COMPARISON OF THE SIMPLEX METHOD WITH GRADIENT ALGORITHMS FOR LASER BEAM CONTROL

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Efficiency and speed of correction for thermal blooming with the use of the simplex method, multidither algorithm, and modified multidither algorithm are compared. These algorithms can be considered as methods for finding an extremum of a function, so their characteristics are compared for the problem of finding a maximum of a given analytic function. The conclusions drawn here are verified by way of examination of the beam control in a linear medium (problem of optimal beam focusing) and beam correction for thermal blooming.

1. INTRODUCTION

Because a scheme of an adaptive optics system is determined by an algorithm for control, the primary problem at the initial stage of system design is a choice of the correction algorithm. In the literature, we can find the description of such algorithms as wave front inversion,[1] phase conjugation,[2] multidither algorithm,[3] *a priory* correction,[4] and simplex method.[5]

All the methods listed above suffer from drawbacks, so the field of their application is limited. In particular, the drawback of phase conjugation is instability of correction when compensating for aberrations of intense laser beams,[6] whereas the sharp decrease of the correction speed with the increase of the degrees of freedom of an adaptive mirror should be considered as a drawback of the gradient methods.

Stability specific to the gradient algorithms and high speed of control are characteristic features of the simplex method whose algorithm was developed and compared with conventional methods by Chesnokov et al.[5,7,8] They have demonstrated the advantages of the simplex method by numerical simulation.

In the similar numerical experiments, we obtained slightly different results. In particular, we found out that by optimizing the multidither algorithm its speed can be considerably increased. Furthermore, as a result of the multidither algorithm modification proposed in Ref. 9 an algorithm can be developed with the parameters close to that of the simplex method.

From this short overview it is seen that the problem of the choice of an algorithm for correction of atmospheric aberrations has not yet been adequately solved, so in the present paper we compare some methods of compensation for thermal blooming.

2. MODEL OF AN ADAPTIVE SYSTEM

Schematic representation of an adaptive optics system whose model was used in numerical experiments is shown in Fig. 1.

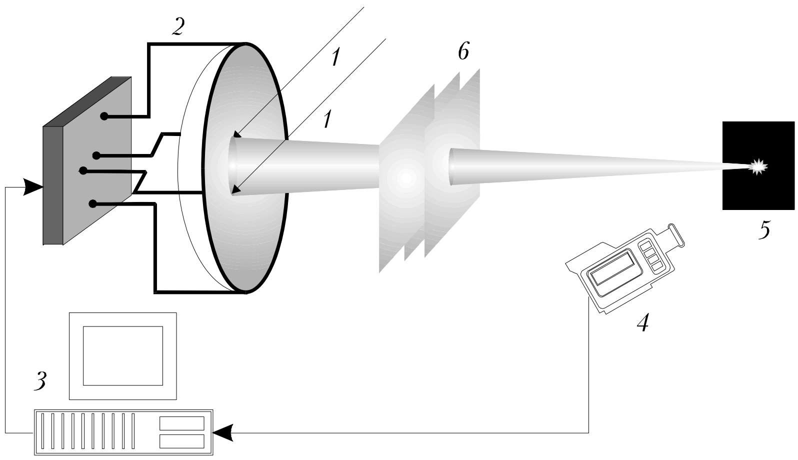

FIG. 1. Block diagram of the adaptive system: laser beam (1), adaptive corrector (2), generator of control signals (3), input system for data characterizing the field distribution (4), object of focusing (5), distributed thermal lens simulated by a set of screens (6).

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In the approximation of stationary refraction, the radiation propagation is described by the formulas [1,2,3]:

$$\begin{cases} 2ik \dfrac{\partial E}{\partial z} = \Delta_{\perp}E + 2 \dfrac{k^2}{n_0} \dfrac{\partial n}{\partial T} TE \ , \\ V \dfrac{\partial T}{\partial x} = \dfrac{\alpha I}{\rho C_p} \ . \end{cases} \quad (1)$$

The first equation describes the propagation of radiation in the parabolic approximation, and the second equation specifies the interaction of radiation with a medium. Here, E is the field complex amplitude, V is the speed of wind (assumed constant for our problem), T is the temperature of the medium, and the other designations are commonly used.

It was assumed that the beam propagated along the OZ axis of the coordinate system, the diffraction length z_d was taken as a spatial scale in the direction of propagation, and the initial radius of the beam in the transverse direction was a_0.

Nonlinear interaction of the beam and the medium is characterized by the parameter

$$R = \frac{2k^2 a_0^2 \alpha I_0}{n_0 \rho C_p V} \frac{\partial n}{\partial T} \ . \quad (2)$$

It should be pointed out that the time variable in its explicit form is absent in the quasistationary approximation used here to describe the interaction. In this approach, a simple numerical model can be applied, but the speed of control can be characterized only indirectly. So for the examined problem, the number of iteration steps required to reach the extremum was used as a parameter characterizing the speed of control (to be more precise, the number of the objective function calls in the process of control).

An adaptive mirror was simulated by a set of the lowest-order Zernike polynomials (tilt, defocusing, and astigmatism, i.e., three degrees of freedom). We also used a model of an elastic plate deformed by four or eight actuators.[10]

3. ALGORITHMS FOR BEAM CONTROL

Multidither algorithm

By this algorithm, a transition from iteration $(n-1)$ to iteration n was made according to the formula

$$\mathbf{F}_n = \mathbf{F}_{n-1} + \alpha \ \mathbf{grad} \ J \ , \quad (3)$$

where $\mathbf{F}_n$ and $\mathbf{F}_{n-1}$ are the vectors of the control coordinates at iteration steps n and $(n - 1)$, J is the objective function, **grad** is the gradient of the objective function calculated with respect to the control coordinates, and α is the gradient step size, which was decreased at unsuccessful iterations, when J was decreased.

When we solve the optical problem, as components of $\mathbf{F}$ we take the coefficients of polynomials or shifts of actuators of the mirror depending on the model of an adaptive corrector. As the objective function J, we take the focusing criterion proportional to the radiation power incident on a receiving aperture with the given radius.

The multidither algorithm is based on the gradient method of finding an extremum (the trial variation method), so with the use of formula (3) we have considered a problem of finding the maximum of an N-dimensional analytic function. In this case, the components of the vector $\mathbf{F}$ are the coordinates of the current point and the criterion J is the value of the function in this point.

Modification of the multidither algorithm

In this algorithm, iterations are performed by the formula[9]

$$x_i^{(n)} = x_i^{(n-1)} + \alpha \ \mathrm{sign}\left(\frac{\partial J}{\partial x_i}\right) , \quad (4)$$

where sign is the signum function. This means that derivatives $\partial J / \partial x_i$ determine only the direction of motion, the displacement at each iteration is completely determined by the parameter α, i.e., iteration steps are identical, and the step size does not depend on the gradient of the function.

Simplex method

In this algorithm, the extremum is approached by successive reflection of the figure called a simplex. In the N-dimensional space, this figure has $(N+1)$ vertices. In 1D case, simplex is a segment of a straight line, in 2D case it is a triangle, and in 3D case it is a tetrahedron.

FIG. 2. Simplex approaching the extremum.

A step from simplex (n–1) to simplex n is made by mirror reflection of the "worstB vertex in a face without this vertex. The center of the simplex approached the extremum as a result of successive reflections of the "worstB vertices. An example of simplex reflections is shown in Fig. 2 for two-dimensional objective function. Here, the "worst" vertex for the first triangle is b, so it reflects in the face $c - a$.

4. DETERMINATION OF THE ALGORITHMIC PARAMETERS TAKING THE PROBLEM OF FINDING THE EXTREMUM OF AN ANALYTIC FUNCTION AS AN EXAMPLE

Prior to an analysis of correction for nonlinear aberrations, let us estimate the speed of the algorithms taking a problem of finding the extremum of the following analytic function:

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \ldots + \frac{x_N^2}{a_N^2} = f^2\,(x_1, x_2, \ldots, x_N) \qquad (5)$$

as an example. A search was made in the domain of positive arguments (search for the maximum). All the algorithms described in Section 3 were considered. The number of the function dimensions was varied from 3 to 8.

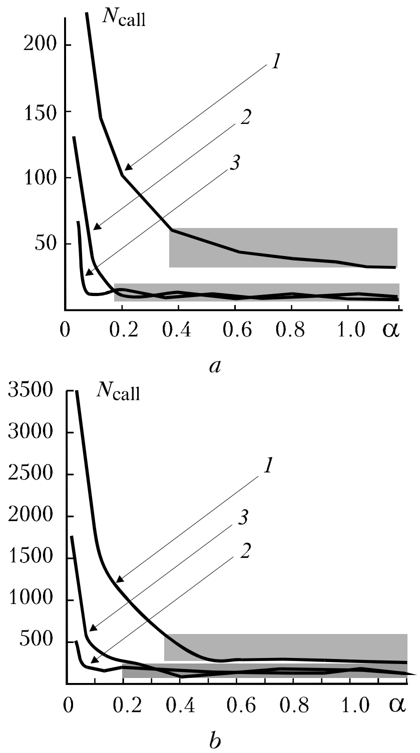

FIG. 3. Dependence of the speed of control on the parameter α (the region of the optimal values of this parameter is hatched on the figure) for 1D (a) and 8D (b) objective functions. Curve 1 is for the trial variation method, curve 2 is for the modified trial variation method, and curve 3 is for the simplex method.

Figure 3 illustrates the dependence of the number of function calls N_{call} on the gradient step size α for 1D and 8D functions. It can be seen that in both cases the behavior of N_{call} is almost the same. In 1D case, the lowest speed is specific to the trial variation method (curve *1* in Fig. 3*a*). The speed of the simplex method is close to that of the modified trial variation method (curves *2* and *3*). At the same time, there is an interval of α where the speed of all algorithms is maximum (but even in this interval the speed of the trial variation method is minimum). This means that to obtain the high speed of control, the gradient step size should be optimized for each algorithm. Optimization is most important for the trial variation method, because for this method N_{call} sharply increases outside the interval of optimal values of α, and the interval itself is narrowest. In the case of the 8D function, the number of objective function calls increases, but all characteristic features, pointed out earlier, remains.

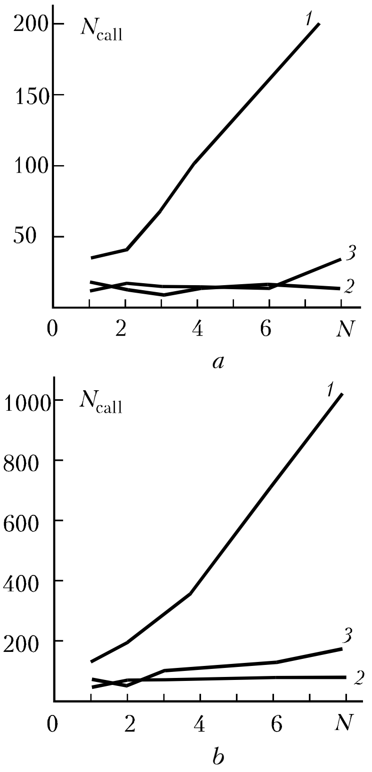

FIG.4. Dependence of the speed of control on the dimension of the basis of control. Numbers of curves and dimensions of the objective functions are the same as in Fig.3.

Dependence of N_{call} on the dimension of function (5) when the number of independent coordinates was varied from 1 to 8 is shown in Fig. 4. Values of α were taken from the optimal interval (Fig. 4*a*). The case of nonoptimal values of the parameter α was also considered (Fig. 4*b*). We can see that the sharpest increase of N_{call} is specific to the trial variation method. The other two algorithms have almost the same speeds of control that decrease slightly when the number of dimensions increases from 1 to 8.

We also considered the dependence of the speed of control on precision of finding the extremum. The results of our analysis are shown in Fig. 5. Figure 5*a* is for optimal values of the parameter α and Fig. 5*b* – for nonoptimal values. In both cases, function (5) was chosen to be eight dimensional.

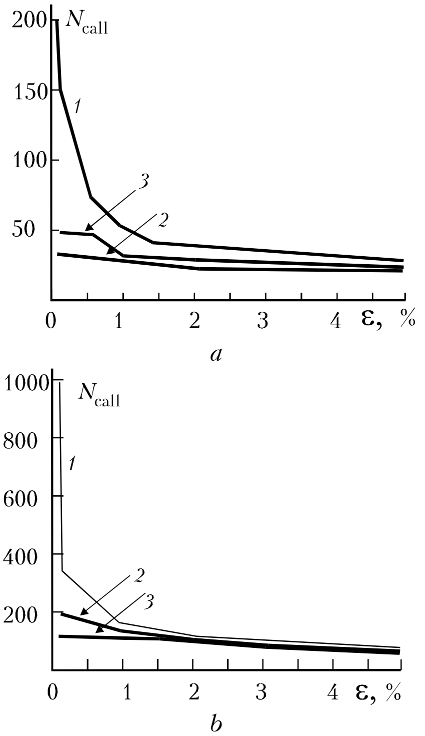

FIG. 5. Dependence of the speed of control on the precision of finding the extremum. Numbers of the curves and dimensions of the objective functions are the same as in Fig.3.

As the precision increases, N_{call} increases most sharply for the trial variation method. The results obtained for the simplex method are close to that for algorithm (4). It should be pointed out here that with the decrease of precision, speeds of algorithms in and out of the interval of optimal values of α differ only slightly. So if the precision of finding the extremum is low, optimization of the gradient step size is not obligatory.

5. CONTROL OF A BEAM IN A LINEAR MEDIUM

Adaptive control of a beam in a linear medium (i.e., when the parameter of nonlinearity R specified by Eq. (2) is equal to zero) means a search for a phase surface with the extremum values of the objective function (in the considered example, the maximum criterion of focusing). A solution of this problem enables us on the one hand, to verify the conclusions drawn in Section 4, on the other, to determine approximately the optimal parameters for beam control in a nonlinear medium.

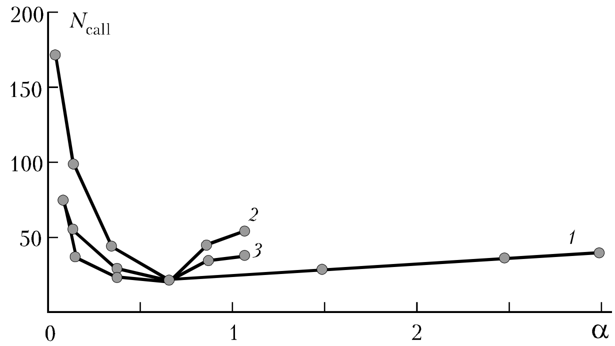

FIG. 6. Speed of control in a linear medium as a function of the parameter α. Curve 1 is for the multidither algorithm, curve 2 is for the modified multidither algorithm, and curve 3 is for the simplex method.

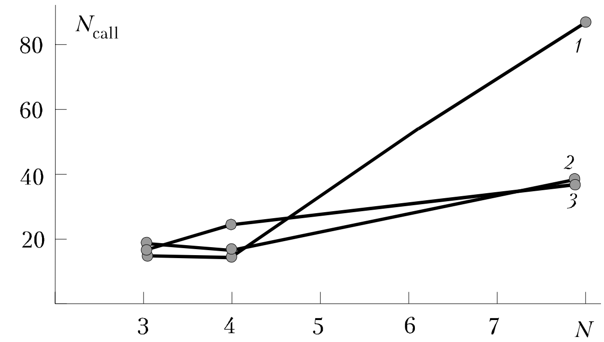

FIG. 7. Speed of control in a linear medium as a function of the number of degrees of freedom of an adaptive corrector. Numbers of curves are the same as in Fig. 6.

In Figs. 6 and 7, the dependence is illustrated of the number of function calls on the parameter α and on the dimension of the basis of control. As in the previous case (finding of the extremum of an analytic function), we can identify the interval of optimal values of α where the algorithms have approximately equal speeds of control. When $\alpha \neq \alpha_{opt}$, N_{call} increases sharply, and the lowest speed of control has the multidither algorithm.

As the number of dimensions of the basis of control increases, the sharpest increase of the number of calls is observed once more for the multidither algorithm (see Fig. 7).

The other characteristics are illustrated by the results presented in Table I. The data are given for the simplex method. The multidither and modified multidither algorithms have almost the same parameters.

We can see that the efficiency of control depends on the type of employed adaptive corrector. For example, for a mirror with four actuators, the criterion J is equal to 0.64 and for a set of Zernike polynomials, $J = 0.79$. The next peculiarity is the dependence of J on

α, which manifests itself when we use the adaptive mirror. In the case of the mirror with four actuators when we go from nonoptimal to optimal values of α, the criterion J increases from 0.57 to 0.64 (by 10%). The increase for the mirror with eight actuators is 21% (from 0.51 to 0.66).

TABLE I. The results of the simplex method application for beam control in a linear medium.

Corrector model: a set of polynomials							
α	0.05	0.1	0.3	0.5	0.7	0.9	1.1
N_{call}	76	41	27	28	26*	34	31
J	0.79	0.79	0.79	0.79	0.79	0.79	0.79
Corrector model: a mirror with 4 actuators							
α	0.2	0.5	1	5	15	18	20
N_{call}	80	63	41	26*	33	21	37
J	0.57	0.63	0.64	0.64	0.64	0.64	0.64
Corrector model: a mirror with 8 actuators							
α	0.1	0.2	0.5	1	5	10	12
N_{call}	–	68	62	24*	36	51	–
J	–	0.51	0.55	0.54	0.65	0.66	–

* Minimum number of the objective function calls

To explain this peculiarity, we should consider the condition of the control termination. Mathematically, the algorithms are organized so that the control is terminated when the increment to the objective function between iterations (n–1) and n is less than or equals to ε (that is, the objective function does not increase). So if the parameter α is nonoptimal and the objective function is flat topped, the rate of growth of J is slowed down with the increase of the iteration order as the extremum is approached. When $\Delta J < \varepsilon$, the control is terminated notwithstanding the fact that the extremum has not yet been reached. It seems that this reason explains the decrease of the criterion J when the parameter α is nonoptimal.

6. CORRECTION FOR STATIONARY ABERRATIONS OF LASER BEAMS

The main features of control in the linear medium are retained when a thermal lens develops on a beam propagation path (the data are presented in Table II). But the resulting values of the criterion of focusing are slightly lower than in the linear medium (i.e., correction is incomplete), and the number of function calls is greater. As in the previous case, it is possible to identify the interval of optimal gradient step size and regions where the parameter α is nonoptimal. When $\alpha = \alpha_{opt}$, the simplex method and the modified multidither algorithm have almost the same speeds of control. Out of this interval, the speed of correction decreases sharply.

For $\alpha = \alpha_{opt}$, the highest speed of control was obtained for the multidither algorithm. Using this method, only 28 calls of the objective function were needed to ascend the hump (see the fourth column of Table II). For other algorithms, N_{call} is almost twice greater than for this method. But the resulting values of J for the multidither algorithm are lower, i.e., it is possible to assume that for the given step size the hump was not reached and the control was terminated according to the condition of interruption. If the step size increases, the values of J also increase, but the speed of control decreases.

TABLE II. Parameters of algorithms intended for correction of thermal blooming. The active element is a mirror with 8 actuators, R = –20.

Simplex method				
α	0.5	2	3	5
N_{call}	75	48	43*	53
J	0.37	0.38	0.40	0.42
Modified multidither algorithm				
α	0.1	1	2	2.5
N_{call}	118	55	46*	55
J	0.35	0.40	0.40	0.39
Multidither algorithm				
α	20	40	60	140
N_{call}	37	28	37*	91
J	0.37	0.37	0.38	0.39

* Minimum number of the objective function calls

In conclusion, let us list the main results once more.

1. Efficiency of correction for stationary thermal blooming is almost the same for all algorithms.
2. The speed of control depends on the gradient step size (or on the length of the edge of the simplex).
3. In the region of optimal parameters, all algorithms have approximately the same speed of control.

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