ON THE ACCURACY OF THE DOPPLER METHOD IN RADIOACOUSTIC SOUNDING OF THE ATMOSPHERE WITH SHORT-WAVE RADIO WAVES

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Frequency measurements performed during radioacoustic sounding of the troposphere using low-frequency sound and decameter radio waves are analyzed. An expression is derived for the power spectrum of the received signal, when a periodic sequence of electromagnetic pulses is scattered by from a single sound pulse, at the point of compensation of wind drift taking into account atmospheric turbulence. It is shown that to achieve a prescribed accuracy in measurements of the temperature of the atmosphere it is necessary to construct antennas with narrow directional patterns.

The method of radioacoustic sounding (RAS) has been widely used in the last few years for contact-free diagnostics of the atmosphere.[1,2] Until recently, however, it- has been assumed that this method is applicable only up to altitudes of the order of 1 ... 3 km (Ref. 1). The fundamental and technical possibilities of developing stratospheric-tropospheric radioacoustic sounding (STRAS) systems are discussed in Refs. 3 and 4. Experimental data on radioacoustic sounding of the atmosphere up to altitudes of 15 ... 20 km are presented in Ref. 5. In Refs. 4 and 5 it is shown that in order to implement STRAS it is necessary to use short-wave radio waves (the optimal wavelengths are $\lambda_s \sim 5$ m, $\lambda_e \sim 10$ m, where λ_s and λ_e are the wavelengths of the acoustic and electromagnetic waves, respectively), the power of the acoustic and electromagnetic radiations must be quite high, and the wind drift must be compensated.

It is well known that in the case of RAS of the atmosphere information about local properties of the medium is obtained from frequency measurements of the received signal. There are a number of works on the frequency spectrum of the scattered signal obtained by RAS of the atmosphere.[1,2,6] The purpose of this paper is to study the frequency measurements performed in STRAS.

We shall employ the following expression for single scattering of an electromagnetic wave by nonuniformities of the dielectric constant[2]

$$E_{sc}(r_0, t) = -\frac{1}{4\pi c^2}\cdot\frac{\partial^2}{\partial t^2}\int\limits_{V_{sc}} \varepsilon_s\left[r', t - \frac{|r_0 - r'|}{c}\right] \times \frac{E_0\left[r', t - \frac{|r_0 - r'|}{c}\right]d^3r'}{|r_0 - r'|}. \tag{1}$$

Here $E_{sc}(r_0, t)$ is the scattered field at the point r_0 at the time t, $\varepsilon_s(r, t)$ is the perturbation of the dielectric constant of the medium by the acoustic field, $E_0(r, t)$ is the undisturbed electromagnetic field, V_{sc} is the scattering volume, and c is the velocity of light. We shall study the interaction of an electromagnetic wave with an acoustic wave in the far zone, and we shall place the transmitting antennas at the origin. We shall assume that the electromagnetic transmitter transmits a periodic sequence of pulses with a repetition frequency ω_r. This makes it possible to solve the problem of decoupling the transmitting and receiving antennas.

To simplify the calculation we shall study the directional patterns of antennas and Gaussian envelopes of the acoustic and electromagnetic pulses. The z axis is directed upwards. Under these assumptions the expression for $E_0(r, t)$ can be written in the following form:

$$E_0(z, \rho, t) = \frac{A_e}{\left[1 + iz/ka_e^2\right]}\cdot\exp\left\{-i\omega t + \frac{ik\rho^2}{2z} - \frac{\rho^2}{2(z\theta_e)^2}\right\}\sum_{n=-\infty}^{+\infty}\exp\left\{-\left[z - \left[t - t_0 - n\frac{2\pi}{\omega_r}\right]\cdot c\right]^2/2l_e^2\right\}, \tag{2}$$

where Ω and k are the frequency and wave vector of the electromagnetic wave; θ_e is the width of the directional pattern of the electromagnetic antenna; a_e is the aperture of the antenna; and, l_e is the spatial scale of the envelope of the electromagnetic pulse.

Under the assumption that the wind blows across the path at a constant velocity V along the path, we obtain for $\varepsilon_s(z, \rho, t)$ (Ref. 1)

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$$\varepsilon_0(z,\ \rho,\ t) =$$

$$= \frac{A_s}{\left[1 + iz/qa_s^2\right]}\cdot \exp\left\{-i\Omega t + iqz + \frac{iq}{2z}\cdot\left[\rho - \frac{V}{v}\cdot z\right]^2\right\}\times$$

$$\times \exp\left\{-\frac{\left[\rho - \frac{V}{v}\cdot z\right]^2}{2(z\theta_s)^2} - \frac{(z - tv)^2}{2l_s^2} + \psi(z,\ \rho - Vt)\right\}, \tag{3}$$

where Ω and q are the frequency and wave vector of the acoustic wave; θ_s is the width of the directional pattern of the acoustic antenna; a_s is the aperture of the antenna; v is the velocity of sound in the medium; l_s is the spatial scale of the envelope of the acoustic pulse; and, $\psi(z,\ \rho)$ is the random increase in the complex phase owing to atmospheric turbulence.

We shall assume that the center of the scattering volume is located at the altitude z_0 and that the function describing the relative contribution of different points of the scattering volume to the scattered field is Gaussian $\exp\left\{-(z - z_0)^2 / 2l\right\}$, where l is the characteristic vertical size of the scattering volume. In the case of short-wave radiation the scale l of the scattering volume is determined by the temperature gradient[7]

$$l \sim \left[2T/\left[q\cdot\frac{\partial T}{\partial z}\right]\right]^{1/2}.$$

Substituting Eqs. (2) and (3) into Eq. (1), after making the substitution of variables $z = z_0 + z'$, $t = t' + t^*$, $t^* = z_0\left(\frac{1}{c} + \frac{1}{v}\right)$, $t_0 = z_0\left(\frac{1}{v} - \frac{1}{c}\right)$ we obtain the following expression for the scattered field:

$$E_{sc}(0,\ \rho_0,\ t) = -\frac{A_e A_s}{4\pi c^2}\times$$

$$\times \frac{\frac{\partial^2}{\partial t^2}\exp\left\{-i(\omega_0 - \Omega)(t^\bullet + t')\right\}}{\left[1 + iz_0/(ka_e^2)\right]\left[1 - iz_0/(q\,a_s^2)\right]}\int\limits_{-\infty}^{+\infty} dz'\,d\rho\ \times$$

$$\times \exp\left\{i\kappa_B(z_0 + z') + \frac{ik(\rho_0 - \rho)^2}{2z_0} + \frac{ik\rho^2}{2z_0} - \frac{iqv}{2z_0c}\times\right.$$

$$\left.\times\ (\rho_0 - \rho)^2\right\}\cdot\exp\left\{\frac{iq}{2z_0}\left[\rho - \frac{V}{v}(z_0 + z')\right]^2 - \frac{\rho^2}{2(z\theta_e)^2} -\right.$$

$$\left. - \frac{\left[\rho - \frac{V}{v}(z_0 + z')\right]^2}{2(z\theta_s)^2} - \frac{\left[z'\left[1 + \frac{v}{c}\right] - vt'\right]^2}{2l_s^2}\right\}\times$$

$$\times \exp\left\{-\frac{z'^2}{2l^2}\right\}\sum_{n=-\infty}^{\infty}\exp\left\{-\left[2z' + c\left[t' - \frac{2\pi n}{\omega_r}\right]\right]^2\Big/2l_e^2\right\}, \tag{4}$$

where $\kappa_B = 2k - q\left(1 + \frac{v}{c}\right)$ is the Bragg detuning parameter.

Next, we shall follow the power spectrum of the scattered signal

$$S(\omega) = \langle E_{sc}(\omega)\cdot E^{\bullet}_{sc}(\omega)\rangle, \tag{5}$$

where

$$E_{sc}(\omega) = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} e^{1\omega t}\cdot E_{sc}(0,\ \rho_0,\ t)\,dt$$

At the point of compensation of the wind drift $\rho_0 = \frac{Vzq}{v\ \kappa}$.

After quite cumbersome calculations, we obtain for $S(w)$

$$S(\omega) = \frac{I_0}{(\Delta\omega)^2}\exp\left\{-\frac{V^2}{v^2}\left[\theta_s^2 + \frac{\theta_e^2}{1 + (qVL\theta_e)^2/(2v^2)}\right]\right\}\times$$

$$\times\sum_{n=-\infty}^{+\infty}\exp\left\{-\left[\kappa_r - q\left[\frac{V}{v}\right]^2\right]\cdot\frac{L^2}{2} - \frac{(n\omega_r l_e)^2}{c^2} -\right.$$

$$\left. - \frac{2\left[\omega - (\omega_0 + n\omega_r - \Omega - \Delta)\right]^2}{(\Delta\omega)^2}\right\}, \tag{6}$$

where

$$(\Delta\omega)^2 = \frac{L^2v^2}{l^2l_s^2} + 2\langle q_1^2\rangle V^2 + \frac{V^2q^2L^4}{2l_s^4\left[\frac{1}{\theta_e^2} + \frac{1}{\theta_s^2} + \frac{(qVL)^2}{2v^2}\right]}, \tag{7}$$

$$\Delta = \frac{L^2v\kappa_r}{2l_s^2}\left[1 - \frac{(VqL)^2}{2v^2\left[\frac{1}{\theta_e^2} + \frac{1}{\theta_s^2} + \frac{(qVL)^2}{2v^2}\right]}\right] -$$

$$- \frac{L^2\kappa_r V^2\langle q_1^2\rangle}{2v\cdot\left[1 + \frac{\theta_s^2}{\theta_e^2} + \frac{(\theta_s qVL)^2}{2v^2} + \langle q_1^2\rangle\cdot(\theta_s z)^2\right]} +$$

$$+ \frac{V^2qL^2\left[\frac{\theta_s^2}{\theta_e^2} + \frac{(\theta_s qVL)^2}{2v^2}\right]}{2vl_s^2\left[1 + \frac{\theta_s^2}{\theta_e^2} + \frac{(\theta_s qVL)^2}{2v^2}\right]}, \tag{8}$$

$$I_0 = \frac{(l_e A_e A_s \omega^2 \omega_r a_e^2 a_s^2 kqL)^2}{64\pi^3 c^6 z_0^2 \left[\frac{1}{\theta_e^2} + \frac{1}{\theta_s^2} + \frac{(qVL)^2}{2v^2}\right]} \times$$

$$\times \frac{1}{\left[1 + \langle q_1^2 \rangle z\right]^2 \Big/ \left[\frac{1}{\theta_e^2} + \frac{1}{\theta_s^2} + \frac{(qVL)^2}{2v^2}\right]},$$

$$\frac{1}{L^2} = \frac{1}{2l^2} + \frac{1}{2l_s^2}, \quad \kappa_r = \frac{(\omega_0 + n\omega_r)}{c} - 2q,$$

$$\langle q_1^2 \rangle = \frac{1}{\rho_c^2} = \left[0.73 c_s^2 q^2 z_0\right]^{6/5},$$

where ρ_c is the transverse coherence radius of the sound wave;[1] c_S^2 is the structure constant of the index of refraction for the sound wave.

The expression (6) for the power spectrum of the received signal at the point of compensation of wind drift for scattering of a periodic sequence of electromagnetic pulses by a single acoustic pulse was obtained under the following assumptions:

$$\omega_r \gg \frac{l}{v}, \quad zq\theta_{s,e} > 1, \quad L_c > L,$$

where $L_c = (1.46\ c_S^2\ q^{7/6} z_0)^{-6/5}$ is the longitudinal coherence radius in the sound wave.[1]

For $\langle q_1^2 \rangle = 0$ and $V = 0$ the spectrum of the central line ($n = 0$) is identical to the expression derived in Refs. 1 and 2. In the case at hand the effect of the wind on the intensity of the received signal is described by the first exponential factor in Eq. (6). It should be noted that this effect is quite weak and is connected with the rotation of the directional pattern of the acoustic antenna owing to the wind by the angle $\theta_M = V/v$. From the expression (7) it follows that turbulence and wind result in broadening of the lines in the frequency spectrum of the received signal, and this in turn affects the accuracy of the method.

As regards the combined effect of turbulence and wind on the displacement of the maximum of a separate line, as follows from Eq. (8) it can be neglected when $\langle q_1^2 \rangle / q^2 \theta_M^2 \ll 1$.

We shall now examine how the local sound velocity can be determined from the frequency spectrum of the received signal. This is best done by analyzing the maximum of the spectrum w_{sc} of the central line with $n = 0$ in the expression (6).

For $\langle q_1^2 \rangle / q^2 \theta_M^2 \ll 1$ Eqs. (6) and (8) yield

$$\omega_{sc} = \omega_0 - \Omega - (2k - q) v\gamma + \frac{\delta V^2 q}{v},$$

where

$$\gamma = \frac{l^2}{l^2 + l_s^2} \frac{\frac{1}{\theta_e^2} + \frac{1}{\theta_s^2}}{\frac{1}{\theta_e^2} + \frac{1}{\theta_s^2} + \frac{(qVL\theta_s)^2}{2v^2}}.$$

$$\delta = \frac{l^2}{l^2 + l_s^2} \frac{\frac{\theta_s^2}{\theta_e^2} + \frac{(qVL\theta_s)^2}{2v^2}}{1 + \frac{\theta_s^2}{\theta_e^2} + \frac{(qVL\theta_s)^2}{2v^2}}.$$

The last term in Eq. (9) is associated with the entrainment of the acoustic wavefronts by the wind.

For $\lambda_s \sim 5$ m and $\partial T/\partial z$ K/km the longitudinal size of the scattering volume $l \sim 200 \ldots 300$ m. At the same time, for short-wave radio waves $l_s \sim 300$ m. Thus unlike the traditional case of RAS,[2] when the condition $l \gg l_s$ is satisfied, in the case at hand under conditions of STRAS $l \sim l_s$, which makes it impossible to determine the sound velocity directly from frequency shift of the line.

If however, the frequency of the sound wave is tuned, then for $\Omega = 2kv \Big/ \left[1 + \frac{V^2}{v^2} \cdot \frac{\delta}{\gamma}\right]$, as follows from Eq. (9), $w_0 = w_{sc} + \Omega$. From this condition it is possible to determine the sound velocity in the atmosphere. It is obvious that in this case the method of RAS becomes significantly less efficient. If, at the same time, corrections for the transverse wind are not made in the frequency measurements, then an error $\delta T \sim \frac{2V^2\delta}{\gamma}$, will be made in the the temperature measurements, and this limits the directional pattern of the transmitting antennas. Thus, if the relative error in measuring the temperature $\delta T \sim 0.1$, $V \sim 10$ m/s, and $L \sim 300$ m, then the following limit is obtained on the width of the directional patterns of both acoustic and electromagnetic antennas:

$$\frac{\theta_e \theta_s}{\left[\theta_e^2 + \theta_s^2\right]^{1/2}} \le 0.2 .$$

Thus, in order to achieve a prescribed accuracy in measuring the temperature by means of STRAS of the atmosphere it is necessary to construct antennas with narrow directional patterns for wavelengths in the range $\lambda \sim 10$ m. When the method of steering of the transmitting electromagnetic antenna is used to compensate the wind drift[3,5] the details involved in determining the temperature profile with the help of frequency measurements will apparently be analogous to those studied above.

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