SYNTHESIS OF A MULTI–APERTURE OPTICAL SYSTEM

P.A. Bakut, I.V. Boyarkina, and I.A. Rozhkov

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This paper deals with the problems in synthesizing optical arrangement of a multi–aperture optical system (MOS). An attempt to construct the exact mathematical description of synthesizing a MOS optical arrangement is undertaken. It is shown that there is a principal circumstance that improves restrictions on the accuracy with which the individual subbeams can be summed at the MOS focus.

The interest to development and design of multi–aperture optical systems (MOS),[1] in which the ideas of aperture synthesis are performed, have grown in recent years in connection with the necessity of obtaining images of astrophysical objects with a high angular resolution. It is caused by the fact that the present standard of optical technology makes it impossible to create continuous light–weight mirrors with diameter more than 8...10 m.[2–3] But one needs the diameters about 20–25 m for constructing images with a high resolution.[4] Therefore, a possible way to overcome difficulties is the creation of the MOS, in which small separate subapertures are spaced from each other by a distance essentially exceeding their diameters. In spite of a great number of papers published on the problem of creation of the MOS, the question about the methodical synthesis of the optical arrangement of the MOS has not been yet considered, excluding Ref. 5. An attempt to consider this question is undertaken in this paper.

The simplest optical arrangement of an optical device, which forms the image of incoherent objects (independently of the configuration of the aperture window) and explains the principle of MOS operation, is presented in Fig. 1*a*. It is evident that this diagram can be equivalently represented in the form of the diagram (Fig. 1*b*) in which the big–size optical element is replaced by subapertures, which are (in the general case) the asymmetric parts of a big quadratic lens or of the equivalent mirror element (devices for summing the light beams from separate subapertures at a common focus). Here $u(\rho)$ is the field in the object plane and $v(\mathbf{r})$ is the field in the focal plane. This diagram has no practical sense, since the constructions made of asymmetric quadratic lenses (or mirrors) are of low accuracy. However, one can imagine on its base that each asymmetric lens can be replaced by a symmetric one and by an optical element for tilting the wave front (for example, a plane mirror). Thus, any MOS can be represented in the form of classical optical arrangements of the image formation, where the images formed by individual classical subapertures, are summed into a single one at a given plane.

Let us give an exact mathematical description. The field in the focal plane $v(\mathbf{r})$ (Fig. 2) can be represented accurate to unessential constants in the form

$$v(\mathbf{r}) \sim \int\limits_A \int u(\rho) \exp\left(-ik\frac{|\mathbf{r}_A-\rho|^2}{2z}\right) \exp\left(ik\frac{\mathbf{r}_A^2}{2F}\right) \times$$

$$\times \exp\left(-ik\frac{|\mathbf{r}-\mathbf{r}_A|^2}{2L}\right) \mathrm{d}\mathbf{r}_A\, \mathrm{d}\rho\,, \qquad (1)$$

where k is the wave number, F is the focal length of the lens, z is the distance between the object plane and the aperture plane, L is the distance between the aperture plane and the focal plane, $\mathbf{r}_A$ and ρ are the radius–vectors of the aperture plane and the object, respectively.

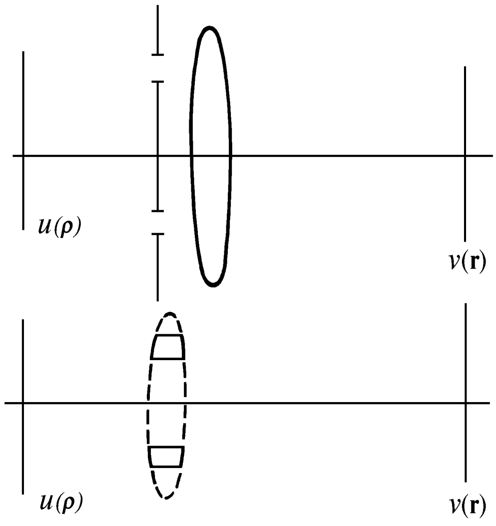

FIG. 1.

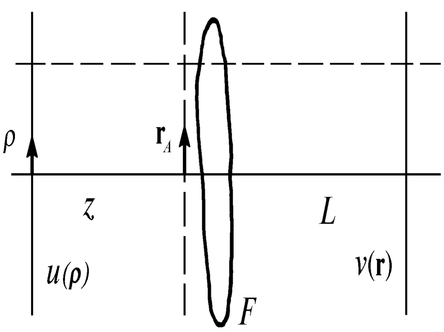

FIG. 2.

In the case of several subapertures (some holes in the screen) the integration over the aperture A is replaced by summing over i apertures (i = 1, ..., n)

$$\nu(\mathbf{r}) = \sum_{i=1}^{n} \int\limits_{Ai}\int u(\rho) \exp\left(-ik\frac{|\mathbf{r}_A - \rho|^2}{2z}\right) \exp\left(ik\frac{\mathbf{r}_A^2}{2F}\right) \times$$

$$\times \exp\left(-ik\frac{|\mathbf{r} - \mathbf{r}_A|^2}{2L}\right) d\mathbf{r}_A\, d\rho\ . \qquad (2)$$

Let us consider the contribution of each subaperture to the total field. To do this it is expedient to pass in the integral over the subaperture to the vectors counted beginning from the optical axis of the ith subaperture

$\rho_i = \rho - \mathbf{R}_i$, $\mathbf{r}_{Ai} = \mathbf{r}_A - \mathbf{R}_i$,

R_i is the radius–vector of the ith subaperture. The origin of the radius–vector is at the center of the subaperture. Then Eq. (2) can be replaced by

$$\nu_i(\mathbf{r}) = \int\limits_{Ai}\int u(\rho_i + \mathbf{R}_i) \exp\left(-ik\frac{|\mathbf{r}_{Ai} - \rho_i|^2}{2z} + ik\frac{|\mathbf{r}_{Ai} - \mathbf{R}_i|^2}{2F} -\right.$$

$$\left. - ik\frac{|\mathbf{r} - \mathbf{R}_i - \mathbf{r}_{Ai}|^2}{2L}\right) d\rho_i\, d\mathbf{r}_{Ai}\ . \qquad (3)$$

An integral over a subaperture is very similar to the field formed by a subaperture with a symmetric lens in the plane at the distance L from the subaperture, excluding the odd, in this case, term $\exp(ik(\mathbf{r}_{Ai}\mathbf{R}_i/F))$. Let us consider the term $\exp(ik(|\mathbf{r}_{Ai} + \mathbf{R}_i|^2/(2F))$ in more detail.

Let us introduce the designation $\mathbf{r} - \mathbf{R}_i = \mathbf{r}_i$. Then one can write down this term as

$$\exp\left(ik\frac{\mathbf{r}_{Ai}^2}{2F}\right) + \exp\left(ik\frac{\mathbf{r}_{Ai}\,\mathbf{R}_i}{F}\right) + \exp\left(ik\frac{\mathrm{R}_i^2}{2F}\right)\ .$$

The third term in this expression is not integrated when substituted into Eq. (3), therefore it can be taken out of the integral sign.

Let us transform the rest exponents in Eq. (3)

$$A = \frac{\mathbf{r}_{Ai}\,\mathbf{R}_i}{F} - \frac{|\mathbf{r}_i - \mathbf{r}_{Ai}|^2}{2L} = \frac{\mathbf{r}_{Ai}\,\mathbf{R}_i}{F} - \frac{\mathbf{r}_i^2}{2L} - \frac{\mathbf{r}_{Ai}^2}{2L} + \frac{2\mathbf{r}_i\,\mathbf{r}_{Ai}}{2L}\ .$$

And finally we derive

$$A = \frac{1}{2L}\left(\frac{L}{F}\mathbf{R}_i + 2\mathbf{r}_i\right)\frac{L}{F}\mathbf{R}_i - \frac{1}{2L}\left|\frac{L}{F}\mathbf{R}_i + \mathbf{r}_i - \mathbf{r}_{Ai}\right|^2 .$$

Substituting this expression into Eq. (2) and removing the unintegrated term from the integrand, we obtain the following expression:

$$\nu(\mathbf{r}) = \sum_{i=1}^{n} \nu_i(\mathbf{r}) \sim \exp\left[ik\left(\frac{1 + \frac{L}{F}}{2F}\mathbf{R}_i^2 + \frac{\mathbf{r}_i\,\mathbf{R}_i}{F}\right)\right] \times$$

$$\times \int\limits_{Ai}\int u(\rho_i + \mathbf{R}_i) \exp\left[-ik\frac{|\mathbf{r}_{Ai} - \mathbf{r}_i|^2}{2z} + ik\frac{\mathbf{r}_{Ai}^2}{2F} -\right.$$

$$\left. - ik\frac{\left|\mathbf{r}_i + \frac{L\,\mathbf{R}_i}{F} - \mathbf{r}_{Ai}\right|^2}{2L}\right] d\rho_i\, d\mathbf{r}_{Ai} =$$

$$= \exp\left[ik\left(\frac{1 + \frac{L}{F}}{2F}\mathbf{R}_i^2 + \frac{\mathbf{r}_i\,\mathbf{R}_i}{F}\right)\right] B. \qquad (4)$$

The expression $B = \nu_i(\mathbf{r}_i)$ is the field formed by a single subaperture with a symmetric quadratic lens with the focal length F. Therefore,

$$\nu(\mathbf{r}) \sim \sum_{i=1}^{n} \exp\left[ik\left(\frac{1 + \frac{L}{F}}{2F}\mathbf{R}_i^2 + \frac{\mathbf{r}_i\,\mathbf{R}_i}{F}\right)\right] \nu_i\left(\mathbf{r}_i + \frac{L}{F}\mathbf{R}_i\right).$$

Assuming $\mathbf{r}_i = \mathbf{r} - \mathbf{R}_i$ and taking into account the condition $\frac{1}{F} = \frac{1}{z} + \frac{1}{L}$, we have

$$\nu(\mathbf{r}) \sim \sum_{i=1}^{n} \exp\left[ik\left(\frac{1 + \frac{L}{F}}{2F}\mathbf{R}_i^2\right) + ik\frac{(\mathbf{r}_i\,\mathbf{R}_i)}{F}\right] \nu_i\left(\mathbf{r} + \frac{L}{F}\mathbf{R}_i\right) =$$

$$= \sum_{i=1}^{n} \exp\left[ik\left(\frac{L\mathbf{R}_i^2}{2zF} + \frac{\mathbf{r}\mathbf{R}_i}{F}\right) \nu_i\left(\mathbf{r} - \frac{L}{z}\mathbf{R}_i\right)\right]. \qquad (5)$$

It can be seen from Eq. (5) that the fields in the focal planes of the subapertures are superposed so that the traces of the optical axes of the subapertures are at the points $\mathbf{r} = L\mathbf{R}_i/z$. The field of each subaperture is multiplied by the phase of the form $\exp(ik\mathbf{r}\mathbf{R}_i/F)$. This can be explained as follows: the optical axes cross the main focal plane of the system at such an angle that the projection of the unit vector of an optical axis on the focal plane is equal to $\mathbf{R}_i/F$. It is natural that $|\mathbf{R}_i|/F < 1$.

Let us consider Fig. 3, where *1* is the main focal plane of the system (the plane of the photodetector position, for example) and *2* is the focal plane of the ith subaperture. The field in the focal plane of the ith subaperture is equal to $\nu_i(\mathbf{r} - \mathbf{R}_i L/z)$ accurate to the phase factor. The spatial distribution of the field in a small vicinity of the focal plane of the subaperture *2* is equal to

$$\nu\left[\left(\mathbf{r} - \mathbf{R}_i\frac{L}{z}\right)_\perp\right] \exp\,(-\,ik\,\mathbf{n}_i\mathbf{r})\ ,$$

where $\left(\mathbf{r} - \mathbf{R}_i\frac{L}{z}\right)_\perp$ is the orthogonal component of the vector $\mathbf{r} - \mathbf{R}_i\frac{L}{z}$ (the component that remains in the plane

2). If the angle between the optical axes that are brought together is small, the orthogonal component only slightly differs from the vector itself and the field in the main focal plane has the form

$$v\left(\mathbf{r} - \mathbf{R}_i \frac{L}{z}\right) \exp\left(- ik\, \mathbf{n}_i \mathbf{r}\right) .$$

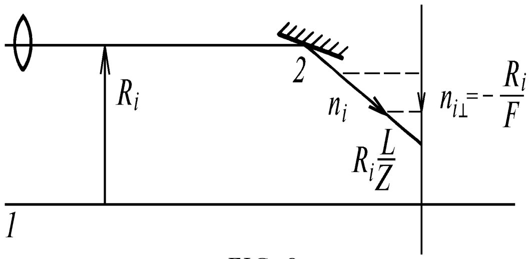

FIG. 3.

By comparing this expression with Eq. (5) it is easy to verify that the projection of the vector $\mathbf{n}_i$ onto the main focal plane must be equal to $|\mathbf{R}_i|/F$. Then we obtain the diagram of bringing the optical axes of the subapertures together. It is shown in Fig. 4. Evidently, all the results presented above are correct in the assumption that $R_i/F \ll 1$.

In conclusion it is necessary to note that the importance of the presence of the phase factor $\exp(ik(LR_i^2/2zF))$ in Eq. (5) is indicative of the necessity of certain equalizing of the lengths of optical paths. This problem is well known for the MOS, a number of papers[5,6] have been devoted to it, therefore it is not included into the subject of this paper. In the particular case of $R_i^2 = \text{const}$ the lengths of optical paths must be merely equal to each other.

The above synthesis of the optical diagram of the MOS makes it possible to consider the peculiarities of the image formation in the MOS from the exact mathematical point of view.

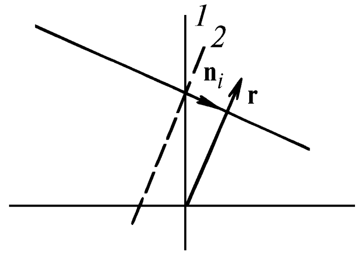

FIG. 4.

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