

COMPENSATION OF FAR-FIELD THERMAL DEFOCUSING OF LASER RADIATION

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Thermal defocusing in the cases of a stationary and moving nonlinear medium is studied. The characteristics of the radiation are calculated in the far zone both in the aberration-free approximation and taking aberrations into account. The effectiveness of phase compensation of nonlinear distortions of radiation is investigated for thin and distributed thermal lenses. The optimal values of the control parameters are found. The theoretical data are compared with known experimental results.

When high-power laser radiation propagates through the atmosphere the amplitude and phase characteristics of the radiation are distorted in different ways. Among nonlinear optical effects which destroy the initial spatial coherence of the laser beam the thermal self-action has the lowest energy threshold for continuous and quasicontinuous radiation.^{1,2} The thermal effects play an important role, in particular, in the propagation of laser radiation in the atmosphere.³⁻⁵ The role of other mechanisms of nonlinearity of the index of refraction of air, for example, the Kerr effect and electrostriction, is negligible for optical pulses of width greater than 10^{-8} s (Refs. 6 and 7).

Thermal defocusing distorts the amplitude profile of the beam and broadens its directional pattern.⁸⁻¹⁰ Degradation of beam quality prevents the transfer of concentrated light energy over large distances. This leads to the problem of developing different methods for suppressing nonlinear distortions of radiation propagating along the path. Compensation is usually performed with the help of optimal control of the amplitude and phase profile of the light beam. This is done with the help of amplitude-phase¹¹⁻¹² and phase¹³⁻¹⁵ correction algorithms. Both adaptive^{1,9,16} and program^{2,15} control of the light-beam parameters can be used.

In many works the propagation of beams on paths whose length is shorter than or of the order of the diffraction length was investigated in the theoretical and experimental study of the possibility of compensation of thermal nonlinearity.^{10,17,18} At the same time, in many problems of practical importance the laser beam strikes a receiving system that is located behind a layer of the nonlinear medium. If, in addition, the receiver is located sufficiently far away from the nonlinear layer, then one must consider the distortion of the directional pattern of the laser radiation or the far-field distribution of the intensity.

Problems of this type can arise in space communication and ranging. The study of the problem of compensation of nonlinear distortions in the far

field, in particular, the angular divergence, began only recently.

The effects of thermal defocusing of a laser beam in thin layers of an absorbing medium were investigated in early works.¹⁹⁻²¹ The continuous-wave and pulsed regimes of defocusing in moving and stationary media, with and without taking into account the nonlinear aberrations, were studied. For this reason, we now have sufficient information about the character of thermal defocusing in bounded layers of a nonlinear medium, and this makes it easier to select methods for suppressing (completely or partially) the nonlinear broadening of the directional pattern of powerful light beams.

There are still only a few theoretical and experimental works in which the far-field characteristics of radiation with a nonuniform propagation path and the behavior of the beam behind the layer of nonlinear medium were studied have been published on this important subject.²²⁻²⁹

In this short review we study the propagation of powerful optical radiation through a stationary and moving defocusing medium with thermal nonlinearity. The far-field characteristics of the radiation are studied both in the aberration-free approximation and taking aberrations into account. The effectiveness of compensating nonlinear distortions with the help of wavefront control at the point of entry into the nonlinear medium is analyzed. The results of theoretical calculations are compared with known experimental data.^{26,27}

1. THERMAL DEFOCUSING. BASIC APPROXIMATIONS AND STARTING EQUATIONS

In nonlinear quasioptics the thermal defocusing of a wave beam, which has a narrow directional pattern and propagates along the z axis, is described in the stationary case by a system of two equations: a parabolic equation for the complex slowly varying amplitude of the beam

$$2ik \frac{\partial E}{\partial z} = \Delta_{\perp} E + k^2 \frac{2}{n_0} \frac{dn}{dT} TE \tag{1}$$

and the heat-conduction equation

$$\rho C_p v \frac{\partial T}{\partial x} - \kappa \Delta_{\perp} T = \alpha \frac{cn_0}{8\pi} E^2, \tag{2}$$

where T is the temperature of the medium, $n = n_0 + dn/dT$ is the index of refraction of the medium, C_p and ρ are the specific heat capacity and density of the medium, v is the velocity of the uniform lateral wind blowing along the x axis, α is the absorption coefficient, and κ is the thermal conductivity.

It is well known that the nonlinear defocusing of a light beam is accompanied by aberrations. However, when the intensities and gradients of the temperatures are sufficiently small, the thermal defocusing can be assumed to be aberration-free. In this approximation the complex amplitude of the Gaussian beam can be represented as follows:

$$E = \frac{E_0}{f(z)} \exp \left[-\frac{r^2}{a_0^2 f^2} - \frac{ikr^2}{2f} \frac{df}{dz} + i\varphi(z) \right]. \tag{3}$$

Here a_0 is the initial radius of the Gaussian beam, f is the dimensionless width of the beam, and E_0 is the amplitude on the axis.

In the case of stationary medium we set in Eq. (2) $v = 1$. Then, using Eqs. (1)–(3), we obtain the starting equation for the width f in dimensionless variables:²³

$$\frac{d^2 f}{d\xi^2} = \frac{\alpha l_d^2}{R_{nl}} \frac{\exp[-\alpha l_d \xi]}{f} + \frac{1}{f^3}, \tag{4}$$

where $\zeta = z/l_d$ is the normalized longitudinal coordinate, $l_d = k \cdot a_0^2$ is the diffraction length, and

$$R_{nl} = 2n_0 \kappa \left| \frac{dn}{dT} \right| \cdot I_0$$

is the nonlinearity parameter.

For a moving medium we shall confine the analysis to velocities of the uniform side wind for which the Peclet number is large: $\gamma = a_0 v / 4\chi > 1$ (χ is the thermal diffusivity of the medium). In this case the temperature field in the beam channel is determined by forced convection, and the effect of heat conduction, which is connected with the second term on the left side of Eq. (2), can be ignored.

The carrying of the heat by the wind out of the region of the beam changes the properties of the thermal lens. The beam spreads out transverse to the wind, so that the size of the beam is now characterized by two quantities: the longitudinal (with respect to the wind) dimensionless width f_2 and the transverse dimensionless f_1 . Correspondingly, the two variables f_1 and f_2 will now appear in Eq. (3) instead of the dimensionless width f . From Eqs. (1)–(3) we obtain, like in the preceding case, equations for f_1 and f_2 in dimensionless variables:²³

$$\frac{d^2 f_1}{d\xi^2} = \frac{l_d^2}{2R_{nl,v}} \frac{\exp[-\alpha l_d \xi]}{f_1^2} + \frac{1}{f_1^3}; \tag{5}$$

$$\frac{d^2 f_2}{d\xi^2} = \frac{1}{f_2^3}, \tag{6}$$

where $R_{nl,v}^2 = \kappa a_0^3 n_0 \sqrt{\pi} v / 2 \left| \frac{dn}{dT} \right| \alpha P_0 \chi \cdot (1 - e^{-1})$ is the nonlinearity parameter, which characterizes the magnitude of the nonlinear refraction taking aberrations into account, and P_0 is the input power.

We shall study the far-field characteristics of radiation after it is passed through a layer of the defocusing medium with a thermal nonlinearity. One of the most important characteristics is the angular divergence of the beam. The total angular divergence is calculated as follows:

$$\vartheta^2 = \left[\frac{df}{d\xi} \right]^2 \bigg|_{\xi=\frac{l_m}{a}} + \frac{1}{f^2} \bigg|_{\xi=\frac{l_m}{a}} \tag{7}$$

In Eq. (7) $\vartheta_0 = \theta / \theta_d$ is the total angular divergence, normalized to the diffraction divergence θ_d ; l_m is the length of the nonlinear medium; and, f is the dimensionless beam width, appearing in Eqs. (4)–(6).

The specific form of the boundary conditions for Eqs. (4)–(6) depends on the method chosen to compensate for the nonlinear distortions. Once these equations have been solved with definite boundary conditions, the total angular divergence of the radiation in the case of a stationary medium as well as the angular divergence, longitudinal and transverse to the wind, for a moving medium can be calculated using Eq. (7).

2. THIN AND DISTRIBUTED NONLINEAR LENSES

The nonuniformity of the thermal nonlinearity consists of the fact that the absorption coefficient depends on the longitudinal coordinate z . The dependence can be continuous or stepped. In any case, we will be dealing with a bounded layer of a nonlinear medium of thickness l_m .

If the layer is sufficiently thin, so that the amplitude profile does not change appreciably, then the nonlinear lens is thin. Let a focused Gaussian beam with a radius of curvature of the wavefront R be incident on the nonlinear medium:

$$E(r) = E_0 \exp \left[-\left[\frac{r}{a_0} \right]^2 - \frac{ikr^2}{2R} \right]. \tag{8}$$

Then the change in the index of refraction accompanying the propagation of the beam through the nonlinear layer results in the appearance of an additional phase factor in the expression (8):

$$\exp \left[i\varphi \exp \left[-2 \left(\frac{r}{\alpha_0} \right)^2 \right] \right], \tag{9}$$

where $\varphi = kl_m n_2 |E|^2$ is the nonlinear increment of the phase on the axis of the beam.

It can be shown³⁰ that the distribution of the field at the exit from the thin nonlinear layer can be represented as a collection of Gaussian beams with the same wavefront curvature and different widths:

$$E(r) = E_0 \sum_{m=0}^{\infty} \frac{(i\varphi)^m}{m!} \exp \left[-\frac{1 + 2m}{\alpha_0^2} r^2 - \frac{ikr^2}{2R} \right]. \tag{10}$$

The simplest method for compensating nonlinear distortions in the case of a thin thermal lens is phase correction. This is done with the help of a controllable lens or an adaptive mirror only at one point of the path (at $z = 0$), and for this reason it is most effective in the case when the diffraction and nonlinear phase increments of the beam can be assumed to be additive.

We shall study the results of such compensation in greater detail. The effectiveness of different methods of phase correction for a medium with wind-type nonlinearity was analyzed in Refs. 28 and 29. The purpose of the compensation is to minimize the far field angular divergence of the radiation behind the thin layer of nonlinear medium.

Numerical calculations performed in the aberration-free approximation made it possible to compare two methods of correction. In the first method a correcting predistortion is introduced into the phase of the radiation:

$$\varphi_c(x, y) = -\frac{1}{2} k \epsilon(x, y, z=0) \int_0^z \exp \left[-\frac{z'}{z_s} \right] dz', \tag{11}$$

where z_s is the effective thickness of the nonlinear layer, normalized to the refraction length. The second method is based on the fact that the intensity at the point of reception of the radiation will be maximum if the initial front of the beam is phase-reversed relative to the front of the wave emitted by the receiver in the far zone and arriving at the radiator.

The process of phase reversal is usually adaptive and iterative, since the correction of the phase front lags behind the changes in the dielectric constant in the layer of the medium. When phase correction is performed by this method the distribution of the phase front in the plane of the source of the adaptive scheme is introduced as a phase predistortion φ_c .

The computational results are presented in Fig. 1. One can see that predistortion makes it possible to achieve close to minimum angular divergence only for an effective thickness $z_s \leq 0.7$; already for $z_s \sim 1$ the angular divergence of the focused beam exceeds the angular divergence of the collimated beam. When only adaptive control (no predistortion) is used it is also possible to achieve close to minimum angular diver-

gence only in a limited region of z_s . The second method of correction proposed in this paper makes it possible to increase the effectiveness of energy transfer for $z_s > 1$ (up to $z_s \sim 2$) too. In this case angular divergence exceeding the minimum by not more than 0.5–3% for $z_s \sim 1.5$ and by not more than 5% for $z_s \sim 2$ can be obtained.

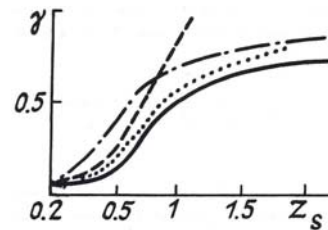


FIG. 1. The angular divergence of the beam γ versus the effective thickness of the nonlinear layer z_s for an optimally focused beam (solid curve) and a collimated beam (dot-dashed curve) as well as for beams with predistortion (dashed curve) and with phase correction and adaptive control (dotted curve).

It is obvious that even for a thin thermal lens it is possible to achieve only partial compensation of nonlinear distortions and to increase somewhat the peak intensity. The effectiveness of compensation depends on how accurately all distortions, including aberrations of high order, are compensated by the controlled element. The conditions of energy transfer can be improved by controlling the phase of the beam as a continuous function of the transverse coordinates.

An even more difficult problem is to study the possibilities of compensating the nonlinear distortions in the case of a distributed thermal lens. Here the effectiveness of compensation is restricted by the propagation of the radiation within the nonlinear layer, since the diffraction and nonlinear phase increments in this case are not additive. This problem is solved with the help of numerical modeling using the aberration-free approximation and systematically taking into account the higher-order aberrations.^{1–15}

To a first approximation the calculation for the distributed thermal lens can be performed with the help of the method of perturbations. If within a thin nonlinear medium the dimensionless width of the beam remains practically constant $f \approx 1$, then for a thick nonlinear lens a correction β ($\beta \ll 1$) to the dimensionless width $f = 1 + \beta$ must be taken into account.

We shall study a stationary medium with thermal nonlinearity. We shall solve in this approximation Eq. (4) with the following boundary conditions:

$$f \Big|_{\xi=0} = 1, \quad \frac{df}{d\xi} \Big|_{\xi=0} = -\vartheta_0, \tag{12}$$

where $\vartheta_0 = \theta_0 / \theta_d$ is the normalized initial divergence of the radiation, i.e., at the point of entry we have a focused beam. The solution has the form²⁶

$$f(\xi) = 1 + \phi_0 \xi + \frac{\xi^2}{2} + \frac{l_d}{R_{nl}} \left[\xi + \frac{1 - \exp(-\alpha l_d \xi)}{\alpha l_d} \right]. \quad (13)$$

Using Eq. (7), we obtain an expression for the nonlinear far-field angular divergence behind the layer of nonlinear medium:

$$\begin{aligned} \phi^2 = & 1 + \phi_0^2 + \frac{2l_d}{R_{nl}} \left[\frac{1 - \exp(-\alpha l_d \xi)}{\alpha l_d} - \xi \exp(-\alpha l_d \xi) \right] + \\ & + \frac{2l_d}{R_{nl}} \phi_0 \left[1 - \exp(-\alpha l_d \xi) \right] + \frac{l_d^2}{R_{nl}^2} \left[1 - \exp(-\alpha l_d \xi) \right]^2. \end{aligned} \quad (14)$$

By finding the minimum of this quantity we calculate the minimum angular divergence with optimal focusing:

$$\begin{aligned} \theta_{min} = & \left\{ \left[\frac{\lambda}{2\pi\alpha_0} \right]^2 + \frac{P_0 \left| \frac{dn}{dT} \right| \lambda^2 l_m}{4\pi^3 \alpha_0^4 \kappa} \left[\frac{1 - \exp(-\alpha l_m)}{\alpha l_m} - \right. \right. \\ & \left. \left. - \exp(-\alpha l_m) \right] \right\}^{1/2}, \end{aligned} \quad (15)$$

where λ is the wavelength of the radiation.

The numerical estimates made using Eq. (15) agree well with experiment.²⁶

3. NUMERICAL AND LABORATORY EXPERIMENTS ON COMPENSATION OF THERMAL DEFOCUSING

a) Aberration-free approximation. The next step in analyzing the effect of a distributed thermal lens on the propagation of radiation and the effectiveness of compensation of distortions is numerical modeling. The values of the main parameters of the beam and the medium in the numerical experiments corresponded to the laboratory experiments,^{26,27} with whose results the numerical results were then compared. In the experiments the compensation was performed by controlling the wavefront of the beam at the point of entry into the nonlinear medium. The radius of curvature of the wavefront was changed with the help of focusing spherical lenses of different optical power.

The overall layout of the experiments is shown in Fig. 2. The radiation of the Ar⁺ laser ($\lambda = 488$ nm, TEM₀₀ mode), having a power of up to 2 W, was directed through a telescope into a vertically oriented cell holding alcohol ($n_0 = 1.36$, $|dn/dT| = 4.2 \cdot 10^{-4}$ deg⁻¹, $\alpha = 0.012$ cm⁻¹, $\kappa = 1.5 \cdot 10^{-3}$ W/cm-deg, and $\chi = 7.94 \cdot 10^{-4}$ cm²/s). This usually made it possible to vary the initial dimensions of the beam from 0.4 to 2.8 mm. A uniform lateral wind was simulated by rotating the cell with a constant angular velocity. The wind velocity was varied from $3.8 \cdot 10^{-2}$ up to $3.8 \cdot 10^{-1}$ cm/s by changing the distance from the axis

of the beam up to the axis of rotation of the cell. The longest dimension of the cell l_m was equal to 75 cm. The divergence of the radiation propagating through the nonlinear medium was measured in the far zone.

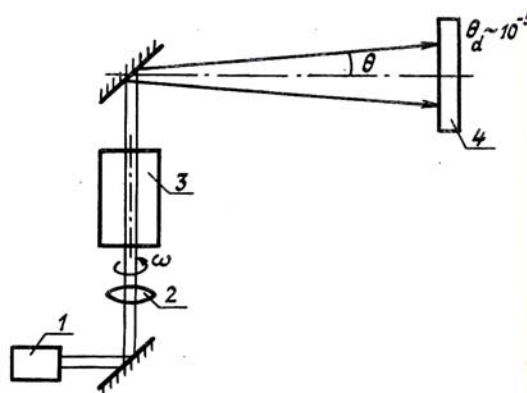


FIG. 2. General scheme of the experiments: 1) Ar⁺ laser, 2) spherical focusing lens, 3) cell, 4) screen.

In the corresponding numerical experiments the angular divergence was calculated using the formula (7) after Eqs. (4)–(6) were solved numerically for the dimensionless beam width with the boundary conditions (12). The dependences of the total angular divergence on the input radiation power, the optical power of the lens, and the wind velocity were calculated. These dependences made it possible to determine the range of values of the beam parameters in which phase compensation of the action of not only a thin but also a distributed thermal lens is possible.

Let us examine the results obtained. Figure 3a shows the dependence of the angular divergence of the radiation θ on the power P_0 with different dioptric power of the lens D^+ and fixed value of the initial radius of the beam for a stationary medium ($\alpha_0 = 0.11$ cm). Figure 3b shows the analogous dependences of the angular divergence transverse to the wind on the initial radiation power for fixed values of the beam radius and wind velocity for a moving medium ($\alpha_0 = 0.084$ cm and $v = 3.8 \cdot 10^{-1}$ cm/s).

These dependences (Figs. 3a and b) are identical: for certain values of the initial focusing and power minimum angular divergence is achieved. The best compensation of the intrinsic defocusing of the beam (dashed curve) corresponds to the envelope of a family of curves (dot-dashed curve). The point C (the point of the minimum of the curve with $D^+ = 1.5$ m⁻¹ and $P_0 = 12$ mW for Fig. 3a; $D^+ = 1$ m⁻¹ and $P_0 = 50$ mW for Fig. 3b) separates the envelope into two different regions. In the first region, situated to the left of the point C, the optimal optical power depends on the input power. Experiment and calculations show that here the beam neck is always located outside the cell, and the dimensionless beam width $f(z) \leq 1$. Such a thermal lens can be regarded as thin. It is obvious that in this region the phase compensation of the angular divergence is effective, and the degree of compensation $\eta = (\theta^2 - \theta_{min}^2) / (\theta^2 - \theta_d^2)$ is close to unity.

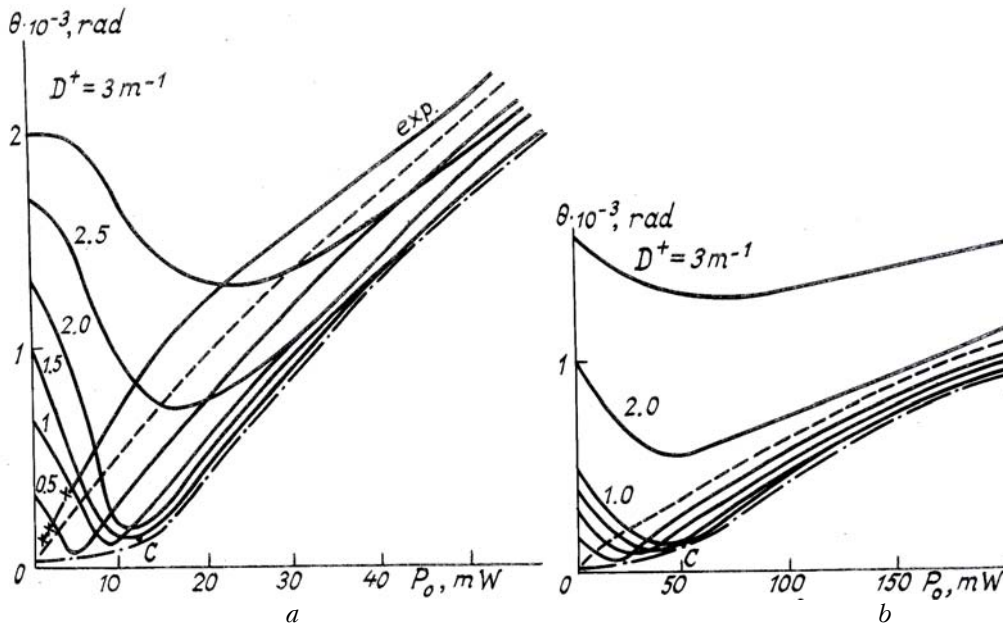


FIG. 3.

In the second region the minimum divergence (or optimal compensation) is achieved for one and the same value of the initial focusing, irrespective of the power. The thermal lens in this region is distributed, so that the effect of the entire lens can be compensated by the phase method. Only phase compensation is possible in the front part of the thermal lens, in front of the neck.

Such dependences were obtained for different values of the initial radius of the beam and wind

velocity. The qualitative behavior of the curves is the same in all cases. The calculations of the angular divergence along the wind for a moving medium show that at some wind velocity, for each specific beam radius, optimization with the help of initial focusing becomes undesirable, and it is better to employ a collimated beam. We also note that the deflection of the beam toward the wind can be compensated by additionally tilting the beam.

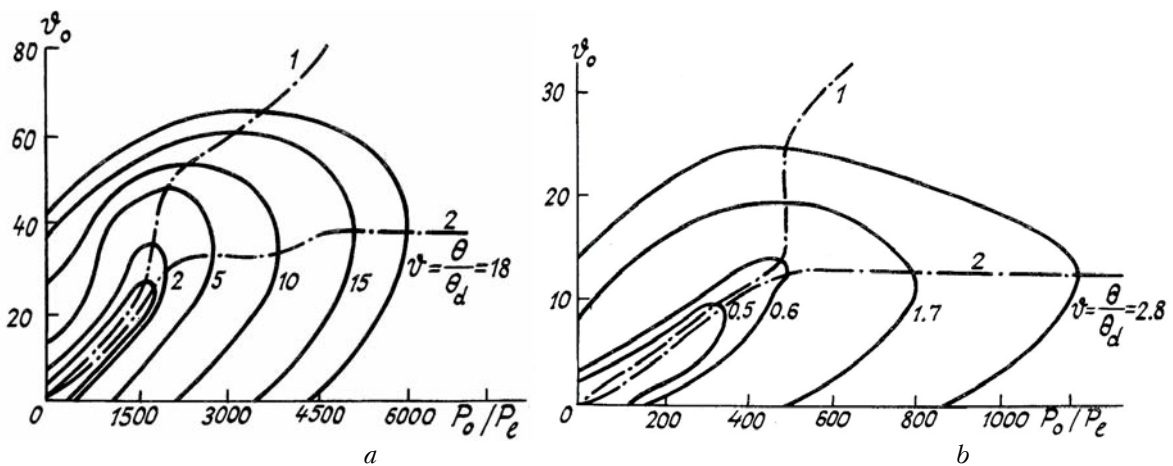


FIG. 4. Contour lines: a) total angular divergence for a stationary medium ($a_0 = 0.11$ cm, $\theta_0 = 7.36 \cdot 10^{-5}$ rad, and $P_1 = 2n_0\kappa\pi a_0^2 / \alpha l_d^2 \left| \frac{dn}{dT} \right|$); b) angular divergence transverse to the wind for a moving medium ($a_0 = 0.084$ cm, $v = 3.8 \cdot 10^{-1}$ cm/s, and $P_1 = \kappa n_0 \sqrt{\pi} a_0^3 v / 2l_d^2 \left| \frac{dn}{dT} \right| \alpha \chi \cdot (1 - e^{-1})$).

The numerical experiments made it possible to construct lines of constant total angular divergence θ in the coordinates $(P_0/P_1, \theta_0)$ for stationary (Fig. 4a) and moving (Fig. 4b) media. These isolines serve as boundaries of regions in which the

total angular divergence does not exceed a definite level with respect to diffraction. For the optimal choice of beam parameters (initial focusing and power) minimum angular divergence is achieved in each region. The curves 1 and 2 correspond to such an

optimal choice. The first curve is employed for selecting the optimal power with a fixed initial focusing and the second curve is used in the opposite case. The region where the curves 1 and 2 are close to one another corresponds to the thin-lens approximation. The distributed nonlinear lens corresponds to the region where the curves diverge. One can see that here the compensation of the thermal divergence is no longer optimal.

All the theoretical data obtained were compared with the results of laboratory experiments.^{26,27} Comparison shows that they agree well with one another. This shows that the chosen theoretical model adequately describes the real physical processes studied.

b) Taking into account nonlinear aberrations.

To obtain a more complete picture of the thermal defocusing and to compensate more effectively the nonlinear distortions which arise it is necessary to perform calculations taking aberrations into account. Spherical aberrations can be taken into account as follows.³¹ When studying spherical aberrations an additional term is introduced into the expression for the eikonal S of an axisymmetric beam. This correction to the eikonal of a plane wave is written in the form of an expansion in even powers of the radial coordinate r :

$$S = S_0(z) + \frac{r^2}{2g(z)} \frac{dg}{dz} + \frac{r^4}{8\alpha_0^2 g^2(z)} \frac{d\beta}{dz}, \tag{16}$$

where $g(z)$ and $\beta(z)$ are aberration functions. The change in the profile of the intensity of an initially Gaussian beam is determined by the following expression

$$I = \frac{I_0}{g^2} \exp \left[-\alpha z - \frac{r^2}{\alpha_0^2 g^2 \left(1 + \frac{r^2}{\alpha_0^2 g^2} \beta \right)} \right] \left(1 + \frac{r^2}{\alpha_0^2 g^2} \beta \right)^{-2}, \tag{17}$$

where I_0 is the initial intensity on the beam axis. The system for determining the aberrational functions $g(z)$ and $\beta(z)$ has the form

$$\frac{d^2 g}{d\xi^2} = \frac{\alpha l_d^2}{R_{n1}} \frac{\exp[-\alpha l_d \xi]}{g} + \frac{1}{g^3} [1 + 12\beta(1 + \beta)]; \tag{18}$$

$$\frac{d}{d\xi} \left[g^2 \frac{d\beta}{d\xi} \right] = -\frac{\alpha l_d^2}{R_{n1}} - \exp[-\alpha l_d \xi] (1 + \beta) - \frac{4}{g^2} \times \{ 2\beta[2 + 5\beta(3 + 2\beta)] \}. \tag{19}$$

Solving Eqs. (18) and (19) numerically and then calculating the intensity using Eq. (17) made it possible to describe the annular aberrational structure of the beam, which was observed experimentally, but could not be calculated in the aberration-free approximation.

This method makes it possible to calculate also the aberrations of higher order, but in order to take

them into account completely a parabolic equation for the complex amplitude of the beam (1) must be solved simultaneously with the equation of heat conduction (2).

In dimensionless variables, for the case of a stationary medium Eqs. (1) and (2) will assume the form

$$2i \frac{\partial \bar{E}}{\partial \xi} = \Delta_{\perp} \bar{E} + [R\bar{T} - i\alpha] \bar{E}; \tag{20}$$

$$-\Delta_{\perp} \bar{T} = \bar{E} \bar{E}^*, \tag{21}$$

where $\bar{x} = x/a_0$, $\bar{y} = y/a_0$, $\zeta = z/l_d$, $\bar{T} = T/T_0$,

$\bar{E} = E/E_0$, $\bar{\alpha} = \alpha \cdot l_d$, and $T_0 = \frac{\alpha P_0}{\kappa \pi}$ is the tem-

perature scale and $R = 8\pi n_0 a_0^2 \frac{dn}{dT} \alpha P_0 / \kappa \lambda^2$ is the nonlinearity parameter.

A focused beam with a Gaussian profile is given at the point of entry into the nonlinear medium:

$$\bar{E}(\bar{x}_0, \bar{y}_0, \xi = 0) = \exp \left[-\frac{(\bar{x}_0^2 + \bar{y}_0^2)}{2} \right] \times \exp \left[-i \frac{(\bar{x}_0^2 + \bar{y}_0^2)}{2R_f} \right], \tag{22}$$

where $\bar{R}_f = R_f/l_d$ is the radius of initial focusing, determined by the spherical lens. The problem is solved in the far zone and the amplitude \bar{E} is a rapidly oscillating function of the variables \bar{x}_0 and \bar{y}_0 , so that for numerical calculations an unrealistically detailed grid must be specified. This is avoided with the help of different coordinate transformations.

In the present case the diffraction transformation was employed.⁵ This transformation takes into account the diffraction divergence of the beam and makes it possible to obtain a solution for both $z = R_f$ and $z > R_f$. In addition, as a result of the diffraction transformation an infinite propagation interval becomes finite. When calculations are performed with a constant step the nodes of the grid automatically become denser near the focal point, where the change in the size and shape of the beam owing to self-action increases significantly.

After diffraction transformations Eqs. (20) and (21) and the boundary conditions (22) acquire the form

$$2i \frac{\partial \bar{E}}{\partial z} = \Delta_{\perp} \bar{E} + \bar{E} \left[2 - \bar{x}^2 - \bar{y}^2 \right] + \frac{\bar{R}_f^2 (1 + \text{tg}^2 z)}{1 + \bar{R}_f^2} \times [R\bar{T} - i\alpha] \bar{E}; \tag{23}$$

$$-\Delta_{\perp} \bar{T} = \bar{E} \bar{E}^*, \tag{24}$$

$$\vec{E} \left[\tilde{x}_0, \tilde{y}_0, \tilde{z} = \arctan \left[-\frac{1}{\bar{R}_f} \right] \right] = \exp \frac{-[\tilde{x}_0^2 + \tilde{y}_0^2]}{2}; \quad (25)$$

where

$\tilde{x} = \bar{x} / \sqrt{D}$, $\tilde{y} = \bar{y} / \sqrt{D}$, $D = \zeta^2 + (1 - \zeta / \bar{R}_f)^2$, $\bar{\Delta}_\perp = D\Delta_\perp$, and $d\tilde{z} / d\zeta = 1 / D$. The problem now reduces to the case of a collimated beam, since in the absence of self-action the solution of Eqs. (23)–(24) corresponds to a beam whose initial shape and size remain constant.⁵

The equations derived above were solved numerically by the method of splitting according to physical factors.³² A 64×64 grid was employed. The total far field angular divergence was calculated in terms of the width of the angular spectrum of the radiation. The results of the calculations for a collimated beam are presented in Fig. 3a. One can see that the computed points agree well with the experimental curve.

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