SCINTILLATION AND WAVE–FRONT MEASUREMENTS

Zeng Zongyong, Weng Ningquan, Zhang Jun, and Wang Yingjian

Institute of Optics and Fine Mechanics, Anhui
China Academia Sinica Hefei Anhui 230031,
Received October, 1992

The probability of signal fading at wave–front measurements due to scintillation is investigated. Numerical simulation of distributions of light intensity and phase fluctuations over the receiving aperture is presented. The calculational results are compared with the measurements under laboratory conditions. This numerical method can be used for the design of optical instruments, test of formulas, and to analyze the turbulence features.

1. INTRODUCTION

Although the limitations imposed on the adaptive control techniques due to the atmospheric turbulence have been thoroughly investigated,[1–5] there exists another effect governing the correction quality. It is the effect of scintillation that influences the accuracy of wave–front measurements. When the turbulence becomes stronger or light intensity weaker, because of a limited dynamic range and sensitivity of wave–front sensors, the probability of observing a zero light intensity would increase and dark regions in the light pattern would appear. As a consequence the wave–front sensor does not receive a signal at some moments at all. The probability of signal missing is important for an adaptive optical system. It is well known that the larger is the number of the wave–front sensors and elements of a deformable mirror, the higher is the efficiency of the correction for corresponding distortions. The smaller is the size of a wave–front sensor, the higher is the probability of detecting the signal fading. The optical communication systems operating in the atmosphere need for large aperture receiving telescopes. In this connection an investigation of the effect of scintillations averaging by the receiving aperture has been carried out.[6,7]

In order to calculate the probability of the missing wave–front measurement, it is necessary to investigate the fluctuations of light intensity in the focal plane of a wave–front sensor, which depend on the average intensity of light at the subaperture of a sensor, the fluctuations of phase difference and its correlation with the intensity fluctuations.

In this paper we mainly discuss the numerical simulation of the irradiance scintillation and make an attempt to find a numerical method which can be used to calculate the probability distribution of the average light intensity over the pupil plane of a sensor. A direct numerical simulation can be based on the statistical features of the intensity and phase of a fluctuating field. A phase screen method may also be used for calculation of the probability distribution. The measurements carried out with a laboratory turbulent medium, which has the statistical characteristics of the homogeneous and isotopic convective turbulence, were used to verify the simulation.

2. NUMERICAL SIMULATION

The light intensity $I(l)$ at the focal plane of a wave–front sensor is

$$I(l)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} E(q_1)E^*(q_2)W(q_1)\,^*(q_2)G(l,\,q_1)\,G^*(l,\,q_2)\,\mathrm{d}q_1\mathrm{d}q_2, \quad (1)$$

where l is the coordinate of the focal plane, E is the field in the incident pupil plane, q_1 and q_2 are the coordinates in the pupil plane, $W(q)$ is the aperture transmittance, G is the Green's function. Let us assume that

$$G=\frac{k}{2\mathrm{p}i\,F}\exp\left[\frac{ik}{2\,F}(l-q)^2\right], \quad (2)$$

where $k = 2\pi/\lambda$, λ is the wavelength, F is the focal length.

For a plane wave with the phase φ we have

$$E = E_0(q)\,\mathrm{e}^{-iz(q)}, \quad (3)$$

and after the change of variables

$$2\,R = q_1 + q_2\,,\ \rho = q_1 - q_2 \quad (4)$$

we can write Eq. (1) in the form

$$I(l)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} E_0\left(R+\frac{\mathrm{r}}{2}\right)E_0^*\left(R-\frac{\mathrm{r}}{2}\right)\times$$

$$\times\exp\left[-i\left(\frac{k}{F}l\rho-\Delta\varphi(R,\,\rho)\right)\right]W\left(R+\frac{\rho}{2}\right)W^*\left(R-\frac{\rho}{2}\right)\mathrm{d}R\mathrm{d}\rho, \quad (5)$$

where $\Delta\varphi(R,\,\rho)=\varphi\left(R+\frac{\rho}{2}\right)-\varphi\left(R-\frac{\rho}{2}\right)$.

When the turbulence in the medium, through which the light propagates, is not very strong, both the logarithm of amplitude $E(R)$ and the phase $\varphi(R)$ are normal random variables and their correlation can be ignored.[8] As a consequence $I(R)$ and its probability density can easily be determined from a numerical solution of Eq. (5). In order to illustrate the effect of scintillations on the wave–front measurements and to check the results of the numerical simulation, distribution of the probability density of the average light intensity over the plane of a subaperture pupil should be calculated, because $I(R)$ can be easily measured.

0235-6880/92/12 843-03 $02.00

If the effect of amplitude fluctuations on the wave–front slope measurements is not strong, Eq. (5) can be reduced to

$$I(l)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} I(R)\exp\left[-i\left(\frac{k}{F}l\rho-\Delta\varphi(R,\rho)\right)\right]\times$$

$$\times\, W\left(R+\frac{\rho}{2}\right)W^{*}\left(R-\frac{\rho}{2}\right)\mathrm{d}R\,\mathrm{d}\rho\ , \qquad (6)$$

where $I(R)=E^2(R)$. The average light intensity in the plane S of a subaperture pupil can be written as the integral over the plane S

$$I(S)=\frac{1}{S}\int\limits_{S} I(R)\,\mathrm{d}R\ , \qquad (7)$$

S is the area of the subaperture pupil.

It is a very hard problem to derive an analytical expression for distribution of the light intensity I probability density. Although the series of logarithms of normal random values have been written analytically, a comprehensive description can only be done using numerical methods. In calculations the random field $x(r)$ of 512 by 512 points with the probability density $P(x)$ has been generated by a computer.

$$P(x)=\left(1/\sqrt{2\pi\sigma_x}\exp\left(-x^2/2\sigma_x^2\right)\right)\ . \qquad (8)$$

The irradiance field $Y(P)$ may be obtained by taking the integral

$$Y(P)=\int\limits_{-\infty}^{\infty} B_{\mathrm{B}}(R-r)\,x(r)\,\mathrm{d}r\ , \qquad (9)$$

where $B_{\mathrm{B}}(r)$ is the correlation between the light intensity logarithms at two points and r is the distance between the two points. It can be shown that the distribution $Y(R)$ is normal, because the weighting coefficients of the modulation function $B_{\mathrm{B}}(R)$ are the same for all $x(r)$. Let us make the transformation

$$\ln I(R)=\sigma_{\ln I}\frac{Y(R)}{\sigma_Y}-\frac{\sigma_{\ln I}^2}{2}\ , \qquad (10)$$

where $\sigma_{\ln I}$ and σ_Y are independent variances of the $\ln I(R)$ and $Y(R)$. There exists the following relation between the probability density $P(Y)$ and $P[\ln I(R)]$.

$$P(Y)\,\mathrm{d}Y=P[\ln I(R)]\,\mathrm{d}[\ln I(R)]\ . \qquad (11)$$

When making calculations of the mean intensity $I(S)$ and of the probability density of the field $P_S(I)$ the field of irradiance $Y(R)$ can be divided into a number of subregions with the diameter d. The field $I(R)$ can also be calculated by inverse transform of a scintillation spectrum with random amplitude. Because the average light intensity at an aperture directly depends on the correlation function, the previous approach to the use of correlation modulation will be used in the numerical simulation process.

A numerical result on $P_S(I)$ for $d=\sqrt{(\lambda L)}$ and $\sigma_{\ln I}^2=0.6$ is shown in Fig. 1. The value of $B_{\mathrm{B}}(\rho)$ measured in a convective turbulence was used for calculating $I(S)$. It should be noted that $B_{\mathrm{B}}(r)$ measured in a weak convective turbulence falls off to the zero level more slowly than theoretical $B_{\mathrm{B}}(r)$.

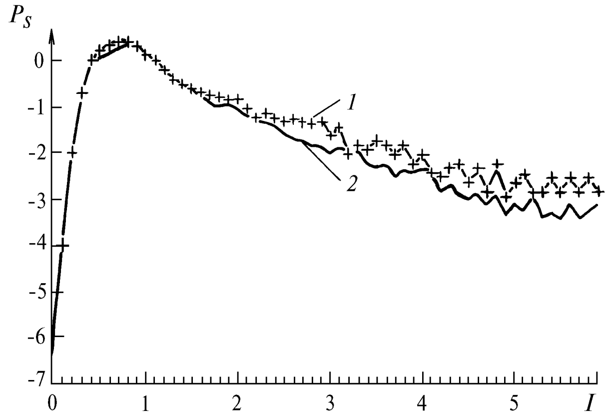

FIG. 1. The probability density of the mean light intensity at the pupil plane of a subaperture. $d=\sqrt{\lambda L}$ *and* $\sigma_{\ln I}^2=0.8$. *1) Measurements and 2) numerical experiment.*

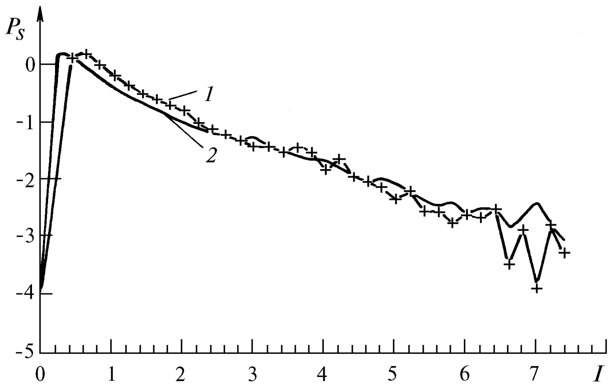

FIG. 2. The probability density of the mean light intensity at the pupil plane of a subaperture, $d=\sqrt{\lambda L}$, $\sigma_{\ln I}^2=2.4$. *Results calculated by the phase screen method are shown by solid curve and measurements are shown by curve with crosses.*

The phase screen method[10] intended for weakening the effects of computation–grid size on the scintillation was used to calculate $I(R)$ and the distribution of the probability density of the mean light intensity $I(S)$. The results are given in Fig. 2. The difference of this numerical experiment from the preceding is that first the simulation of turbulence is performed and then the field passed through the turbulent medium is obtained by solving the wave equation.

3. MEASUREMENTS

A 2–m long tank filled with a medium possessing the properties of a convective turbulence has been used to model the atmospheric turbulence. In this laboratory model the turbulence had a wide inertial range and was stationary. The measurements of fluctuations of the arrival angles showed that the fluctuations of the phase difference were normal and did not correlate with the light intensity. The observed spikes and quasi–ordered structure can be interpreted like the turbulence in the convective boundary layer of the atmosphere. A beam of an He–Ne laser passed through the turbulent medium and two folding mirrors were used to increase the length of the propagation path. The light field passed through a round subaperture with the diameter d was recorded and its parameters were calculated. The probability density of the measured radiance scintillations can be represented by a lognormal function. The probability densities of the measured mean light intensity are shown in Figs. 1 and 2.

4. DISCUSSION

In Fig. 1 $d = \sqrt{\lambda L}$, $\sigma^2_{\ln I} = 0.8$, and the turbulence is weak. The probability of zero mean light intensity is close to zero. The maximum mean intensity reaches 8. The result of numerical simulations well agrees with the measurements. If a wave–front sensor has a sufficiently wide dynamic range and the subaperture diameter is $d = \sqrt{\lambda L}$ the effects of scintillations can be ignored. When the turbulence is strong, the correlation function $B_{\rm B}(r)$ falls off to zero more slowly than in the case of a weak turbulence. Hence, as shown in Fig. 2 for $\sigma^2_{\ln I} = 2.4$, the probability of the zero mean intensity of light decreases more slowly. The probability density calculated by the phase screen method disagrees with the measurements. Although the calculated probability density of scintillations is a normal function in this case the correlation function is determined inaccurately.

The numerical simulations based on statistical features of the light intensity and phase are quite simple and can be widely used techniques. Based on these methods one can determine the probabilities of signal fading. In addition, these methods can be useful when validating some analytically described features such as the influence of the outer scale of a turbulence on measurements of a beam center of gravity and the effect of aperture averaging of scintillations.

But this direct numerical simulations cannot correctly describe an actual light field. In an unstable atmosphere and in our laboratory experiments on turbulence, the light field pattern looks like a net with nail–like knots. Analogous net structure is obtained with the phase–screen method. What statistical features does this net–like structure represent? It is an interesting problem. Are there other features of the irradiance field, that correlate with some characteristics of turbulence such as intermittence of the fine structure, quasi–ordered structure, and so on? We have described a static scheme for the irradiance field. What information is in the movement and variability of light pattern except for the average transverse wind velocity component?

REFERENCES

1. R.L. Fante, J. Opt. Soc. Am. **66**, 730–735 (1976).
2. D.P. Greenwood, J. Opt. Soc. Am. **69**, 549–553 (1979).
3. H.T. Yura, et al., J. Opt. Soc. Am. **2**, 765–773 (1985).
4. M.T. Tavis, et al., J. Opt. Soc. Am. **4**, 57–59 (1978).
5. R.G. Frehlich, J. Opt. Soc. Am. **7**, 2119–2140 (1990).
6. S.J. Wang, et al., J. Opt. Soc. Am. **73**, 831–837 (1983).
7. H.T. Yura, et al., Appl. Opt. **22**, 1608–1609 (1983).
8. J.W. Strohbehn, *Modern Theories in the Propagation of Optical Waves in a Turbulent Medium*, *Laser Beam Propagation in the Atmosphere* (Berlin, Springer–Verlag, 1978).
9. R. Barakat, J. Opt. Soc. Am. **66**, 211–216 (1976).
10. Y. Wang, et al., *High Power Laser and Particle Beams* (to be published).