DIVERGENCE OF A LASER BEAM IN A REGULAR NONLINEARLY REFRACTIVE MEDIUM

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Received July31, 1991

Based on the technique of effective beam parameters are found the conditions of existence of the exact aberration solutions for the effective width, radius of phase front curvature, and limiting divergence. The effect of similarity of processes under the strong nonlinear distortions for different classes of a collimated beam self–action and mechanisms of nonlinear interaction are revealed. Regimes for formation of the limiting divergence in the primarily homogeneous and inhomogeneous nonlinear refractive media are studied. Relationships between the parameters of inhomogeneous path and the initial parameters of laser beams are determined for the case of weak nonlinear distortions.

Divergence is one of the main characteristics of laser sources.[1] It can be transformed considerably due to the self–action effects at the radiation propagation through nonlinear media.[4,5] Almost all results of the previous studies concerning this problem were obtained based on the aberrationless approximation.[2,3] However, the beam aberration distortions play a principle role in nonlinear interactions and their account can be performed only using more rigourous consideration of the problem. To do this, a technique based on the equations for an effective beam parameters is developed. Therefore, obtaining the quantitative relationships connecting the angular characteristics of laser radiation in a nonlinear medium with beam parameters and characteristics of regular medium inhomogeneities along the path as well as studying the regimes of formation a radiation directional pattern can be of certain interest. This paper deals with the nonlinear refractive effects in the regular homogeneous and inhomogeneous media with linear absorption.

1. Beam divergence can be introduced as an effective width of the radiation angular spectra (directional diagram)

$$\theta_e = \left[\frac{k^{-2}\displaystyle\iint_{-\infty}^{\infty} \mathbf{k}^2 G(\mathbf{k}, z, t)\, d^2\mathbf{k}}{\displaystyle\iint_{-\infty}^{\infty} G(\mathbf{k}, z, t)\, d^2\mathbf{k}}\right]^{1/2}, \tag{1}$$

where $G(\kappa, z, t) = |E_\kappa|^2$ is the laser radiation angular spectrum; $|E_\kappa| = (2\pi)^{-2} \displaystyle\iint_{-\infty}^{-\infty} E(\mathbf{R}, z, t) \exp(i\kappa\mathbf{R})\, d^2R$ is the Fourier transform of the complex field amplitude E; z and $\mathbf{R} = (x, y)$ are the longitudinal and transverse coordinates; t is time; and k is the wavenumber in the medium. If $E = A\exp(i\varphi)$, where A and φ are the real amplitude and phase of the wave, then Eq. (1) results in

$$\theta_e^2(z, t) = \frac{1}{k^2\rho_e^2(z, t)} + \frac{R_e^2(z, t)}{F_e^2(z, t)}. \tag{2}$$

In Eq. (2) $R_e^2 = \dfrac{1}{P}\displaystyle\iint_{-\infty}^{-\infty} R^2 I(\mathbf{R}, z, t)\, d^2R$ is the squared effective beam radius; I and P are the radiation intensity and power, respectively; the coherence scale

$$\rho_e = \left[k^{-2}\iint_{-\infty}^{\infty} (\nabla_\mathbf{R} A)^2 d^2R \Big/ \iint_{-\infty}^{\infty} I(\mathbf{R}, z, t)\, d^2R\right]^{-1/2} \tag{3}$$

characterizes the beam diffraction peculiarities in the nonlinear medium, and the scale

$$F_e = kR_e \times \left[\iint_{-\infty}^{\infty} (\nabla_\mathbf{R}\varphi)^2 I(\mathbf{R}, z, t)\, d^2R \Big/ \iint_{-\infty}^{\infty} I(\mathbf{R}, z, t)\, d^2R\right]^{-1/2} \tag{4}$$

has a sense of the effective radius of the beam phase front curvature.

Divergence of a beam in a nonlinearly refractive medium is related to the other effective beam parameters through the system of equations,[4] which can be represented in the form

$$\frac{dR_e^2}{dz} = 2\frac{R_e^2}{F_{e1}}, \tag{5a}$$

$$\frac{d}{dz}\frac{R_e^2}{F_{e1}} = \theta_e^2 + \frac{1}{2P(z)}\iint_{-\infty}^{\infty} \mathbf{R}\nabla_\mathbf{R}\tilde{\varepsilon}\, I\, d^2R, \tag{5b}$$

$$\frac{d\theta_e^2}{dz} = \frac{k^{-1}}{P(z)}\iint_{-\infty}^{\infty} \nabla_\mathbf{R}\tilde{\varepsilon}\nabla_\mathbf{R}\varphi I\, d^2R, \tag{5c}$$

$$\frac{dP}{dz} = -\alpha(z)\, P\,, \tag{5d}$$

where $\tilde{\varepsilon} = \tilde{\varepsilon}(I)$ is the dielectric constant perturbation and α is the absorption coefficient of the medium. The scale

$$F_{e1} = kR_e^2 \times$$

$$\times \int_{-\infty}^{\infty}\!\!\int I(\mathbf{R}, z, t)\, d^2R \Big/ \int_{-\infty}^{\infty}\!\!\int \mathbf{R}\nabla_{\mathbf{R}}\, \varphi I(\mathbf{R}, z, t)\, d^2R \tag{6}$$

as well as F_e determines the weighted average radius of the beam phase front curvature. The difference between the scales F_e and $|F_{e1}|$ indicates the presence of aberrations in the wavefront. Coincidence of the scales is possible when the phase is a square–law characteristic of **R** near the beam centre of gravity that corresponds to the so–called aberrationless approximation. In the most general case, according to definitions (4) and (6), one can show that $F_e \le |F_{e1}|$. The sign of the scale F_{e1} indicates either focusing ($F_{e1} < 0$) or defocusing ($F_{e1} > 0$) of a beam.

If the beam is not axially symmetric, its centre of gravity can lie off the beam axis. The radius–vector of the beam centre of gravity displacement $\mathbf{R}_c = \frac{1}{P}\int_{-\infty}^{-\infty}\!\!\int \mathbf{R}I(\mathbf{R}, z, t)\, d^2R$ is defined by the equation

$$\frac{d^2\mathbf{R}_c}{dz^2} = \frac{1}{2P(z)}\int_{-\infty}^{\infty}\!\!\int \nabla_{\mathbf{R}}\tilde{\varepsilon}\; I\, d^2R\,, \tag{5e}$$

As follows from Eq. (2) the limiting beam divergence, i.e., θ_e in the long range zone is

$$\theta_{e\infty} = \frac{R_e(\infty)}{F_e(\infty)} = \left(\frac{1}{2}\frac{d^2R_e^2(\infty)}{dz^2}\right)^{1/2}. \tag{7}$$

In accordance with Eq. (5c) the nonlinear component $\theta_{n\infty} = |\theta_{e\infty}^2 - \theta_e^2(0)|^{1/2}$ of the limiting divergence of the beam is

$$\theta_{e\infty} = \left[\frac{k^{-1}}{P}\int_0^{\infty} dz \int_{-\infty}^{\infty}\!\!\int \nabla_{\mathbf{R}}\tilde{\varepsilon}\nabla_{\mathbf{R}}\varphi I\, d^2R\right]^{1/2}. \tag{8}$$

In the general case this characteristic can be sought after only from numerical solution of the problem on the beam propagation through a nonlinear medium.

2. A salient feature of Eq. (5) is that in Eqs. (5b) and (5c) there are two additional unknown functions which are the integral characteristics of the radiation and medium. This peculiarity is entirely associated with the aberration character of the beam self–action and disappears in the aberrationless approach to solution of the problem.[3] However, there exists a class of practical problems for which the above difficulties can be overcome in the aberration case. Some of these problems will be considered below.

Let us define the conditions under which Eq. (5) can be solved. To do this, let us pass from the system of three differential equations to an equation of the third order for R_e^2

$$\frac{d^3R_e^2}{dz^3} = \frac{2k^{-1}}{P}\int_{-\infty}^{\infty}\!\!\int \nabla_{\mathbf{R}}\,\tilde{\varepsilon}\;\nabla_{\mathbf{R}}\varphi I\, d^2R +$$

$$+\frac{d}{dz}\left[\frac{1}{P}\int_{-\infty}^{\infty}\!\!\int \mathbf{R}\nabla_{\mathbf{R}}\tilde{\varepsilon} I\, d^2R\right] = \Phi(z)\,. \tag{9}$$

It is obvious that the first integral in Eq. (9) can be sought after in terms of quadrature if the right side of this equation (function $\Phi(z)$) is known. The function $\Phi(z)$ can be determined for a number of cases without solving of the wave equation.

It appears that for a nonlinear medium of the Kerr type, where $\tilde{\varepsilon} = \varepsilon_2 I$, the self–action always results in $\Phi(z) = 0$. It is equivalent to the existence of the invariant

$$\theta_e^2(z, t) + \frac{1}{2P(z)} \times$$

$$\times \int_{-\infty}^{\infty}\!\!\int \mathbf{R}\nabla_{\mathbf{R}}\tilde{\varepsilon}\,(\mathbf{R}, z, t)\, I(\mathbf{R}, z, t)\, d^2R = \text{const}. \tag{10}$$

From Eq. (10) it follows that

$$R_e^2(z) = R_e^2(0) + \frac{dR_e^2(0)}{dz}\, z + \frac{1}{2}\frac{d^2R_e^2(0)}{dz^2}\, z^2\,; \tag{11}$$

$$F_{e1}(z) = \frac{2R_e^2(z)}{\left(\dfrac{dR_e^2(0)}{dz} + \dfrac{d^2R_e^2(0)}{dz^2}\, z\right)}. \tag{12}$$

The solution for the beam divergence in a Kerr–type medium can be found for its limiting value alone

$$\theta_e^2(\infty) = \frac{1}{2}\frac{d^2R_e^2(0)}{dz^2} = \theta_e^2(0) - \frac{\varepsilon_2}{2P}\int_{-\infty}^{\infty}\!\!\int I^2(0)\, d^2R\,. \tag{13}$$

Having determined, according to Ref. 4, the length of the refractive nonlinearity

$$L_n = R_e(())\left[\frac{1}{2P(0)}\left|\int_{-\infty}^{\infty}\!\!\int \mathbf{R}\nabla_{\mathbf{R}}\tilde{\varepsilon}\,(z{=}0)\, I\,(z{=}0)\, d^2R\right|\right]^{-1/2}, \tag{14}$$

we can represent relation (13) in the form $\theta_e^2(\infty) = \theta_e^2(0) \le \theta_n^2$, where

$$\theta_n = R_e(0)/L_n\,. \tag{15}$$

In the case of focusing media ($\varepsilon_2 > 0$) Eq. 15 is valid at $\theta_e(0) \le \theta_n$. One can see from Eq. (13) that the law of radiation propagation through a Kerr–type nonlinear medium agrees with the analogous law for a linear medium but with different diffraction–limited divergence. It should be noted that solution (11) was first obtained in Ref. 9 by different method. The solutions for R_e^2 and F_{e1} similar to Eq. (11) and (12) can be obtained for the case of arbitrary nonlinearity but only for short propagation distances $z \ll L_n$ when the relation $\Phi(z) \cong 0$ is valid.

3. The problem concerning the Kerr nonlinearity is useful for the analysis of behavior of self–action of a beam having an arbitrary intensity profile. Similarity in the behavior of the relative effective radius $R_e(z)/R_e(0)$ of beams having different profiles takes place in the Kerr defocusing medium under conditions of strong nonlinear distortions (the parameter of nonlinearity $P^2 = L_d^2/L_n^2 \gg 1$, where L_d is the beam diffractional length).

The similarity means that for a beam with a plane phase front the ratio $R_e(z)/R_e(0)$ depends only on the dimensionless parameter of nonlinearity distortions z/L_n and the form of this dependence is the same for all types of beams:

$$\frac{R_e^2(z)}{R_e^2(0)} \cong 1 + \frac{z^2}{L_n^2}\,. \tag{16}$$

This similarity property is characteristic of other types of nonlinear media as well. It takes place under conditions of strong nonlinearity ($P^2 \gg 1$) at short ($z < L_n$) and long ($z \gg L_n$) distances. It happens so that in the both first and second cases the solution for the effective parameters has a form as in the case with the Kerr–type media. Moreover, in the second case the limiting beam divergence is being formed in the medium at distances less than the propagation length.

As was noted above, in the majority of cases the limiting divergence can be calculated numerically. Therefore, let us analyze the results of calculations using the data from the papers in which self–action of beams of different intensity profiles under conditions of stable wind nonlinearity was studied.[10,11]

The calculations were carried out for the beam having the intensity upon entrance into the medium given by

$$I(\mathbf{R},\,0) = I_0(m)\,\exp\left[-\left(\frac{x}{R_0}\right)^m - \left(\frac{y}{R_0}\right)^m\right], \tag{17}$$

and $R_e^2(0) = 2R_0^2\,\Gamma(1/m)\,\Gamma(3/m)$,

$$L_n = \left[\frac{c_p\rho\nu}{|\partial\tilde{\varepsilon}/\partial T|_p\alpha}\,\frac{32R_0^2 2^{1/m}\Gamma(3/m)}{m^2P(0)}\right]^{1/2},$$

where $m = 2, 4, 6, \ldots$ is the parameter determining the beam shape; $I_0(m)$ is the maximum intensity which is being chosen depending on the beam shape, so that the initial power of beams is one and the same, R_0 is the initial radius of a Gaussian beam at the level e^{-1}; Γ is the gamma–function; c_p and ρ are the isobar heat capacity and density of the medium, respectively; υ is the transverse component of the wind velocity; and, $(\partial\tilde{\varepsilon}/\partial T)_p$ is the isobar derivative of the dielectric constant with respect to temperature.

Figure 1 shows the views of the function $\eta = \dfrac{(R_e^2(z) - R_c^2(z) - R_e^2(0))^{1/2}}{R_e(0)}\dfrac{L_n}{z}$ depending on the distortion parameter $(z/L_n)^2$ values. The circles correspond to the beams with the different intensity profiles. Since the propagation channel in the problem concerning the wind nonlinearity is not axially symmetric, it is worth saying about the beam divergence near the direction of propagation of the beam gravity centre $\theta_c = \mathrm{d}\mathbf{R}_c/\mathrm{d}z$, i.e., about the value $\tilde{\theta}_e = (\theta_e^2 - \theta_c^2)^{1/2}$. The relative radius $(R - R)^{1/2}$ is determined only by the relative limiting divergence at long distances ($z > L_n$) in the case of a strong nonlinearity[4]

$$(R_e^2 - R_c^2)^{1/2} \cong [\theta_e^2(\infty) - \theta_c^2(\infty)]^{1/2}z\,. \tag{18}$$

As a consequence, at $z > L_n$ the function η must behave as $\eta = \tilde{\theta}_{n\infty}L_n/R_e(0)$, where $\tilde{\theta}_{n1\infty} = (\theta_{e1}^2(\infty) - \theta_e^2(0))^{1/2}$ is the nonlinear component of the relative limiting divergence

$$\frac{(R_e^2(z) - R_c^2(z) - R_e^2(0))^{1/2}}{R_e^2(0)} \cong \frac{z^2}{L_n^2} \text{ and } \eta \cong 1.$$

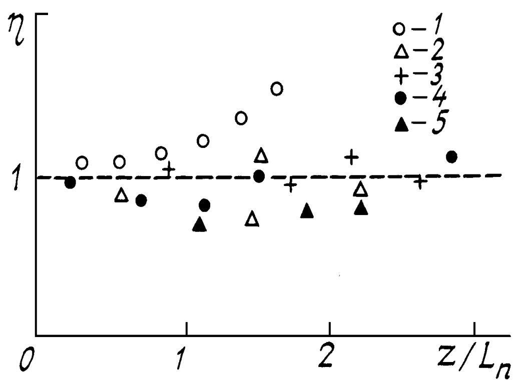

FIG 1. Formation of the relative limiting divergence of the beam with different intensity profiles (17) in the nonlinear medium with the stationary wind flow for $m = 2$ *(1, 4), 4 (2, 5), 6 (3); (1–3) according to the data from Ref. 10, (4,5) according to the data from Ref. 11.*

Since the dependence $\eta(z/L_n)$ for $z>L_n$ has the same form for all types of beams this is indicative of similarity in the character of their self–action. At the same time one can see that the value η calculated numerically is sufficiently close to $R_e(0)/L_n$ for all types of the beams.

The above facts show that the qualitative analysis of the problem is possible even if the exact solution for the effective beam parameters cannot be found. Such an analysis allows one to calculate the aberration scales of the self–action problem and to generalize the results of numerical experiments. Note that it cannot be done based on the analysis of the equation of quasioptics only and using the definitions of the effective beam parameters.

4. The above–considered case of forming of the limiting beam divergence in a medium with strong distortions belongs to a practically important class of the self–action problems, which are associated with the radiation propagation through a nonlinearly refractive layer. For this class the exact solutions of Eqs. (5) also exist but beyond the limits of the nonlinear layer ($\Phi(z) = 0$ at $z > z^*$). The point z^* we shall call the boundary of the nonlinear layer. In accordance with Eq. (9) the value z^* determines the range at which the limiting nonlinear beam divergence $\theta_n(z^*) = \theta_{n\infty} = \mathrm{const}$ is formed. A nonlinear refractive layer can be formed due to inhomogeneity of the propagation channel caused by variations in the thermodynamic parameters and concentrations of gases along the path as well as by the self–action leading to decrease of the nonlinearity inside the medium because of the beam defocusing. The vertical and slant paths in the atmosphere[5-8] and the paths in laboratory experiments

where the radiation is received beyond the nonlinear medium are examples of such inhomogeneous paths.

The solution for the effective beamwidth beyond the nonlinear layer can be written as

$$R_e^2(z) = R_e^{*2}\left[\left(1 + \frac{z - z^*}{F_{e1}}\right)^2 + \right.$$

$$\left. + \frac{(z - z^*)^2}{k^2 r_e^{*2} R_e^{*2}} + \delta^*(z - z^*)^2\right], \quad (19)$$

where $\delta^* = \frac{1}{F_e^{*2}} - \frac{1}{F_{e1}^{*2}} \geq 0$ is the characteristic of aberrations and the values on the nonlinear layer boundary are marked by asterisk.

From Eq. (19) it follows that the structure of solutions for R_e^2 in the aberration ($\delta > 0$) and aberrationless ($\delta = 0$) cases are always different. Therefore, the conclusions one can arrive at based on the aberrationless approach to analysis of the problem on transformations of the effective beam parameters are not always versatile.

In the region of $z \gg z^*$, $|F_{e1}^*|$ the effective beam radius is determined only by the limiting beam divergence. Let us now analyze the situation when the radiation propagating through a nonlinear layer undergoes weak amplitude distortions. It takes place when $L_n \gg z^*$. In a practical case of a wide beam self–action ($z^* \ll L_d$) the phase gradient in Eq. (8) can be written in the geometrical optics approximation:

$$\nabla_{\mathbf{R}}\, \varphi = \frac{k}{2} \int_0^z \nabla_{\mathbf{R}} \tilde{\varepsilon}(z)\, dz\,, \quad (20)$$

and it is quite sufficient to calculate the parameters of a nonlinear lens appearing in the medium under the regime of weak distortions ($z^* < L_n$) using a fixed field approximation when $I(z < z^*) = I(0)$. It should be noted that the perturbation of the medium dielectric constant can be represented as

$$\tilde{\varepsilon}(\mathbf{R}, z) = \tilde{\varepsilon}_m(z)\,\overline{\varepsilon}(\mathbf{R})$$

for the majority of the mechanisms of optical nonlinearity.

Here $\tilde{\varepsilon}_m(z)$ is determined by the medium properties along a path and by the characteristic parameters of a beam, while $\tilde{\varepsilon}(R)$ is determined by the type of radiation source and by the beam intensity profile. Taking into account these remarks one obtains for the effective radius of the phase front at the boundary of a nonlinear layer the following relation

$$F_{e1}^* \cong R_e^2(0) \left(\int_0^{z^*} \tilde{\varepsilon}(z)\, dz\right)^{-1} \times$$

$$\times \frac{1}{2P(0)} \int_{-\infty}^{\infty}\!\!\int \mathbf{R}\nabla_{\mathbf{R}}\tilde{\varepsilon}\,(\mathbf{R}, 0) I(\mathbf{R}, 0)\, d^2R\,. \quad (21)$$

Relation (21) can be reduced to the form

$$F_{e1}^* \cong \tilde{L}_n^2 / L_{\text{eff}}\,, \quad (22)$$

where

$$L_{\text{eff}} = \int_{-\infty}^{\infty} (\tilde{\varepsilon}_m(\mathbf{z})/\tilde{\varepsilon}_m(0))\, dz \quad (23)$$

is the scale characterising the longitudinal inhomogeneity of the propagation channel.

By analogy, for the effective radius of curvature (4) we have

$$F_e^* \cong \tilde{L}_n^2 / L_{\text{eff}}\,, \quad (24)$$

where the scale

$$\tilde{L}_n = R_e(0) \left| \frac{1}{4P(0)} \int_{-\infty}^{\infty}\!\!\int (\nabla_{\mathbf{R}}\tilde{\varepsilon})^2\, I(\mathbf{R}, 0)\, d^2R \right| \quad (25)$$

has the meaning of a refraction nonlinearity length like L_n but it differs from L_n because of the aberrations of the wavefront.

The nonlinear component of the laser beam limiting divergence (8) on an inhomogeneous path in the case of weak distortions has the form

$$\theta_{n\infty} = \theta_n^* = \frac{R_e(0)\, L_{\text{eff}}}{\tilde{L}_n^2}\,. \quad (26)$$

Under conditions of strong nonlinear distortions the transformation of the effective beam parameters will occur in the same way as for beams propagating through a homogeneous medium, when $\theta_{n\infty} \cong R_e(0)/L_n$.

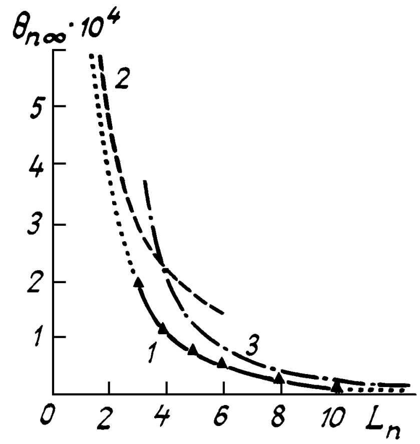

FIG. 2. Nonlinear component of limiting beam divergence as a function of nonlinearity length at self–action of a long laser pulse propagating along a vertical path in the atmosphere; Curve 1 (solid) presents the data calculated by radiation transfer equation[5], Curve 2 is the asymptotic for the case of strong nonlinear distortions (15), and Curve 3 is the asymptotic for the case of weak nonlinear distortions (26). Close triangles show the data calculated by approximation formula (27).

Thus two limiting regimes of the effective beam parameters transformation are possible in an inhomogeneous nonlinearly refractive medium. These are the regimes of weak and strong nonlinear distortions. The limiting divergence of a beam in the regime of weak nonlinear distortions makes approximately an L_{eff}/L_n fraction of the same value for the case of strong distortions.

The above–noted peculiarities of the limiting nonlinear divergence of a beam are illustrated by an example of the problem dealing with thermal blooming of a Gaussian beam propagating along a vertical path in the atmosphere. The asymptotic curves (15) and (26) for the nonlinear component of the limiting divergence are shown in Fig. (2). The results of numerical calculations for the corresponding problem[5] are also presented in this figure. The calculations have been done for a long pulse with $\lambda = 10.6$ μm and the diffraction length $L_d = 592$ km. The distribution of thermodynamical parameters of the medium along the path corresponds to the summer mid–latitude model of the atmosphere. In this case $L_{eff} = 3.24$ km. Since in Ref. 5 as well as in other papers well known to authors no solutions that exactly correspond to the regimes of weak ($L_n < L_{eff}$) and strong ($L_n > L_{eff}$) nonlinear distortions have been obtained we made an extrapolation of the calculational data to these regimes. It was revealed that the function $\theta_{n\infty}(L_n)$ is close to the dependence $R_e(0)/L_n$ in the case of strong distortions though the extrapolated values of $\theta_{n\infty}$ are lower (for example when $L_n = 2$ km and $\theta_{n\infty} = 0.85\, R(0)/L_n$). In the range ($L_n > L_{eff}$) the solution has a tendency to approach to asymptotics for weak nonlinear distortions (26). In the range of moderate distortions ($L_n \cong 1$–$3\, L_{eff}$) the calculated dependence satisfies the approximation formula

$$\theta_{n\infty} = \frac{R_e(0)}{L_n}\frac{L_1}{L_{\text{n}}} \quad \text{where} \quad L_1^{-1} = L_n^{-1} + L_{\text{eff}}^{-1}\,. \tag{27}$$

The laser beam self–action processes occurring on an inhomogeneous path under the action of other nonlinear interaction mechanisms can be described quantitatively in a similar way.

In conclusion we should like to briefly summarize the main results obtained in this paper. The conditions under which the exact solutions for the effective width, effective radius of the phase front curvature and limiting divergence of a laser beam have been determined and the form of these solutions derived. It was revealed that in the case of an intense laser radiation propagating through an inhomogeneously refractive medium a nonlinear layer is being formed where the limiting directional pattern of the beam is formed. In the regime of strong distortions for initially inhomogeneous path the limiting divergence is equal to its value for a homogeneous path. But it depends on the scale of the path inhomogeneity in the weak distortions regime. The formation of a nonlinear layer causes the similarity effect in the behavior of the relative effective radius under the self–action of collimated beams in the regime of strong nonlinear distortions.

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