SPLINE APPROXIMATION METHOD FOR OPTIMIZING THE NUMBER OF SPATIAL MODES OF AN PHASE CONJUGATE ADAPTIVE OPTICAL SYSTEM

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Received April 11, 1991

Analytical formulas for the rms error, averaged over the receiving aperture, of reconstruction of the phase front with an account of the measurement noise of a Hartmann phase–front sensor are derived based on the spline approximation method using the normalized parabolic B–splines. An estimate of the correlation matrix of the vector of controlling signals of a flexible adaptive mirror is obtained for the Kolmogorov spectrum of turbulence. As a result, an optimal recurrent estimate for the controlling signal vector of the flexible adaptive mirror based on a piesoceramic plate is written.

At present flexible adaptive mirrors based on piesoceramic plates are being widely used in the creation of adaptive optical systems designed to compensate for nonstationary phase distortions of the light beams propagating through a turbulent atmosphere. This is caused by two reasons. First, such mirrors have a wide frequency band and a large dynamic range of phase correction.[1] Second, M.A. Vorontsov et al.[2] proposed an efficient procedure for estimating the distribution of the residual error of phase-front approximation with such mirrors as functions of the shape and number of the controlling electrodes. It should also be noted that the response functions of such mirrors can be close to orthogonal Zernike polynomials, for which the authors of Ref. 3 have derived analytical formulas, permitting one to substantiate the choice of the number of spatial modes for phase correction as functions of the required accuracy of the phase-front approximation under conditions of the Kolmogorov spectrum of phase fluctuations. By virtue of a specific nature of quadratic detection, the interference and the Hartmann sensors are usually used in the phase-conjugate adaptive optical systems as sensors of the phase front. This is due to the fact that the Hartmann method, which makes it possible to determine the value of the phase-front distortions based on the displacements of images of the object at the foci of the subapertures covering uniformly the aperture, appears to be the most promising method.[4] In this case, based on the measured average tilts of the phase front within the subaperture Ω_{ij}, which are proportional, in general, to the quantities

$$k^{-1}\frac{\partial\Phi(x_i,y_i)}{\partial x},\quad k^{-1}\frac{\partial\Phi(x_i,y_i)}{\partial y},\quad i,j=\overline{1,n}\ ,\tag{1}$$

where k is the wave number, $\Phi(x, y)$ is the function describing the phase distribution over the aperture, and $n^2 = M$ is the number of the subapertures, the vector of controlling signals of an adaptive optical system can be calculated.[5] As shown in Refs. 2 and 3, an increase in the number of spatial modes, corrected by an adaptive optical system, results in a decrease of the phase–front approximation error. However, due to the fact that the least–squares method is employed in order to calculate the components of the vector of the controlling signals in real systems while the output signals from the phase–front sensor can be represented in the form

$$\frac{k^{-1}\partial\Phi(x_i,y_j)}{\partial x}+n_{ij}^{x},\quad \frac{k^{-1}\partial\Phi(x_i,y_j)}{\partial y}+n_{ij}^{y},\tag{2}$$

where n_{ij}^{x}, and n_{ij}^{y} are the components of the measurement noise, with increase in the number of the spatial modes N and in the variance of the noise σ^2, the error in calculation of the vector of controlling signals caused by these factors will increase. A technique for optimizing the choice of the number of spatial modes of a phase-front corrector when these modes are described by orthogonal Zernike polynomials has been developed in Ref. 4. For the response function of an arbitrary shape this problem has not yet been solved.

This paper is devoted to the development of a method for optimizing the choice of the number of spatial modes of an adaptive optical system using a system of normalized parabolic B–splines with an account of the measurement noise of the local phase-front tilts.

Let us consider this problem as follows. Assume that on the aperture of radius R the Hartmann sensor measures the values of the partial derivatives of the phase front (2), which are proportional to the local phase–front tilts averaged over the subaperture Ω_{ij}. In so doing, we will use the Kolmogorov model of the turbulent atmosphere and will accept the hypothesis of "frozen–in" turbulence. We will describe the adaptive phase-front corrector in terms of a linear combination of its spatial modes

$$U(x,\ y,\ \mathbf{A}) = \sum_{m} a_i\,\Psi_i(x,y),\tag{3}$$

where $U(x, y, \mathbf{A})$ is the response of the mirror to the vector of controlling actions **A** with components a_i, $\Psi_i(x, y)$ is the lth spatial mode of the adaptive corrector, and N is the number of spatial modes.

Determining the components of the vector **A** by means of the least–squares method and ignoring the noise components n^x and n^y, we can write a system of standard equations in the following form:

$$D\mathbf{A} = \mathbf{F},\tag{4}$$

where D is the quadratic matrix with elements d_{kl},

$$d_{kl}=\left(\frac{\partial\Psi_k(x,y)}{\partial x},\frac{\partial\Psi_l(x,y)}{\partial x}\right)+\left(\frac{\partial\Psi_k(x,y)}{\partial y},\frac{\partial\Psi_l(x,y)}{\partial y}\right),$$

$$l,k=\overline{1,N}\ ,$$

A is the sought–after column vector of the controlling actions, and **F** is the column vector in the right side of Eq. (4) with components f_k,

0235-6880/91/12 886-04 $02.00

$$f_k = \left(\frac{\partial\Phi(x,y)}{\partial x}, \frac{\partial\Psi_k(x,y)}{\partial x}\right) + \left(\frac{\partial\Phi(x,y)}{\partial y}, \frac{\partial\Psi_k(x,y)}{\partial y}\right).$$

We hereafter will denote by parentheses the scalar product

$$(q_1, q_2) = \int_s\int q_1 q_2 \mathrm{d}x\mathrm{d}y = \int_s\int q_1 q_2 \mathrm{d}^2\mathbf{r}, \quad (5)$$

where $\mathbf{r} = \sqrt{x^2 + y^2}$. We will prescribe the statistical properties of the noise components n^x and n^y in the form

$$M[n_{ij}^x] = M[n_{ij}^y] = 0, \quad (6)$$

$$M[n_{ij}^x n_{rs}^x] = \begin{cases} \sigma^2 & \text{at } i = r \text{ and } j = s, \\ K & \text{at } i = r \text{ or } j = s, \end{cases}$$

$M[n_{ij}^x n_{rs}^y] = 0$ for arbitrary i, j, r, and s.

where $M[\cdot]$ is the symbol of mathematical expectation and the averaging hereafter is carried out over the ensemble of realizations.

One can easily verify the validity of Eq. (7) in the following way. Let the phase–front tilts be measured with the help of the Hartmann sensor with quadrant photodetectors and let the statistical characteristics of the output noise n_i of each photodetector under condition of a strong signal obey the following relations:

$$M[n_i] = 0, \; M[n_i n_j] = \begin{cases} \sigma_n^2 & \text{at } i = j, \\ K_1 & \text{at } i \neq j, \end{cases} \quad (8)$$

where K_1 is the correlation coefficient.

Taking into account the fact that the signals proportional to the local tilts U_x and U_y are usually obtained in the form

$$U_x = (U_1 + U_2) - (U_3 - U_4);$$
$$U_y = (U_1 + U_2) - (U_3 + U_4), \quad (9)$$

and using relations (8), it is not difficult to verify that

$$M[n^x] = M[n^y] = 0;$$

$$M[n^x n^y] = M[\{(n_1 + n_2) - (n_3 + n_4)\} \times$$
$$\times \{(n_1 + n_3) - (n_2 + n_4)\}] = 0;$$

$$M[n^x n^x] = M[\{(n_1 + n_2) - (n_3 + n_4)\} \times$$
$$\times \{(n_1 + n_3) - (n_2 + n_4)\}] = 4(\sigma_n^2 - K_1), \quad (10)$$

and when $K_1 = 0$, $M[n^x n^x] = 4\sigma_2^n$.

Relations (10) mean that the measurement errors of the phase–front tilts are uncorrelated even in the case in which the noise of the quadrant photodetectors is correlated.

With an account of expression (2), the system of linear equations (4) for estimation of the vector $\mathbf{A}^*$ can be written in the form

$$\mathbf{A}^* = D^{-1}\mathbf{F}^*, \quad (11)$$

where $\mathbf{A}^* = \mathbf{A} + \Gamma$, Γ is the column vector of the estimate of the errors in determining the coefficients $\mathbf{A}$ and $\mathbf{F}^*$ is the column vector with components

$$f_k^* = \left(\frac{\partial\Phi(x,y)}{\partial x} + n^x, \frac{\partial\Psi_k(x,y)}{\partial x}\right) + \left(\frac{\partial\Phi(x,y)}{\partial y} + n^y, \frac{\partial\Psi_k(x,y)}{\partial y}\right).$$

Let us find the statistical characteristics of the components of the vector Γ, namely, $M[\gamma_k]$ and $M[\gamma_k, \gamma_i]$. Taking into account the linearity of system (11), we can write

$$M[\gamma_k] = M\left[\sum_{i=1}^{N} d_{ki}^{-1} f_i^*\right] =$$

$$= \sum_{i=1}^{N} d_{ki}^{-1}\left\{\left(M[n^x], \frac{\partial\Psi_i(x,y)}{\partial x}\right) + \left(M[n^y], \frac{\partial\Psi_i(x,y)}{\partial y}\right)\right\} = 0, \quad (12)$$

where d_{kl}^{-1} are the elements of the inverse matrix D^{-1}, f_i^* are the components of the vector $\mathbf{F}^*$,

$$M[\gamma_k \gamma_i] = M\left[\sum_{i=1}^{N} d_{ki}^{-1} \times \right.$$

$$\times \left\{\frac{1}{n^2}\sum_{s=1}^{n}\sum_{p=1}^{n} n_{sp}^x \cdot \frac{\partial\Psi_k(x_s, y_p)}{\partial x} + n_{sp}^y \cdot \frac{\partial\Psi_k(x_s, y_p)}{\partial y}\right\} \times$$

$$\times \sum_{i=1}^{N} d_{ji}^{-1} \times$$

$$\left. \times \left\{\frac{1}{n^2}\sum_{r=1}^{n}\sum_{t=1}^{n} n_{rt}^x \cdot \frac{\partial\Psi_j(x_r, y_t)}{\partial x} + n_{rt}^y \cdot \frac{\partial\Psi_j(x_r, y_t)}{\partial y}\right\}\right]. \quad (13)$$

In formula (13) the continuous scalar product has been replaced by its discrete analog, since the Hartmann sensors, as has already been indicated above, measure the average tilts of the phase front only at the aperture points. In this case, the total number of the subapertures M is proportional to n^2. When considering the multipliers inside the braces of quadratic form (13) in detail, it turns out that by virtue of satisfying conditions (6) and (7) their product will be nonzero only when the subscripts sp and rt coincide. Thus, we can write

$$M[\gamma_k \gamma_i] = \frac{\sigma^2}{n^4}\sum_{l=1}^{N}\sum_{j=1}^{N} d_{kl}^{-1} d_{ij}^{-1}\left[\sum_{s=1}^{n}\sum_{p=1}^{n} \frac{\partial\Psi_k(x_s, y_p)}{\partial x} \times \right.$$
$$\left. \times \frac{\partial\Psi_j(x_s, y_p)}{\partial x} + \frac{\partial\Psi_k(x_s, y_p)}{\partial y} \cdot \frac{\partial\Psi_j(x_s, y_p)}{\partial y}\right], \quad (14)$$

or based on Eq. (4)

$$M[\gamma_k \gamma_i] = \frac{\sigma^2}{M}\sum_{l=1}^{N}\sum_{j=1}^{N} d_{kl}^{-1} \cdot d_{ij}^{-1} \cdot d_{kj}. \quad (15)$$

In a matrix form Eq. (14), with the fact that D is symmetrical taken into account, can be written in the form:

$$M[\gamma_k, \gamma_i] = \frac{\sigma^2}{M} D^{-1} D D^{-1} = \frac{\sigma^2}{M} D^{-1}. \quad (16)$$

Thus, in order to calculate the covariation matrix elements of errors in the estimate of the vector **A** taking into account the measurement noise of the phase–front sensor, one must know the elements of the matrix D. In calculating the characteristics of a real adaptive system with a flexible mirror, it is necessary to analyze two cases. If we can describe the system of spatial modes of a phase–front corrector with the help of analytic functions, then the calculation of the elements of the matrix D in general is not difficult. And if we fail to express analytically the spatial modes of a phase–front corrector with sufficient accuracy, but their experimental measurements are available,[3] then it is expidient to use the methods of numerical integration and differentiation. In this case, we can obtain good results using a well developed apparatus of spline functions.

Let us analyze this problem in ample detail. Let the synthesized or experimentally measured spatial modes of a phase-front corrector be well–known. They can always be represented in terms of a system of normalized parabolic 5-splines specified at the nodes of an immobile grid.[6,9]

$$\Psi_k(x,y) = \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} b_{ij}^k B_{2i}(x, \bar{x}_i B_{2j}(y, \bar{y}_j; \qquad (17)$$

where

$$B_{2i}(x,\bar{x}) = \frac{1}{2}\left[\frac{3}{2} + \frac{x-\bar{x}_i}{h}\right]^2 \cdot \mathrm{B}_{0i-1} +$$

$$+ \left[\frac{3}{4} - \left(\frac{x-\bar{x}_i}{h}\right)^2\right] \cdot \mathrm{B}_{0i} + \frac{1}{2}\left[\frac{3}{2} + \frac{x-\bar{x}_i}{h}\right]^2 \cdot \mathrm{B}_{0i+1};$$

$$+ B_{2j}(y,\bar{y}) = \frac{1}{2}\left[\frac{3}{2} + \frac{y-\bar{y}_j}{h}\right]^2 \cdot \mathrm{B}_{0i-1} +$$

$$+ \left[\frac{3}{4} - \left(\frac{y-\bar{y}_j}{h}\right)^2\right] \cdot \mathrm{B}_{0j} + \frac{1}{2}\left[\frac{3}{2} + \frac{y-\bar{y}_j}{h}\right]^2 \cdot \mathrm{B}_{0i+1};$$

$$+ B_{0i} = \begin{cases} 1 \text{ for } x \in [x_i, x_{i+1}] \\ 0 \text{ for } x \notin [x_i, x_{i+1}] \end{cases}, \quad B_{0j} = \begin{cases} 1 \text{ for } y \in [y_j, y_{j+1}] \\ 0 \text{ for } y \notin [y_j, y_{j+1}] \end{cases},$$

where h is the grid step.

In such a representation the set of coefficients of the two–dimensional parabolic B–spline uniquely describes the kth spatial mode of the phase–front corrector. The system of the coefficients for the fcth spatial mode can be calculated by well-known methods, if one knows its values at the nodes of the spline collocation.[6] On account of Eqs. (4) and (16), the values of the partial derivatives of the kth spatial mode $\frac{\partial\Psi_k(x,\ y)}{\partial x}$ can be given by

$$\frac{\partial \Psi_k(x,y)}{\partial x} = \sum_{i=0}^{n} \sum_{j=0}^{n} \left\{ \left[\left(\frac{3}{2} + \frac{y-\bar{y}_j}{h}\right)^2 \times \right.\right.$$

$$\times \left(\frac{3}{4}\left(b_{i+1,j-1}^k - b_{i+1,j+1}^k\right) + \frac{1}{2}\left(\frac{x-\bar{x}_i}{h}\right) \cdot \left(b_{i-1,j-1}^k + b_{i+1,j-}\right)\right] +$$

$$+ \left(\frac{3}{4} - \left(\frac{y-\bar{y}_j}{h}\right)^2\right) \left(\frac{3}{2}\left(b_{i-1,j}^k + b_{i+1,j}^k\right) + \right.$$

$$+ \left.\left.\left(\frac{x-\bar{x}_i}{h}\right)\left(b_{i-1,j}^k - 2b_{i,j}^k - b_{i+1,j}^k\right)\right]\right]$$

$$+ \left(\frac{3}{2} - \frac{y-\bar{y}_j}{h}\right)^2 \left(\frac{3}{4}\left(b_{i-1,j+1}^k - b_{i+1,j+1}^k\right) + \right.$$

$$\left. + \frac{1}{2}\left(\frac{x-\bar{x}_i}{h}\right)\left(b_{i-1,j+1}^k - 2b_{i,j+1}^k - b_{i+1,j+1}^k\right)\right]\right\} \qquad (18)$$

Let us introduce the notations

$$b_{i-1,j-1}^k - 2b_{i,j-1}^k + b_{i+1,j-1}^k - 2b_{i-1,j}^k + 4b_{i,j}^k + 4b_{i+1,j}^k +$$

$$+ b_{i-1,j+1}^k - 2b_{i,j+1}^k - b_{i+1,j+1}^k = a_1^k;$$

$$b_{i-1,j-1}^k - b_{i+1,j-1}^k - 2b_{i-1,j}^k - 2b_{i+1,j}^k + b_{i-1,j+1}^k + b_{i+1,j+1}^k = a_2^k;$$

$$b_{i-1,j-1}^k - 2b_{i,j+1}^k + b_{i+1,j-1}^k - b_{i-1,j+1}^k + 2b_{i,j+1}^k + b_{i+1,j+1}^k = a_3^k;$$

$$b_{i-1,j-1}^k - b_{i+1,j-1}^k - b_{i-1,j+1}^k - b_{i+1,j+1}^k = a_4^k;$$

$$b_{i-1,j-1}^k - 2b_{i,j+1}^k + b_{i+1,j-1}^k + b_{i-1,j+1}^k - 2b_{i+1,j+1}^k = a_5^k;$$

$$b_{i-1,j-1}^k - b_{i+1,j}^k + 2b_{i+1,j}^k + b_{i-1,j+1}^k + b_{i+1,j+1}^k = a_6^k. \qquad (19)$$

After simple, though quite cumbersome calculations, it can be shown that the elements of the matrix D can be calculated in terms of the splines according to the following formula:

$$\int\int \frac{\partial\Psi_k(x,y)}{\partial x} \cdot \frac{\partial\Psi_l(x,y)}{\partial x} \mathrm{d}x\mathrm{d}y =$$

$$= \frac{1}{h^4} \sum_{i=0}^{n} \sum_{j=0}^{n} \left(\frac{h^2}{256\cdot 15} a_1^k \cdot a_1^l + \frac{9(h\bar{x}_i + h^2)}{40\cdot 16} a_2^k \cdot a_2^l + \right.$$

$$+ \frac{h^2}{9\cdot 16} \cdot \left(\frac{3}{16} a_1^k \cdot a_5^l + \frac{3}{16} a_5^k \cdot a_1^l + \frac{9}{8} a_3^k \cdot a_3^l\right) +$$

$$+ \frac{(2\bar{x}_i + h)}{36} \cdot \left(\frac{81}{16} a_4^k \cdot a_4^l + \frac{27}{64} a_6^k \cdot a_2^l + \frac{27}{64} a_2^k \cdot a_6^l\right) +$$

$$\left. + \frac{(2\bar{y}_i + h)}{324} \cdot a_5^k \cdot a_5^l + \frac{81h^2}{256} a_6^k \cdot a_6^l \right]. \qquad (20)$$

The scalar product

$$\int\int \frac{\partial\Psi_k(x,y)}{\partial y} \cdot \frac{\partial\Psi_l(x,y)}{\partial y} \cdot \mathrm{d}x\mathrm{d}y \qquad (21)$$

can be calculated in an analogous way.

The response of the phase–front corrector reconstructed from the measurements of the Hartmann sensor with an account of the measurement noise, can be written in the form

$$U(x,y,\mathbf{A}) = \sum_{l=0}^{N} (a_l + \gamma_l)\,\Psi_l(x,y)\,. \tag{22}$$

In what follows the error due to the phase–front corrector can be determined by

$$\Delta U(x,y,\mathbf{A}) = \sum_{l=0}^{N} \gamma_l\,\Psi_l(x,y)\,. \tag{23}$$

It is obvious that $M[\Delta U] = 0$, and in order to obtain the final estimate of the error due to the measurement noise, we must calculate $M[\Delta U^2]$ from the formula

$$M[\Delta U^2] = M\left[\sum_{k=1}^{N} \gamma_k\,\Psi_k(x,y) \sum_{i=1}^{N} \gamma_i\,\Psi_i(x,y)\right] =$$

$$= \sum_{k=1}^{N} \sum_{i=1}^{N} M[\gamma_k\,\gamma_i]\,\Psi_k(x,y)\,\Psi(x,y)\,. \tag{24}$$

By subsituting Eq. (15) into Eq. (24) we finally obtain

$$M[\Delta U^2] = \frac{\sigma^2}{M} \sum_{k=1}^{N} \sum_{i=1}^{N} d_{ki}^{-1}\Psi_k(x,y)\,\Psi_i(x,y)\,. \tag{25}$$

After averaging Eq. (25) over the aperture S we can derive the formula for the rms error of correction

$$\sigma_{cor}^2 = \frac{\sigma^2}{M} \sum_{k=1}^{N} \sum_{i=1}^{N} d_{ki}\,b_{ki}\,, \tag{26}$$

where b_{ki} are the scalar products

$$b_{ki} = \int\int_S \Psi_k(x,y)\cdot\Psi_i(x,y)\,\mathrm{d}x\mathrm{d}y\,.$$

The coefficients d_{ki} in Eq. (26) can be calculated from Eqs. (18), (19), and (20). The values of b_{ki} can be calculated in terms of the B–splines. It is obvious that the specific values of the spline coefficients will depend on the shape of the phase-corrector spatial modes $\Psi_k(x, y)$.

Analysis of Eq. (26) shows that, as the number of the spatial correction modes increases, the error caused by the measurement noise with variance σ^2 also grows. This is due to the fact that in general rtonnegative terms are added to sum (26).

The error variance of the phase-front approximation in the case of the Kolmogorov spectrum of turbulence for the adaptive mirror corrector with arbitrary response functions σ_{ap}^2 can be easy calculated based on an efficient technique, which has been developed in Refs. 2 and 3. For this reason, the number of spatial modes of the phase corrector in a real adaptive system must be chosen by means of the well–known optimization methods from the condition that the total variance be minimum

$$\min_N \sigma_\Sigma^2 = \sigma_{\text{ap}}^2 + \sigma_{\text{cor}}^2\,. \tag{27}$$

In order to calculate the matrices of the second–order moments of the spatial modes of the phase–front corrector W with elements $M[a_i\,a_j]$, we can make use of the relation

$$M[a_i\,a_j] = \alpha_{ik}\left(\frac{D}{r}\right)^{5/3}, \tag{28}$$

where

$$\alpha_{ik} = -\frac{55}{D^4\pi^2}\int\int \Psi_i(r_1)\cdot(\Psi_k(r_2))\left(\frac{|r_1 - r_2|}{D}\right)^{5/3}\mathrm{d}^2r_1\mathrm{d}^2r_2\,.$$

However, in practice it would require much computation time. Taking into account the fact that appropriate relations for Zernike polynomials in the case of the Kolmogorov spectrum of the phase fluctuations have been obtained in Ref. 7, the results of those calculations can be used to estimate the coefficients W_{ik} for an arbitrary phase–front corrector. Let us write the corrector response in the form

$$U(x,y,\mathbf{A}) \approx \sum_{i=1}^{N} a_i \sum_{i=1}^{N_z} g_{ij}\,Z_j(x,y)\,, \tag{29}$$

where g_{ij} are the coefficients of the expansion of the ith spatial mode of the phase–front corrector in a system of orthogonal Zernike polynomials, N_z is the number of orthogonal Zernike polynomials, and $z_j(x, y)$ are Zernike polynomials. As $N_z \to \infty$ relation (29) becomes an identity. Changing the order of summation operations, we can write relation (29) in the form

$$U(x,y,\mathbf{A}) = \sum_{j=1}^{N_z} Z_j(x,y) \sum_{i=1}^{N} a_i g_{ij} = \sum_{j=1}^{N_z} Z_j(x,y)\,C_j\,, \tag{30}$$

where C_j are the components of the vector $\mathbf{C}$, $\mathbf{C} = G\mathbf{A}$, and G is the $N \times N_z$ matrix of transition from Zernike basis to the basis of the spatial modes of the phase–front corrector. The matrix of the second-order moments of the phase–front expansion in the system of spatial modes of the phase-front corrector can be then written in the form

$$W = \mathbf{C}^{\mathrm{T}} A_1 \mathbf{C}, \tag{31}$$

where A_1 is the correlation matrix for Zernike coefficients and the components a_{1ij}. Thus, for a 13–electrode corrector based on a piesoceramic plate, the transition matrix has been calculated in Ref. 8. Using the results of these calculations and the results published in Ref. 7 we can calculate the matrix of the second-order moments of the phase-front expansion. Knowing the covariation matrix W and the correlation matrix for the measurement noise G, we can write the recurrent algorithm for optimizing discrete estimate of the vector $\mathbf{A}$ in the ith step[10]

$$\mathbf{A}_i^* = \mathbf{A}_{i-1}^* + K_{i-1}\left[\frac{\sigma^2}{M}I + DK_{i-1}\right]^{-1}\cdot\left(\mathbf{F} - DA_{i-1}^*\right)\,, \tag{32}$$

$$K_i = \left[I + \frac{\sigma^2}{M}K_{i+1}D\right]^{-1} K_{i-1}\,, \tag{33}$$

where K_i is the covariation matrix for the estimate $\mathbf{A}$, $K_1 = W$, and I is the unit matrix.

CONCLUSIONS

The developed method for optimizing the choice of the number of spatial modes with an account of the noise of the Hartmann sensor makes it possible to restrict the number of degrees of freedom of an arbitrary phase–front corrector, starting from the specific conditions of operation of an adaptive optical system. In so doing, it turns out that, in order to reduce the variance of the error caused by the measurement noise, one must, in general, increase the number of the quadrant photodetectors of the Hartmann sensor. Due to the use of the spline–approximation method, a possibility arises to calculate, with sufficient degree of accuracy, by means of the numerical analytical method, the elements of the matrix D of a normal controlling system in terms of linear combination of the coefficients of normalized B–spline. Meanwhile, in order to construct a spline, one can use both the values of the response functions at the collocation nodes and the values of their partial derivatives as an a priori information.[6] It should be noted that when considering the measurement noise we took into account only the thermal noise with zero mean, which, in general, corresponds to the case of a strong signal. The quantum noise and the noise due to the signal integration during a finite time of measuring the phase–front local tilts, should be taken into account individually, e.g., using the technique used in Ref. 4. It should be noted also that, along with adaptive mirrors based on a piesoceramic plate, this method can be employed for optimizing the adaptive optical systems with membrane mirrors controlled by various actuators.

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