Correction of turbulent distortions based on the phase conjugation in the presence of phase dislocations in a reference beam

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Results of numerical studies on the correction of laser beam turbulent distortions under conditions of wave front dislocations are presented. To detect singular points, two algorithms are used and their accuracy estimated. Beam controlling based on phase conjugation is considered. When analyzing the correction, a flexible mirror and an ideal corrector, which allows reproduction of the phase surface with almost no restrictions, are used as control elements. The efficiency of turbulence compensation by these two types of correctors is compared. The peculiarities arising in the adaptive system due to the use of a mirror are shown.

Introduction

In this paper we consider, based on the methods of numerical experiment, an adaptive control over a laser beam that has passed through the layer of a turbulent medium and compare the efficiency of correcting distortions with the use of an ideal corrector allowing reconstruction of the wave front with no restrictions and the adaptive mirror modeled by a thin flexible plate.

It is known from literature that, as the coherent radiation propagates through the atmosphere, phase singularities (sometimes called dislocations) arise in the fine structure of the fields. The development of these singularities is explained by the interference among different parts of the optical field.[1] Regardless that there exists several types of singularities according to the classifications available, the condition for their existence is the same, namely, the presence of isolated points with zero intensity, i.e., the points, at which both the real and imaginary parts of the complex amplitude of the field vanish and the phase is consequently undefined. It was noted[2] that the behavior of the most well known screw-type dislocations or optical vortices is similar, to a certain degree, to that of charged particles. These dislocations can turn around the beam axis, approach each other, move away, and die at collisions. It is thought possible that dislocations can transform from one type to another, for example, from edge to screw dislocations, from edge dislocation to the dislocations of mixed edge-screw type, etc.

As numerical analysis of interference of the fields from several sources of plane or spherical waves shows,[3,4] dislocations can be localized and classified and their properties can be determined, but this analysis does not allow us to follow up the processes of appearance and annihilation of the dislocations. The dynamics of these processes was considered in solving the problem on diffraction of a beam with a distorted wave front[5] and in the problem on nonlinear refraction of a beam with the initially parabolic phase profile,[6,7] as well as in solving some problems on the behavior of a speckle field in a randomly inhomogeneous medium.[8,9] In general, we can say that in spite of a rather large number of papers devoted to the study of phase singularities, some problems connected with the peculiarities of adaptive correction under these conditions are not sufficiently studied yet.[10] Studying the effect of dislocations on the efficiency of control and developing an optimum wave front corrector can apparently be considered as urgent problems of modern adaptive optics. In this paper, we are going to discuss just these problems.

1. Turbulent distortions of a beam along the propagation path

Propagation of a beam through a randomly inhomogeneous atmosphere was described by a parabolic equation[11]:

$$2ik\frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + 2k^2\tilde{n}(x, y)E, \qquad (1)$$

where E is the complex amplitude of the field; z is the propagation direction; x and y are the coordinates in the plane normal to the propagation direction; k is wave number. In numerical solution of Eq. (1), the longitudinal coordinate is normalized to the diffraction length $Z_\text{d} = ka_0^2$, the cross coordinates are normalized to the initial radius of the beam a_0, $\tilde{n}(x, y)$ is a random field of the refractive index fluctuations that is characterized by the structure function $D_n(\rho)$ (Ref. 12) obeying the “2/3”-power law

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$$D_n(\rho) = C_n^2 \rho^{2/3}, \quad (2)$$

where C_n^2 is the structure constant, $\boldsymbol{\rho} = (x, y)$; $\rho = |\boldsymbol{\rho}|$ are coordinates of a point. The spectral density of fluctuations $\Phi_n(\kappa)$ was described by the Karman model

$$\Phi_n(\kappa) = 0.489 r_0^{-5/3} (k_0^2 + k_m^2)^{-11/6};$$

$$r_0 = (0.423 k^2 \int_0^L C_n^2(l) \mathrm{d}l)^{-3/5}; \quad (3)$$

$$k_0 = 2\pi / l_0, \quad k_m = 2\pi / L_0.$$

Here r_0 is the Fried radius used in this paper as a characteristic of turbulence; l_0 and L_0 are the inner and the outer scales of turbulence, respectively; L is the thickness of the turbulent layer. In the numerical model used, the turbulence was represented by a random phase screen set at the beam path (Fig. 1*a*). The position of the screen varied; in particular, the calculations have been made for the screen situated just in the plane of the emitting aperture ($Z_\mathrm{s} = 0$) and in the middle of the path ($Z_\mathrm{s} = 0.5Z$). In controlling the beam in the observation plane, the criterion of focusing

$$J(t) = \frac{1}{P_0} \iint \rho(x, y)\, I(x, y, z_0, t)\, \mathrm{d}x\mathrm{d}y$$

was measured. This criterion has the meaning of a relative fraction of the beam power falling within the aperture of radius a_0. Here P_0 is the total beam power; ρ is the aperture function $\rho(x, y) = \exp[-(x^2 + y^2)/a_0^2]$.

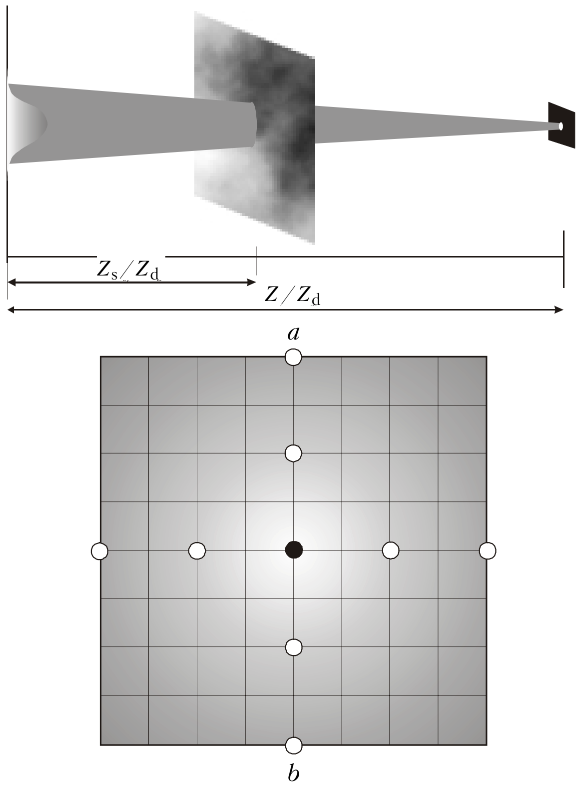

Fig. 1. Scenario of the numerical experiment on localization of dislocations (*a*) and model of flexible mirror being a part of an adaptive phase conjugation system (*b*).

One of the realizations of the random screen and the corresponding intensity distribution of the beam having passed through this screen are shown in Fig. 2.

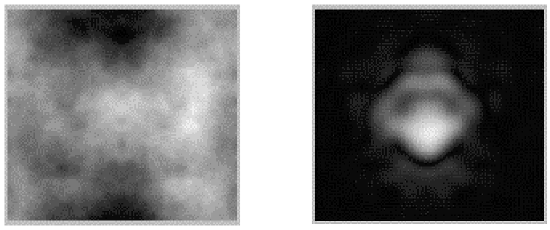

Fig. 2. Turbulent screen set on the beam path and intensity distribution of the beam.

As a control element in the adaptive system, we used an ideal corrector and a flexible mirror with nine servoactuators shown in Fig. 1*b*. Mirror deformations $W(x, y)$ were described by the equation of a biharmonic type[13]:

$$D\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial^2 x \partial^2 y} + \frac{\partial^4 W}{\partial y^4}\right) = f(x, y), \quad (4)$$

where $f(x, y)$ is the distributed shear load; $D = E_\mathrm{Y} h^3 / [12(1 - \sigma^2)]$ is cylindrical inflexibility; σ is the Poisson coefficient; E_Y is the Young's modulus; h is the plate thickness. The equation was solved by the method of finite elements.[14]

The control of the beam has been realized based on the algorithm of phase conjugation,[15] according to which the beam phase in the emission plane is

$$\varphi(x, y) = -\psi(x, y), \quad (5)$$

where $\psi(x, y)$ is the phase of the reference beam.

2. Algorithms for detection of dislocations

As known[7] the presence of dislocations in the phase profile of the reference beam affects considerably the efficiency of correction for the turbulent distortions, when a flexible mirror with a continuous reflecting surface is used as a control element. Therefore, to estimate the efficiency of compensation, we need to analyze the regularities of the appearance of dislocations, as well as to reveal the dependence of their number and location points on the parameters of the medium and the path length.

For this research, we used two algorithms for detection of the wave front singularities. In the first algorithm, the intensity distribution of the beam having passed through the turbulent layer was considered in the observation plane. Points with the zero intensity were determined in this distribution, then the points corresponding to the zero intensity were found in the phase profile, and the phase increment at go-round of points along a closed contour was calculated. If an outlier roughly equal to 2π was observed in this calculation, then a point was assumed dislocation.

In the second algorithm, only the phase profile of the beam was studied. The algorithm was based on the assumption that the phase has only one break with the value of 2π in the region of dislocations and it is topologically continuous and monotonically increasing at all other points of the contour about the dislocation. All points of the central area of the surface were checked for the correspondence to these signs. It should be noted here that the research was based on numerical methods, according to which continuous functions of the amplitude and phase are represented by their grid analogs. In the discrete representation, the methods have limited accuracy, in particular, we should take into account that points with zero intensity may fall between grid nodes, that is, they are not detected with absolute reliability. Therefore, in realizing the first algorithm, we determined points of local minima in the intensity distribution, rather than zero points.

The next source of such errors is random noise arising at the edges of the computational grid when specifying the exponential function and making Fourier transform. Just the presence of this noise explains the fact that the density of dislocations at the edges of the grid several times exceeds that in the central areas. To eliminate this factor, singularities were calculated only in the central area roughly equal to two energy radii of the beam.

The accuracy of determination of singular points in the phase profile of the beam based on the two algorithms described above is illustrated in Fig. 3. It can easily be seen that the algorithms detect almost all dislocations in the central area. However, in both of the cases, some non-singular points are detected as well. The number of such points detected by the second algorithm (Fig. 3*b*) is larger than for the first one (Fig. 3*a*). This can be explained only by the errors connected with the discrete representation of the functions, we failed to exclude completely (to improve the reliability of computations, we used grids of different dimension and all variables in the program were set with double accuracy). In general, we can say that both algorithms overestimate somewhat the number of dislocations and the second algorithm is likely characterized by the lower reliability as compared with the first one.

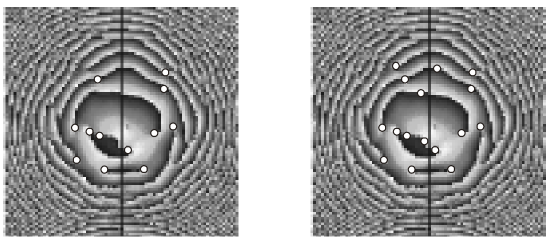

Fig. 3. Phase profile of the beam propagating through a turbulent medium and dislocation points detected by the two algorithms.

The statistics of dislocations on the beam path is illustrated in Fig. 4. The plots show the number of dislocations in different cross sections of the path at different characteristics of the phase screen (different Fried radius). From these plots we can see the following regularities: the number of singular points increases with the increase of the turbulence intensity. At high intensity of the distortions, the number of dislocations increases drastically as the path length increases, and this increase is smoother at the lower intensity. In both cases, as the path length increases, the number of dislocations increases not unlimitedly, but tends to some constant value (has a tendency to saturation).

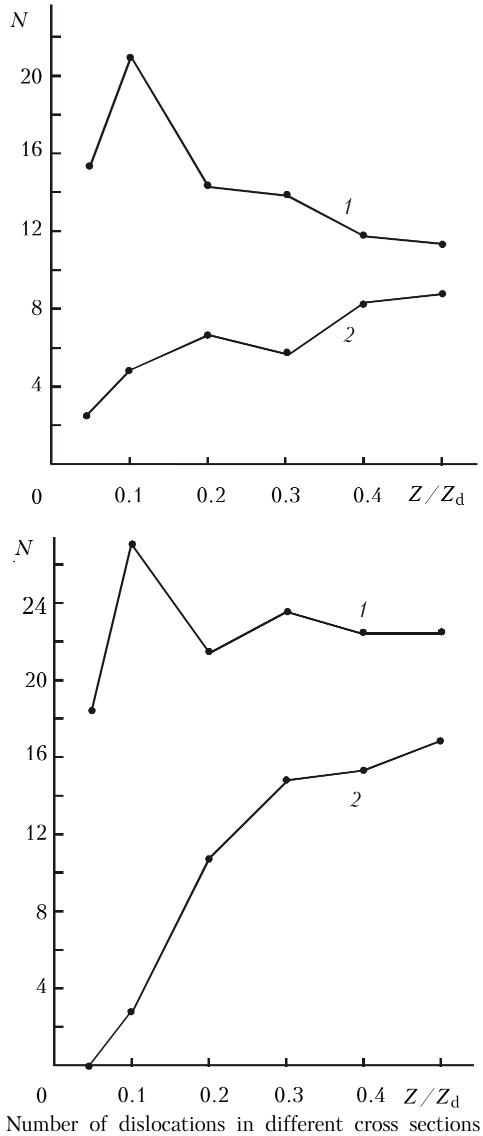

Fig. 4. Number of dislocations in different cross sections of the beam path for two algorithms as a function of the longitudinal coordinate z: r_0 = 0.05 (curve *1*) and r_0 = 0.1 (curve *2*).

3. Influence of dislocations on the efficiency of phase conjugation

In realizing the phase conjugation algorithm, the phase of the reference beam in the plane of the emitting aperture was specified in the following way:

$$\varphi(x, y) = \mathrm{Arg}[E(x, y)].$$

Since the argument of a complex value is not a single-valued function, this equation can be used only for the surface determined in the interval $[-\pi, +\pi]$. At the boundaries of this interval, the calculated phase changes in a jump from $+\pi$ to $-\pi$ (or vice versa from $-\pi$ to $+\pi$). To eliminate this restriction, we should find the total phase

$$\varphi(x, y) = \mathrm{Arg}[E(x, y)] + 2\pi m,\ m = 0, \pm 1, \pm 2, \ldots,$$

i.e., to determine the integer number m for each of the field points.

The studied adaptive system was used with two different control elements: an ideal corrector allowing reproduction of the wave front with no limit and a flexible mirror, whose model is shown in Fig. 1*b*. In the latter case, displacement of each servoactuator was set equal to the phase shift at the corresponding point. Besides, for the points of actuator fixation, the number m was determined, i.e., breaks introduced by the function Arg were taken into account (surface joining was made). The need in this joining is illustrated by the data presented in the Table, which gives the values of the focusing criterion J obtained for one of the realizations of the phase screen without control (first column) and at phase conjugation (2nd, 3rd, and 4th columns) with the use of a collimated reference beam. Correction was performed on the path with the length $0.5Z_{\rm d}$ at $r_0 = 0.1$, and the turbulent screen in these numerical experiments was set in the beginning of the path, i.e., there were no dislocations in the recording area (the distance between the screen and the recording plane was equal to zero). The tabulated data show that compensation for the distortions under these conditions is characterized by high efficiency when using an ideal corrector (doubling of the focusing criterion), while the mirror without phase joining not only fails to provide for adaptive focusing, but also leads to a decrease in the irradiance in the observation plane. If the joining operation is included in the algorithm, then the mirror provides for the results close to that given by the ideal corrector (4th column of the Table).

Table. Results (focusing criterion) of turbulence correction based on phase conjugation, DC

No control	Ideal PhC	Mirror without phase joining	Mirror with phase joining
0.26	0.51	0.10	0.40

For one of the realizations of the phase screen, the data characterizing the efficiency of the correction are shown in Fig. 5*a*. The values of the focusing criterion were obtained as a result of control at different intensity of the turbulent distortions (the screen was set in the middle of the path, what led to appearance of dislocations in the plane of the emitting aperture). Under conditions of low turbulent distortions (r_0 varying from 0.07 to 0.1), the flexible mirror provided for higher values of the criterion efficiency as compared with the ideal corrector. This can be explained by the presence of dislocations in the reference signal and peculiarities of the surface joining algorithm. At joining, the number m is determined, whose value depends on the number of phase breaks with the value of 2π on the straight line between the mirror center and the point of an actuator. In the presence of singular points, this number is overestimated (breaks are caused not only by calculation of the phase as an argument of a complex value, but also by dislocations). Therefore, the mirror actuator displaces by the value larger than that dictated by the control algorithm. As a result, the beam is re-focused. However, because the reference beam is collimated and the phase conjugation reduces the forward beam to the same parameters as for the reference beam, re-focusing of the forward beam with respect to the reference signal leads to higher concentration of the radiation energy in the observation plane. In the area of strong distortions (r_0 ranging from 0.01 to 0.07) at a large number of singular points in the phase of the reference beam, the quality of the correction achievable with the use of a mirror is much lower than that with the use of the ideal corrector.

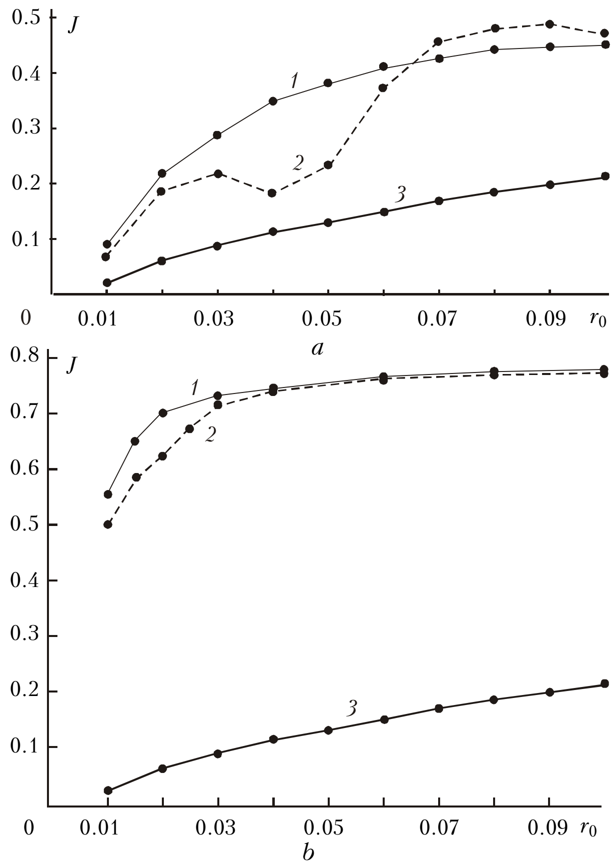

Fig. 5. Focusing criterion J as a function of the Fried radius in the case of control based on phase conjugation with collimated (*a*) and defocused (*b*) reference beam for the system with ideal corrector (curve *1*), flexible mirror (*2*), no adaptive control (*3*).

The limit values of the criterion J (highest efficiency of the correction) are achieved in the case of a defocused beam used as a reference one. In this case, the forward beam in the non-distorting medium is reduced to the focused one as a result of phase conjugation. In the presence of turbulence on the propagation path, the higher values of the criterion are obtained with the use of an ideal corrector, and the increase of the field concentration due to re-focusing when using the mirror is impossible in principle.

The data obtained in such experiments are shown in Fig. 5*b*. For all values of the Fried radius, the values of the criterion in the adaptive system including the mirror are lower than those for an ideal corrector. The decrease in the efficiency is especially noticeable in the region of strong fluctuations, i.e., in the region where the number of dislocations is maximum.

The values of the criterion *J* obtained as a result of control and averaged over 50 realizations are shown in Fig. 6. The plots follow the tendencies characteristic of a single realization of the screen. In particular, it can be seen that in the first case at low intensity of turbulence the mirror provides for higher results (likely, because of dislocations on the edges leading to re-focusing). As the turbulent distortions intensify, the efficiency of correction with the use of the mirror decreases, while the ideal corrector provides for higher values of the criterion. In the second case (defocused reference beam), the value of the criterion obtained with the use of the mirror is lower in the entire range of the parameter r_0.

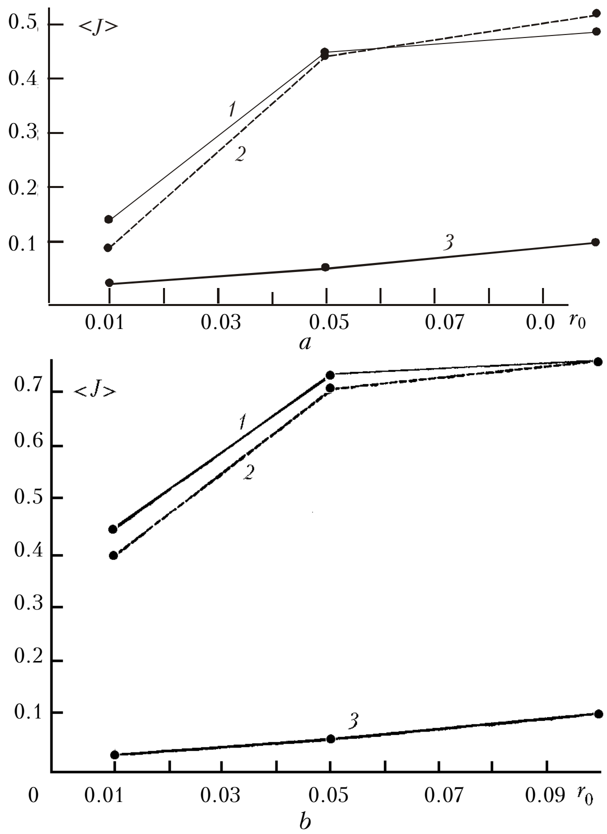

Fig. 6. Values of the focusing criterion averaged over 50 realizations <*J*> obtained with the control (screen in the middle of the path): collimated (*a*) and defocused (*b*) reference beam.

To confirm the fact that nonmonotonic decrease of the focusing criterion in some realizations at the increasing intensity of turbulent distortions is caused just by dislocations, numerical experiments were conducted with the phase screen in the plane of the emitting aperture. In this case there are no singular points in the phase of the reference beam (radiation was defocused). The results of control for one realization and the values of the criterion averaged over 50 realizations are shown in Fig. 7. It can be seen that for the ideal corrector the results of adaptive focusing are independent of r_0, that is, distortions are completely compensated for. If the mirror is included in the system, the values of the criterion decrease monotonically with the increasing intensity of distortions and they are always lower than the results obtained for the ideal corrector.

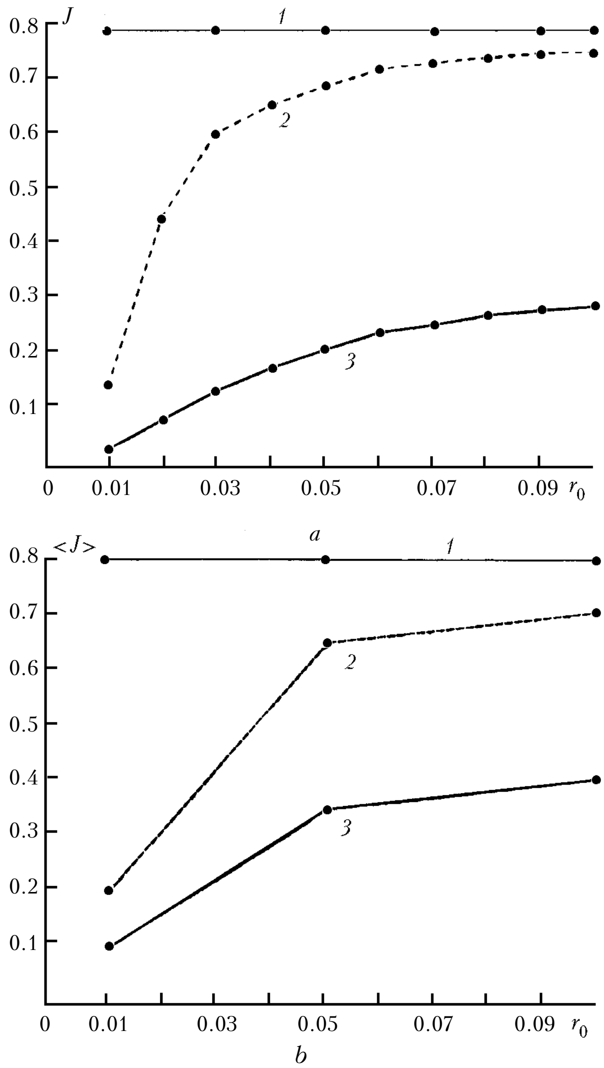

Fig. 7. Focusing criterion *J* with control for one realization (*a*) and averaged over 50 realizations <*J*> (*b*) for the system with ideal corrector (curve *1*), flexible mirror (*2*), no adaptive control (*3*).

In general, we can conclude that at low intensity of the turbulent distortions and, consequently, small number of dislocations the mirror with nine actuators provides for quite high efficiency of adaptive correction

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comparable with the efficiency achieved with the use of an ideal corrector. As the intensity of distortions increases, the use of this mirror becomes inexpedient. In this case, it is necessary to increase the number of actuators.

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