# Theoretical description of LD-WM-CW ladar

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Some theoretical models of a laser-diode-based CW ladar are analyzed. The results of numerical simulation of an LD-WM-CW ladar are presented. Recommendations are given on the parameters of laser radiation modulation.

Among various methods of gas analysis by use of measurements along open paths, most sensitive and selective method is the spectroscopic method based on wavelength modulation (WM) of radiation of a laser diode (LD) within the contour of an absorption line of a gas under study. With all its advantages, this method does not provide range.

Traditionally, monostatic lidars with spatial resolution use pulsed sensing method. A disadvantage of this method is the necessity of using high-power pulsed solid-state or gas lasers that are highly expensive and bulky. Besides, they induce noise and often require cooling with water.

When using continuous-wave (CW) modulation laser lidars (ladars), it becomes possible to construct less expensive, lighter-weight, and smaller-size systems. The existing CW aerosol ladars operating at a single wavelength do not allow determination of the gas composition of the atmosphere. The CW DIAL ladars provide measurements of the spatial distribution of gas concentration. However, they have lower concentration sensitivity than the methods of modulation spectroscopy, because the concentration is measured using only two points on the absorption line profile.

Thus, for studying the gas composition of the atmosphere, development of a ladar that combines the advantages of a CW lidar and modern modulation spectroscopic methods is very promising.

In this paper, a CW spectroscopic laser-diode-based ladar with spatial resolution (LD-WM-CW ladar) is analyzed theoretically.[1,2] The functional scheme of the LD-WM-CW ladar is shown in Fig. 1.

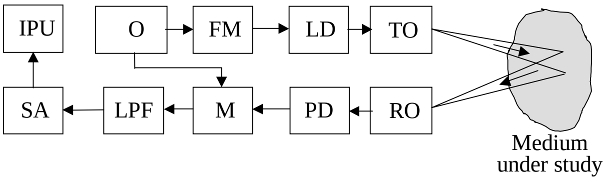

**Fig. 1.** Functional diagram of a CW spectroscopic ladar with spatial resolution: linear frequency modulation oscillator (O), frequency modulator (FM), laser diode (LD), transmitting optics (TO), receiving optics (RO), photodetector (PD), mixer (M), low-pass filter (LPF), spectrum analyzer (SA), and information processing unit (IPU).

There are several mathematical models for description of the modulation spectroscopic methods.[3–5] However, all the methods are constructed for the case of frequency modulation of radiation by a simple harmonic signal. Let us analyze the mathematical models of the LD-WM-CW ladar by developing theoretical principles and describing the peculiarities of interaction of the frequency-modulated laser radiation with a gas analyzed on an open atmospheric path.

One of the approaches to analytical description of the LD-WM-CW ladar is based on description of a current frequency of laser radiation and interaction of this radiation with the gas studied[3,5]:

$$T_L = \exp\{-2\tau\gamma_L^2/[(\nu(t)-\nu_0)^2+\gamma_L^2]\}. \qquad (1)$$

Here $T_L$ is the spectral transmittance of the gas under study assuming Lorentz profile of an absorption line; $\nu_0$ is the central frequency of the gas absorption line; $\nu(t)$ is the current frequency of the sounding radiation; $\gamma_L$ is the half-width of the absorption profile of the gas under study; $\tau = \int\limits_0^R N(r)\,\sigma\,\mathrm{d}r$ is the optical depth of the path along the laser beam; $N(r)$ is the concentration of the gas under study; $R$ is the distance to the studied section of the path; $\sigma$ is the absorption cross section of the gas under study at the absorption line center.

Such a model of the LD-WM-CW ladar was analyzed in Ref. 1. This model allows the main properties of the system to be revealed and the analytical equations for the spatial distribution of the gas under study to be derived. However, the above-mentioned method of description of the LD-WM-CW ladar is valid only for the cases that the width of the absorption line of a gas far exceeds the deviation of linear frequency modulation of laser radiation that provides for spatial resolution of measurements.

Another method describing the LD-WM-CW ladar is to consider interaction of spectral components of sounding radiation with the gas within the contour of its absorption line.[5–7] In this approach, the effects arising in situations that deviation of linear modulation is comparable with the half-width of the absorption line $\gamma_L$ can be taken into account.

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According to this approach, the electromagnetic filed of modulated radiation can be presented as

$$E(t) = E_0 \exp[i\varphi(t)] = \\ = \sum_{n=-\infty}^{\infty} E(n\, w_\mathrm{m}) \exp(i\, n\, w_\mathrm{m}\, t), \qquad (2)$$

where $E_0$ is the amplitude of the electromagnetic field; $\varphi(t) = \int_0^t \nu(t')\, dt'$ is the time dependence of the phase of the electromagnetic field; $i$ is imaginary unit; $w_\mathrm{m}$ is the modulation frequency.

After interaction of radiation described by Eq. (2) with the gas under study, the electromagnetic field strength of the lidar return signal takes the form[5]:

$$E_\mathrm{T}(t) = \sum_{n=-\infty}^{\infty} E(n\, w_\mathrm{m}) \exp\,(i\, n\, w_\mathrm{m}\, t)\, T_\mathrm{L}(n\, w_\mathrm{m}). \qquad (3)$$

Here

$$T_\mathrm{L}(\nu) = \exp[\,-\,\delta(\nu) - i\,\phi(\nu)] \qquad (4)$$

is the complex transmission function characterized by the absorption coefficient $\delta(\nu)$ and the phase shift $\phi(\nu)$ of the radiation.

Different absorption of spectral components of the optical radiation described by Eq. (3) gives rise to periodic variation of the radiation intensity $I_\mathrm{T}(t)$ proportionally to the square absolute value of the electromagnetic field strength vector.

The signal at the photodetector output is proportional to the intensity of the incident radiation described by the following equation[6]:

$$I_\mathrm{T}(t) = c\,(E_\mathrm{T}(t)^*\, E_\mathrm{T}(t))/8\pi, \qquad (5)$$

where $c$ is the speed of light; $E_\mathrm{T}(t)^*$ is the complex conjugate function for $E_\mathrm{T}(t)$.

Time dependence of the photodetector current was calculated using methods of numerical simulation. In numerical calculations, the input data for the model were frequency-modulated oscillations of the electromagnetic field of laser radiation for one period of linear variation of the frequency $T_\mathrm{m}$ according to Eq. (2). Then, using fast Fourier transform, the spectrum of these oscillations was determined. The spectrum of oscillations of the electromagnetic field after interaction with the gas was calculated by Eq. (3). Then the process of photodetection described by Eq. (5) was simulated.

Figure 2 shows the spectrum of the received lidar return signal due to interaction of the electromagnetic field of the modulated radiation with the gas. This spectrum was obtained in numerical simulation at different ratios of the deviation of linear frequency variation to the width of the gas absorption line.

Figure 3 shows the signals at the photodetector output for the cases of reception of return signals having the spectra shown in Fig. 2.

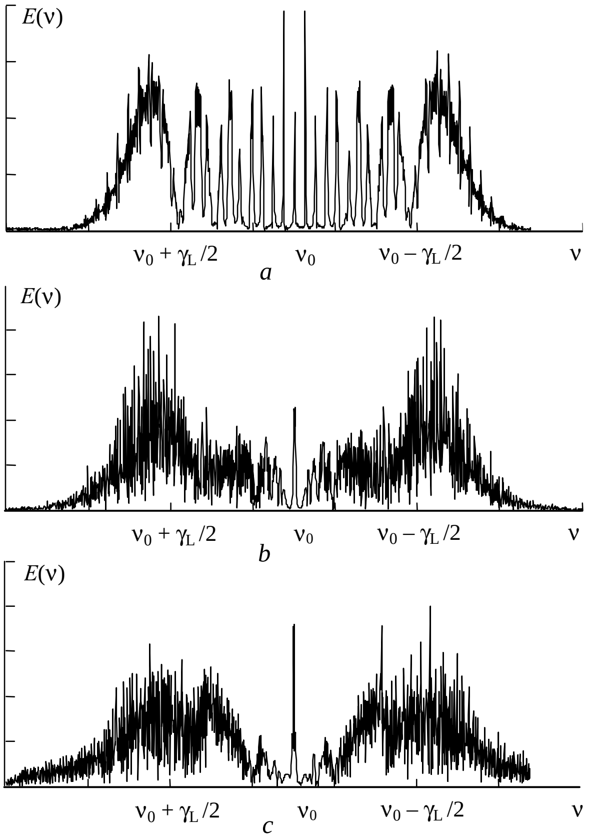

**Fig. 2.** Simulated spectrum of lidar return signal after interaction with the gas studied: deviation of linear frequency variation $W = 0.01\ \gamma_\mathrm{L}$ (*a*), $W = 0.3\ \gamma_\mathrm{L}$ (*b*), and $W = \gamma_\mathrm{L}$ (*c*).

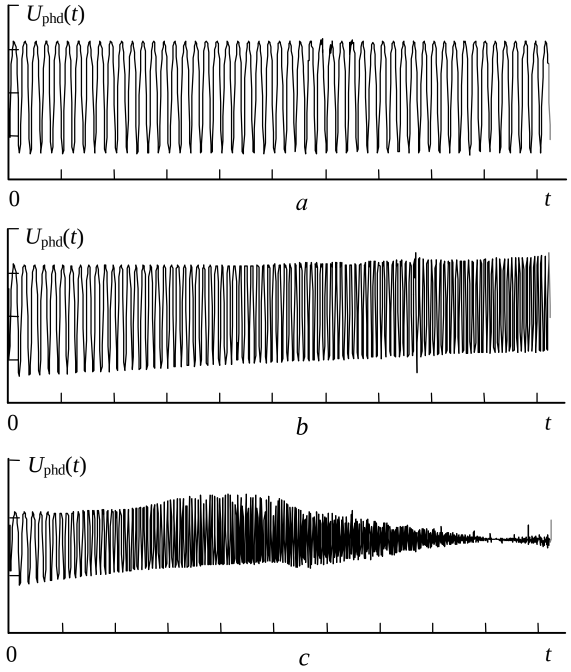

**Fig. 3.** Simulated signals at photodetector output: deviation of linear frequency variation $W = 0.01\ \gamma_\mathrm{L}$ (*a*), $W = 0.3\ \gamma_\mathrm{L}$ (*b*), and $W = \gamma_\mathrm{L}$ (*c*).

It should be taken into account that the half-width of the absorption profile $\gamma_L$ of different gases under standard atmospheric conditions varies about several gigahertzs (for example, for ammonium in the region of 1.5 μm it is about 3 GHz) (Ref. 1). To provide for spatial resolution about several meters, the deviation of linear frequency variation should be several tens of megahertzs. Thus, the situation that the deviation of linear frequency variation is much less than the width of the absorption profile of the studied gas ($W \ll \gamma_L$) is typical of the LD-WM-CW ladar.

As is seen from Fig. 3, the interaction of the frequency-modulated electromagnetic field of the LD-WM-CW ladar with a gas within the absorption line contour gives rise to a signal with linearly varying frequency at the photodetector output.

In the cases that the deviation of the subcarrier frequency becomes comparable with the half-width of the absorption line $W \approx \gamma_L$ (Fig. 3*b* and *c*), the signal amplitude changes for the period of linear frequency variation $T_m$, i.e., additional amplitude modulation of the spectral envelope appears. The period of this modulation is equal to the period of linear frequency variation $T_m$. The presence of the parasitic amplitude modulation is explained by the fact that the sections of absorption profile, spectral components of the sounding radiation interact with, change for the period $T_m$ of linear frequency variation.

The parasitic amplitude modulation widens the spectrum of ranging frequencies for the range-resolvable sections of the sounding path and, thus, deteriorates the spatial resolution of measurements. The situation that the deviation of the subcarrier is comparable with the half-width of the absorption line $W \approx \gamma_L$ can occur in attempts to measure the spatial distribution of a gas having a very narrow absorption line with high spatial resolution less than 1 m.

At the subcarrier deviation much less than the half-width of the gas absorption line $W \ll \gamma_L$ (Fig. 3*a*; this situation is typical of the LD-WM-CW ladar), the amplitude of oscillations with linear frequency variation at the photodetector output is constant during the entire period of linear modulation $T_m$.

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