, at this stage, to a simplified situation. Considering this situation, it is easy to obtain the results, which are not numerically accurate but giving the general pattern with clear qualitative regularities.

The exact solutions to the problem of thermoelastic stresses in cylindrical or spherical bodies with axially or centrally symmetric temperature distribution are considered in detail in the theory of thermoelasticity.[15–18] It is shown (see, for instance, Ref. 18, Chapter 7) that the maximum stresses occur under heating as tangential components σ_t on the body surface. Let us suppose that the symmetric profile of the temperature distribution has the form of the power dependence on the relative radial coordinate $x = r/R : T(x) = T(0) + x^\nu\,\Delta T$, where ν is the exponent characterizing the steepness of the profile; $\Delta T = T(1) - T(0)$. Then the maximum values of σ_t are expressed through the mechanical properties of the substance and the parameter ν:

$$\sigma_t^{\text{cyl}} = -\frac{\nu}{\nu+2}\,\Delta T\,\frac{2G\,(1+\gamma)}{1-\gamma}\,\alpha_T,$$

$$\sigma_t^{\text{sphere}} = -\frac{\nu}{\nu+3}\,\Delta T\,\frac{2G\,(1+\gamma)}{1-\gamma}\,\alpha_T, \qquad (5)$$

where G is the displacement module.

Our numerical calculations of the temperature fields inside cylindrical and spherical ice particles with $R > 15–20$ μm have revealed the following: if particles are irradiated by high-intensity laser radiation with $\lambda = 10.6$ μm, the temperature distribution profile in the irradiated hemisphere along the radius coinciding with the incident beam direction really has the form of exponential function with different ν and ΔT. The values of ν and ΔT depend on the radius of a particle, the radiation intensity, and the maximum temperature; and these values vary during heating. The maximum temperature $T_{\max}$ (on the particle surface) and the maximum temperature differences ΔT between the center and the surface take place just on this radius. Since ice is a brittle substance, it is natural to suppose that the point with $T_{\max}$ is just the point where the cracking limit is achieved. There comes a time when the absolute value of σ_t achieves or exceeds the critical value of the ultimate strength σ_{cr} of the substance.

Although the temperature distribution is evidently not symmetric, we nevertheless apply the solution (5) obtained for an ideally symmetric situation to the radius. This is the main assumption of the paper. Then we can find the critical temperature difference, at which the destruction occurs:

$$\Delta T_{\text{cr}}^{\text{cyl}} \ge \left|\frac{\sigma_{\text{cr}}\,(1-\gamma)}{2G\,\alpha_T\,(1+\gamma)}\,\frac{(\nu+2)}{\nu}\right|,$$

$$\Delta T_{\text{cr}}^{\text{sphere}} \ge \left|\frac{\sigma_{\text{cr}}\,(1-\gamma)}{2G\,(1+\gamma)\,\alpha_T}\,\frac{(\nu+3)}{\nu}\right|. \qquad (6)$$

Thus, the problem is reduced to solution of inequalities (6) with allowance for the following factors: the mechanical properties of ice (γ, G, α_T, σ_{cr}) varying with temperature and, consequently, in time; the degree of steepness of the temperature profile ν depending on the particle size, the radiation intensity, and the maximum temperature reached by a given time. The value of ν is determined from solution of the problem on the dynamics of the temperature field inside a particle, i.e., from solution of the heat transfer equation with the corresponding initial and boundary conditions.

6. VALUES OF THE MECHANICAL CONSTANTS

Since a heated particle undergoes a stress in compression, we have taken the generalized data[12,19] on the most probable values of σ_{cr}^{comp} (varying from 1.6 to 4.0 MPa as the temperature varies from 273 to 248°C) as the limit stresses. This limit remains practically constant at lower temperatures.

As follows from numerous experimental data, the Poisson coefficients for ice vary not very widely depending on concrete samples and measurement conditions. Reference 12 recommends to use the coefficient $\gamma \approx 0.36$ for freshwater ice. The tabulated data[12] on the temperature dependence of the linear expansion coefficient for ice can be approximated by the expression $\alpha_T\,[\text{K}^{-1}] = 1.8859{\cdot}10^{-8}\,T^{1.4181}$ accurate to 4%. The displacement module G varies approximately from 3.07 to 3.36 hPa according to the data of different authors obtained for different samples.[12] If we take the mean value $G = 3.335$ hPa, then $M(T) = 1.4973{\cdot}10^4\,T^{-1.4181}$ for the temperature from 210 to 250 K and $M(T) = 5.8991 \cdot 10^3 - 69.737\,T + 0.27523\,T^2 - 3.623{\cdot}10^{-4}\,T^3$ for the temperature from 250 to 273 K; where $M(T)$ is the first, i.e., “mechanicalB factor in the right-hand side of Eq. (6). This more complicated form of the relation for elevated temperatures is caused by the fact that the value of the critical stress σ_{cr} in compression is already not constant within this interval, but temperature-dependent in a rather complicated way (Ref. 12, Table 4.9).

7. COMPUTED RESULTS

Before exposure to high-power laser radiation, a particle was supposed to be in the thermal equilibrium with the ambient gaseous medium at the temperature T_{amb} = 210 K. Computations were performed for the intensity of laser radiation $I = 2{\cdot}10^3 - 10^8$ W/cm^2. Under these conditions, a particles of any size break down far before its temperature achieves the ice melting point. When analyzing the results, most attention was paid to comparison of spherical and cylindrical particles.

The similar heat release inside large cylindrical and spherical particles leads to very similar character of their heating under exposure to laser radiation with λ = 10.6 μm. Some examples are shown in Fig. 1. The behavior of the curves in Fig. 1 confirms that the temperature profile at the main radius can really be described by an exponential function. Small deviations are observed only for not very large particles at relatively low intensities; they are most marked (within 5%) near the irradiated surface. Thus, curves like those presented in Fig. 1, especially for particles with $R > 20$ μm, confirm the validity of the main assumption.

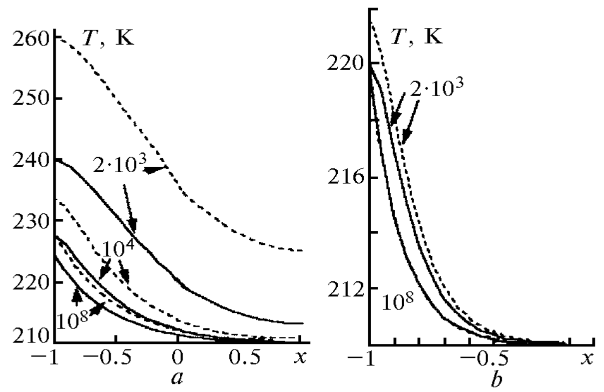

FIG. 1. Temperature distribution T, in K, *along the main diameter of ice cylindrical (solid curves) and spherical (dashed curves) particles by the time of their destruction: R* = 15 μm *(a) and* 50 μm *(b). The figures at the curves show the radiation intensity I, in* W/cm^2. *The direction of radiation propagation is from left to right.*

It should be noted that, depending on particular conditions, the critical temperature difference ΔT_{cr} can vary within a wide range (in contrast to constant ΔT_{cr} = 10 K accepted in Refs. 4 and 5).

As one could suppose based on Eq. (6), destruction of spherical particles requires somewhat higher critical temperature difference ΔT_{cr} between the irradiated surface and the center than in the case with cylindrical particles. The data on the time of destruction for spherical and cylindrical particles under the same conditions (Fig. 2) also confirm the conclusion that requirements for destruction of cylindrical particles are not so rigid as compared to spherical particles. It follows from the calculated results that the time t_{destr} from the beginning of exposure to radiation with the intensity $I = 10^4 - 10^8$ W/cm^2 to the destruction of particles of both types can be described by the relation $\log(t_{destr}) = a + b \log I$. For cylindrical particles of radii R = 15–70 μm, the coefficients a and b take values from −1.59 to −1.95 and from −1.032 to −1.017, respectively. For spherical particles, the corresponding coefficients vary from −1.44 to −1.94 and from −1.032 to −1.013. The difference between the values of t_{destr} for a sphere and a cylinder decreases with increasing R.

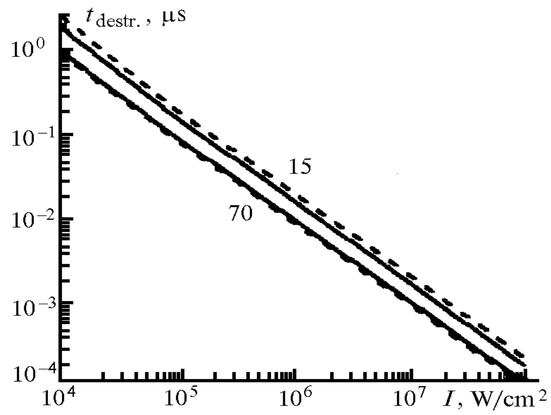

FIG. 2. Time t_{destr}, *in* μs, *from the beginning of irradiation to destruction of ice cylindrical (solid curves) and spherical (dashed curves) particles versus the radiation intensity I, in* W/cm^2. *The figures at the curves indicate the particle radius R, in* μm.

Now let us turn to analysis of the main characteristic of the thermal destruction of ice particles under the exposure to high-power laser radiation, namely, to the energy absorbed by particles from the beginning of irradiation to the time of achieving the destruction conditions. It is determined by the relation $E_{abs} = I t_{destr} C_{abs}$, where C_{abs} is the absorption cross section (for an infinite cylinder, the cross section is calculated per unit length). For a correct comparison of the results obtained for particles of different shape, C_{abs} should be normalized to the particle volume: $q_{abs} = C_{abs}/V$ (for an infinite cylinder, by the volume is meant the volume of its part with a unit length). The calculated results for the value of q_{abs} are presented in Fig. 3. The shape of the curves in Fig. 3 permits the following conclusions to be drawn:

a) as a rough estimate, we can assume that the value $q_{abs} \approx 10$ μJ/μm^3 is sufficient to break down large ($R > 15$ μm) ice particles in the considered range of radiation intensity;

b) spherical particles require higher q_{abs} values, as compared to the cylindrical ones, as could be expected according to the theory of thermoelasticity (see, for instance, Refs. 14–18). However, the difference is not large (by 1.3–1.7 times), and it decreases with increasing R;

c) the dependence of q_{abs} on I is very weak at the radiation intensity higher than $2 \cdot 10^4$ W/cm^2.

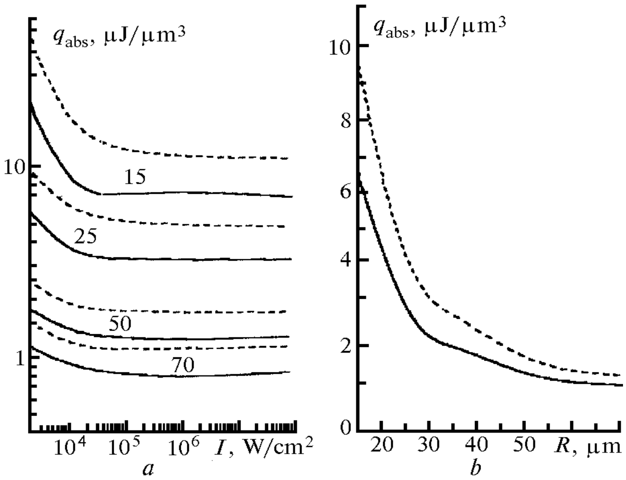

FIG. 3. Volume-averaged energy q_{abs}, in μJ/μm^3, absorbed by ice cylindrical (solid curves) and spherical (dashed curves) particles for the time from the beginning of irradiation to destruction of a particle versus the radiation intensity I, in W/cm^2, (a) and the particle radius R, in μm, at $I = 10^8$ W/cm^2 (b). The figures at the curves (a) indicate the particle radius R, in μm.

Since a sphere and an infinite circular cylinder are very different and, in some sense, extreme geometrical models of ice particles, the close results obtained for these models allow us to suppose, at least at the qualitative level, that ice particles of other (intermediate) shapes break down under exposure to high-power laser radiation following approximately the same regularities.

REFERENCES

1. A.A. Zemlyanov, Atmos. Oceanic Opt. **8**, Nos. 1–2, 44–57 (1995).
2. O.A. Volkovitskii, L.N. Pavlova, and O.G. Petrushin, *Optical Properties of Crystal Clouds* (Gidrometeoizdat, Leningrad, 1984), 199 pp.
3. P.N. Svirkunov and L.P. Semenov, Trudy Ins. Exp. Meteorol., issue 11(54), 3–18 (1975).
4. A.P. Prishivalko, L.P. Semenov, L.G. Astaf'eva, and S.T. Leiko, Inzh. Fiz. Zh. **54**, No. 1, 103–108 (1988).
5. L.G. Astaf'eva, A.P. Prishivalko, L.P. Semenov, and S.T. Leiko, Opt. Atm. **1**, No. 2, 63–67 (1988).
6. A.P. Prishivalko, *Optical and Thermal Fields inside Light Scattering Particles* (Nauka i Tekhnika, Minsk, 1983), 190 pp.
7. S.G. Warren, Appl. Optics **23**, No. 8, 1206–1225 (1984).
8. V.A. Babenko, G.P. Ledneva, and I.R. Katseva, Vesci Akad. Nauk BSSR, Ser. Fiz. Math. No. 2, 70–73 (1987).
9. P.W. Barber and S.C. Hill, *Light Scattering by Particles: Computational Methods* (World Scientific, Singapore, 1990), 262 pp.
10. W.J. Lentz, Appl. Optics **15**, No. 3, 668–671 (1976).
11. J.P. Mason, Comp. Phys. Comm. **30**, No. 1, 1–11 (1983).
12. V.V. Bogorodskii and V.P. Gavrilo, *Ice. Physical Properties. Modern Methods of Glaciology* (Gidrometeoizdat, Leningrad, 1980), 384 pp.
13. E.M. Lifshits and L.P. Pitaevskii, *Physical Kinetics* (Nauka, Moscow, 1979), 287 pp.
14. L.D. Landau and E.M. Lifshits, *Theory of Elasticity* (Nauka, Moscow, 1987), 246 pp.
15. S.P. Timoshenko and J.N. Goodier, *Theory of Elasticity* (McGraw-Hill, New York, 1970).
16. B.A. Boley and J.H. Weiner, *Theory of Thermal Stresses* (John Wiley and Sons, New York–London, 1960).
17. L.D. Kovalenko, *Introduction to Thermoelasticity* (Naukova Dumka, Kiev, 1965), 204 pp.
18. A.A. Pomerantsev, *Thermal Stresses in Rotationally Symmetric Bodies of Arbitrary Shape* (State University, Moscow, 1967), 104 pp.
19. I.G. Petrov, Trudy AANII **331**, 4–41 (1976).