Mean power of a partly coherent laser beam scattered at an atmospheric layer

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The mean power of a partly coherent laser beam scattered at an aerosol atmospheric layer is analyzed numerically. The combined transceiving optical scheme with a circular exit aperture and a round receiving aperture is considered. It is shown that the mean power of the scattered radiation depends on the initial spatial coherence of the beam and varies with distance to the scattering layer because of diffraction at the exit aperture.

During propagation through the atmosphere, laser beams experience distortions due to turbulent fluctuations of the refractive index of air. To correct these distortions, modern optical systems include a feedback loop with a reference source, whose radiation carries information about wave distortions on a propagation path and is used for adaptive control over parameters of propagating laser beams. Either specialized artificial reflectors or independent laser sources are commonly used as reference sources.

Along with turbulent distortions arising upon propagation in the atmosphere, high-power laser radiation experiences phase fluctuations and uncontrollable field distortions at the part of formation of the output beam between a source and the transmitting aperture. This worsens spatial coherence of laser beams and leads to additional beam broadening and reduction of the efficiency of laser energy transfer to the given distance. When propagating along high-altitude paths, these "natural" fluctuations of laser radiation may prevail over atmospheric distortions, and the problem of their removal becomes urgent. To correct distortions arising at the stage of laser beam formation, it was proposed to use a natural target – atmospheric aerosol – as a source of the reference wave.[1] The laser beam radiation scattered by aerosol particles carries information about beam distortions and can be used to control output parameters of the beam.

This paper presents the results of calculation of the mean received power (intensity flux) of a partly coherent pulsed laser radiation scattered at an atmospheric layer as a function of angular divergence of the irradiating beam and spatial coherence of the initial field. The calculations are performed for the combined optical scheme of a transceiving system with a circular exit and a round receiving apertures. The model of a phase screen with the Gaussian phase correlation function is used as a model of distortions of the initial field.

1. Computational relationships

Consider an optical system with the combined transceiving optical scheme and a pulsed laser source (see, for example, Fig. 1 in Ref. 2, as well as Ref. 3). Assume that the control signal for correction of distortions is generated based on measurements of the mean power P_s of radiation backscattered at an atmospheric layer lying at a distance z from the source. The laser irradiates the aerosol layer through the circular exit aperture with the radii a, $b = a/M$, where $M > 1$, is a numerical coefficient. The field formed by the exit aperture has a wavefront curvature F with angular half-divergence of the beam $\alpha = a/F$. The backscattered radiation is received on a round aperture with the radius a_0 and the focal length f. A photodetector of the radius a_d lies in the focal plane of this aperture. The scattering occurs in a thin layer of the thickness Δz, so that $\Delta z \ll z$, randomly filled with particles having sizes smaller than the radiation wavelength $r_s < \lambda$.

The field of the partly coherent beam at the exit aperture can be represented in the following form:

$$U(\rho) = A\exp\left[i\frac{k\rho^2}{2F} + i\varphi(\rho)\right], \tag{1}$$

where A is the field amplitude; $e^{i\varphi(\rho)}$ is the phase screen; $\varphi(\rho)$ is a random phase with a zero mean and the Gaussian correlation function

$$\langle\varphi(\rho_1)\varphi^*(\rho_2)\rangle = B_s(\rho_1 - \rho_2) = \sigma^2 e^{-(\rho_1-\rho_2)^2/2l^2};$$

σ^2 is the variance of phase fluctuations; l is its correlation scale; angular brackets denote averaging over an ensemble. The model (1) uniquely determines the radius of the spatial coherence ρ_c of the field generated by the source.[4] Below the factor A is omitted.

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The field of the irradiating beam at a distance z in the parabolic approximation is determined by the integral[5]:

$$U(\mathbf{R},z)=\frac{\exp(ikz+ikR^2/2z)}{i\lambda z}\int\limits_{S_{ab}}U(\rho)\exp\left(\frac{ik}{2z}(\rho^2-2\mathbf{R}\rho)\right)d\rho, \quad (2)$$

where S_{ab} is the surface of the circular exit aperture; z is the longitudinal coordinate; $\mathbf{R}$ is the transversal radius vector in the plane perpendicular to the direction of propagation.

The presence of phase fluctuations in the initial field affects the mean value of the received power through the mean intensity of the irradiating beam. According to Eqs. (1) and (2), the mean intensity of the field at the scattering layer can be written in the form

$$\langle I_i(\mathbf{R},z)\rangle=\frac{1}{\lambda^2 z^2}\int\limits_{S_{ab}}\int\limits_{S_{ab}}\langle U(\rho_1)U^*(\rho_2)\rangle\times$$
$$\times\exp\left(\frac{ik}{2z}\left[\rho_1^2-\rho_2^2-2\mathbf{R}(\rho_1-\rho_2)\right]\right)d\rho_1 d\rho_2, \quad (3)$$

where

$$\langle U(\rho_1)U^*(\rho_2)\rangle=U_0(\rho_1)U_0^*(\rho_2)\exp\left\{-\frac{1}{2}D_s(\rho_1,\rho_2)\right\}, \quad (4)$$

$$D_s(\rho_1,\rho_2)=2\left[B_s(0)-B_s(\rho_1,\rho_2)\right]$$

is the structure function of phase of the initial field.

Under the condition $r_s<\lambda$, the intensity of pulsed radiation, backscattered at an atmospheric layer and received in the focal plane of the lens $z=-f$, according to Refs. 2 and 3, can be represented in the form

$$\langle I_s(\rho_0,f)\rangle=\frac{\sigma_s}{\lambda^2 f^2}\int d\mathbf{R}\,\langle I_i(\mathbf{R},z)\rangle I_r(\mathbf{R},\rho_0,z), \quad (5)$$

where $\sigma_s=\frac{\alpha_s^2}{4\pi}$ is the backscattering cross section; α_s is the scattering amplitude, and the function

$$I_r(\mathbf{R},\rho_0,z)=\frac{1}{z^2}\left|\int\limits_{S_{a0}}d\rho'\exp\left\{ik\left(\frac{\rho'^2}{2z}-\frac{\rho_0\rho'}{f}-\frac{\rho'\mathbf{R}}{z}\right)\right\}\right|^2 \quad (6)$$

can be treated as an intensity of the collimated beam with the effective radius equal to the radius of the receiving lens, which propagates from the transceiver plane in the direction of the scattering layer at an angle ρ_0/f; S_{a_0} is the surface of the receiving lens.

The mean power of the received radiation is the integral of the intensity (5) over the photodetector area S_d:

$$P_s(f)=\int\limits_{S_d}d\rho_0\,\langle I_s(\rho_0,f)\rangle. \quad (7)$$

Below we omit the variable f in Eq. (7) for $P_s(f)$ assuming that the photodetector always lies in the focal plane.

2. Mean power of scattered radiation

The integrals (3) and (6) have finite domains of integration S_{ab} and S_{a_0}, and it is more convenient to calculate them by the numerical Fast Fourier Transform procedure, using the fact that the integral (5) is a convolution integral of the functions (3) and (6) [Ref. 2].

We can write the mean intensity (5) and the mean received power (7) of the radiation scattered at the layer as:

$$\langle I_s(\rho_0,f)\rangle=\frac{\sigma_s}{\lambda^2 f^2}\int\limits_{-\infty}^{+\infty}d\mathbf{s}\Phi_i(\mathbf{s},z)\Phi_r(\mathbf{s},\rho_0,z)\exp\left(2\pi i\mathbf{s}\rho_0\frac{z}{f}\right), \quad (8)$$

$$P_s=\frac{\sigma_s}{\lambda^2 z^2}\int\limits_{S'_d(z)}d\mathbf{r}_0\int\limits_{-\infty}^{+\infty}d\mathbf{s}\Phi_i(\mathbf{s},z)\Phi_r(\mathbf{s},\mathbf{r}_0,z)\exp(2\pi i\mathbf{s}\mathbf{r}_0), \quad (9)$$

where $\Phi_i(\mathbf{s},z)$ and $\Phi_r(\mathbf{s},\mathbf{r}_0,z)$ are the spectra of the intensities (3) and (6);

$$\mathbf{r}_0=\rho_0\frac{z}{f};\quad S'_d(z)=S_d\frac{z^2}{f^2}$$

is the geometric projection of the photodetector to the scattering layer.

The algorithm for calculation of the spectra of the intensities (3) and (6) is considered for the intensity of the irradiating beam (3), taken as an example, since for the intensity of the "secondary" beam (6) the spectrum is calculated similarly. By definition, the spectrum of the intensity $\langle I_i(\mathbf{R},z)\rangle$ is

$$\Phi_i(\mathbf{s},z)=\frac{1}{(2\pi)^2}\int\limits_{-\infty}^{+\infty}d\mathbf{R}\langle I_i(\mathbf{R},z)\rangle\exp(-2\pi i\mathbf{R}\mathbf{s}). \quad (10)$$

The substitution of the Eq. (3) for $\langle I_i(\mathbf{R},z)\rangle$ in Eq. (10) gives the Dirac delta function $\delta(\lambda z\mathbf{s}+\rho_1-\rho_2)$ for the integral with respect to the variable $\mathbf{R}$, that is,

$$\Phi_i(\mathbf{s},z)=\int\limits_{S_{ab}}\int\limits_{S_{ab}}d\rho_1 d\rho_2\,\delta(\lambda z\mathbf{s}+\rho_1-\rho_2)\times$$
$$\times\exp\left(\frac{ik\mu}{2z}(\rho_1^2-\rho_2^2)+\sigma^2\left[\exp\left(-\frac{(\rho_1-\rho_2)^2}{2l^2}\right)-1\right]\right), \quad (11)$$

where $\mu=1-z/F$.

The domain of integration S_{ab} in Eq. (11) is the ring surface (output aperture), and the direct calculation of this integral at the substitution $\rho_2=\lambda z\mathbf{s}+\rho_1$ is a rather complicated problem. However, if each integral over S_{ab} in Eq. (11) is written as a difference of two integrals over the surfaces of the circles a and b with the radii S_a and S_b, that is,

$$\iint\limits_{S_{ab}\,S_{ab}} = \iint\limits_{S_a\,S_a} - \iint\limits_{S_b\,S_b} + 2\mathrm{Re}\left[\iint\limits_{S_a\,S_b}\right], \qquad (12)$$

then we can easily obtain the equation common for all integrals in Eq. (12). Introduce an auxiliary function f_x (a form-factor of a circle), where x is the circle radius; $f_x(\rho_j) = 0$, if the point of integration is beyond the circle surface $|\rho_j| > x$, and $f_x(\rho_j) = 1$, if it lies inside the circle, $|\rho_j| < x$, $j = 1, 2$. Then, with the use of f_x for transition to infinite integration limits in Eq. (12) after the replacement $_{1,2} = \mathbf{r} \mp \frac{\lambda z \mathbf{s}}{2}$, any of three double integrals $\Phi_{i,ab}(\mathbf{s}, z)$ in the right-hand side of Eq. (12) over any pair of the surfaces S_aS_a, S_bS_b, and S_aS_b can be represented in the form

$$\Phi_{i,ab}(\mathbf{s},z) = \exp\left(\sigma^2\left[\exp\left(-\frac{1}{2}\left(\frac{\lambda z \mathbf{s}}{l}\right)^2\right) - 1\right]\right)\times$$

$$\times\int\limits_{-\infty}^{+\infty} \mathrm{d}\mathbf{r}\, f_a\left(\mathbf{r} - \frac{\lambda z \mathbf{s}}{2}\right) f_b\left(\mathbf{r} + \frac{\lambda z \mathbf{s}}{2}\right)\exp(2\pi i \mu \mathbf{r}\mathbf{s}). \qquad (13)$$

Using the fact that the vector of the spatial frequency $\mathbf{s} = \mathbf{e}_\perp s_\perp + \mathbf{e}_\| s_\|$, where $\mathbf{e}_\perp$, and $\mathbf{e}_\|$ are unit vectors, can be oriented arbitrarily, we assume $s_\perp = 0$, $s_\| = |\mathbf{s}| = s$, thus removing one of the integration variables from the exponent. As a result, the integral in Eq. (13) with respect to this variable is calculated analytically and with respect to the remained variable – numerically. The substitution of the calculated integrals of type (13) into Eq. (12) allows us to calculate the integral over the ring surface S_{ab} with the external and internal radii a and b and thus to find the spectrum Φ_i described by Eq. (11). In the case of the receiving aperture $a = b = a_0$ only one term remains in place of the sum of integrals in Eq. (12).

The scenario (12), (13) of spectra calculation was used in computation of the mean power of the signal scattered at the layer in the plane z by the equation

$$P_s = \frac{\sigma_s}{\lambda^2 z^2}\sum_{n'=0}^{N-1}\sum_{m'=0}^{N-1} S'_\mathrm{d}(z, n'\Delta r_0, m'\Delta r_0)\times$$

$$\times\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\Phi_i(m\Delta s, n\Delta s, z)\Phi_r(m\Delta s, n\Delta s, z)\times$$

$$\times\exp\left[2\pi i\left(\frac{mm'}{N} + \frac{nn'}{N}\right)\right](\Delta s \Delta r_0)^2, \qquad (14)$$

where N is the number of nodes of the computational grid; Δr_0 and Δs are, respectively, the spatial resolution and the resolution in the spatial frequencies. In the longitudinal direction, the computation by Eq. (14) was performed with a step sufficient for resolution of oscillations in the mean intensity of the irradiating beam $\langle I_i \rangle$ arising at the "diffraction" part of the path $[0, z_\mathrm{d}]$, where $z_\mathrm{d} = a^2(1 - M^{-2})/\lambda$ is the effective diffraction length for the circular exit aperture.

3. Calculated results

The variation of $\langle I_i \rangle$ on the beam axis along the propagation path is shown in Fig. 1, where I_0 is the initial intensity of the field at the circular aperture in the absence of fluctuations.

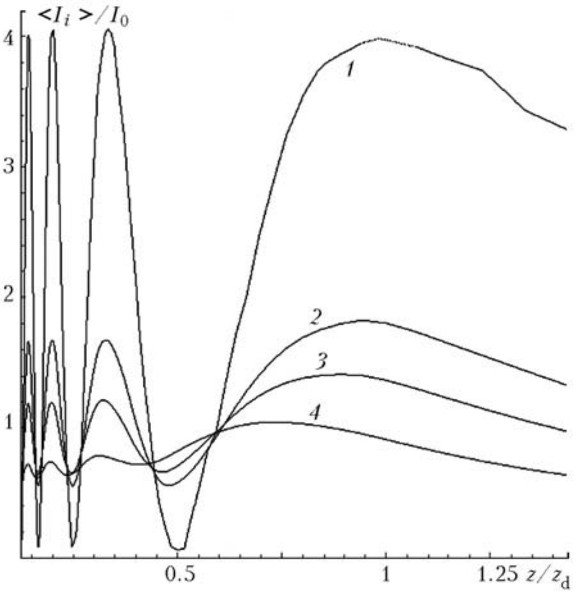

Fig. 1. Mean intensity on the axis of the collimated irradiating beam, circular exit aperture $M = 3$: without fluctuations (*1*), $\rho_c/a = 0.6$, 0.42, and 0.3 (*2*–*4*).

It follows from Fig. 1 that as fluctuations appear in the initial field, the amplitude of diffraction intensity variations on the axis of the irradiating beam decreases. In the cross plane, the diffraction pattern of the intensity distribution is blurred at the partial spatial coherence of the field at the exit aperture of the source, and the ring beam broadens (Fig. 2).

The intensity distribution of the backscattered radiation in the focal plane of the receiving telescope as a function of the distance to the scattering layer is shown in Fig. 3*a*. It can be seen from Fig. 3 that diffraction variations in the intensity of the irradiating beam lead to analogous variations in the intensity of the scattered radiation in the plane $z = -f$. As this takes place, the photodetector, depending on its field of view $\alpha_0 = a_\mathrm{d}/f$, intercepts only a part of the diffraction pattern in the focal plane, if the distance z, at which the scattering layer lies, is shorter than the distance z_{α_0} (Fig. 3*b*).

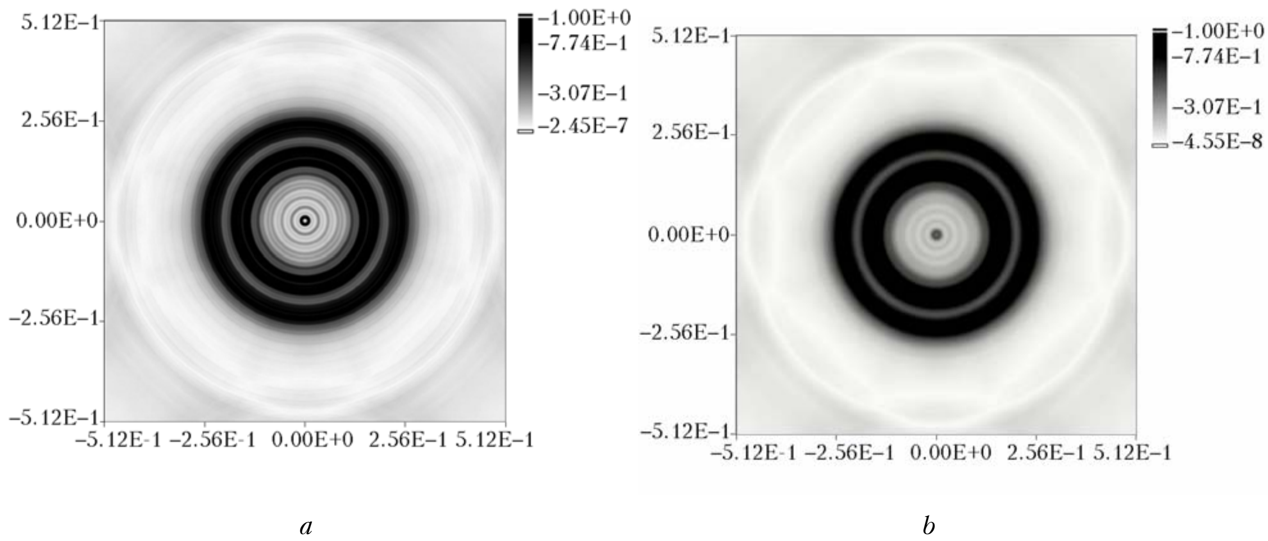

Fig. 2. Intensity distribution of the collimated irradiating beam at a distance $z/z_{\text{d}} = 0.0625$: without fluctuations (*a*); $\rho_c/a = 0.42$ (*b*).

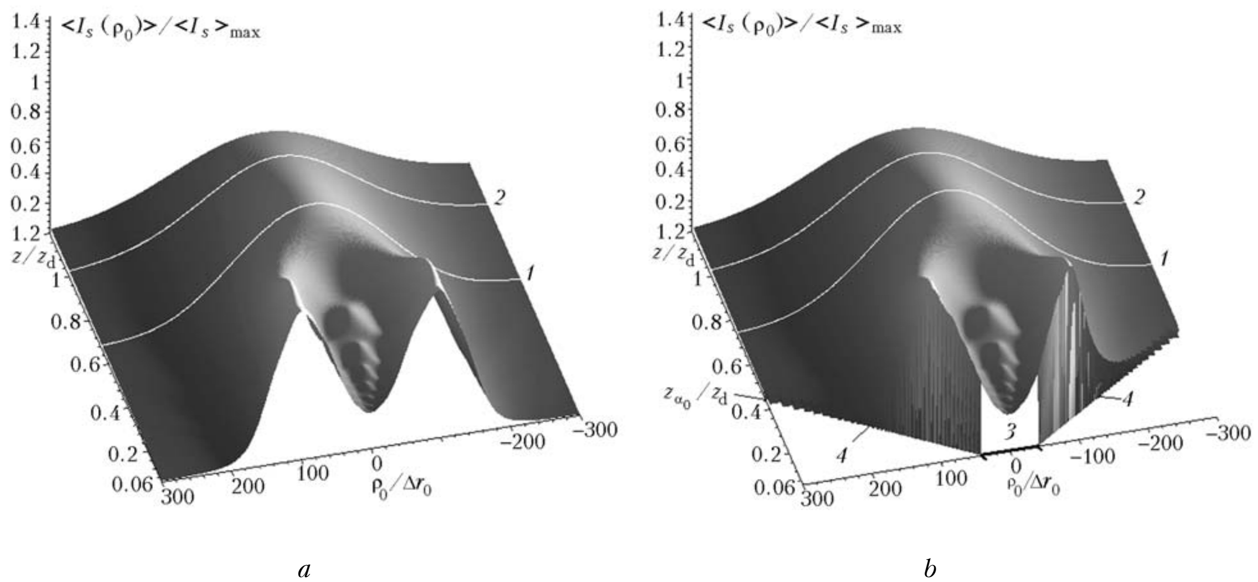

Fig. 3. Intensity distribution $\langle I_s(\rho_0, f)\rangle/\langle I_s\rangle_{\max}$ of the collimated beam backscattered radiation in the focal plane of the receiving objective as a function of the distance z to the scattering layer; $\langle I_s\rangle_{\max}$ is the maximal intensity; the last maximum on the optical axis (*1*); diffraction length of the exit aperture z_{d} (*2*); projection of the photodetector at a distance $z = 0.06z_{\text{d}}$ normalized to the spatial step of the computational grid (*3*); boundaries of the photodetector projection with the increasing distance to the scattering layer (*4*); the photodetector's field of view is $\alpha_0 = 5\cdot 10^{-5}$ rad.

Figure 4 shows the calculation results for the mean received power of the collimated beam scattered radiation as a function of the distance to the scattering layer at the constant detector's field of view and different spatial coherences of the initial field.

The values of $P_s(z)$ in Fig. 4 are normalized to the maximal value of the mean received power $P_{s_{\max}}$ in the absence of fluctuations of the initial field. It can be seen that the mean received power $P_s = P_s(z)$ as a function of the distance to the layer first increases with the increase of z to some critical value and then decreases. As the distance to the scattering layer increases, the photodetector intercepts the increasing part of the diffraction intensity distribution in the focal plane (see Fig. 3*b*), and P_s increases. However, this process continues only until the extinction of the scattered radiation intensity begins to affect the received power to the greater extent than the increase in the intensity due to the more complete interception of the focal spot by the photodetector. The worsening of the spatial coherence leads, on the one hand, to blurring of the diffraction pattern and to the increase of the irradiated area at the diffraction pattern center and, on the other hand, to the higher extinction of the scattered radiation intensity due to the additional divergence of the laser beam, irradiating the layer. As a consequence, the distance to the scattering layer, at which the mean received power is maximal, decreases.

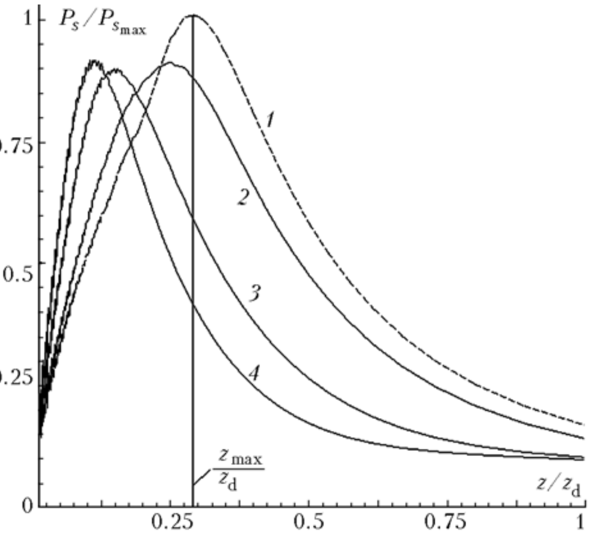

Fig. 4. Mean received power as a function of the distance to the scattering layer at different fluctuations of the initial field; collimated irradiating beam: coherent source (*1*); radius of spatial coherence ρ_c = 4, 1.8, and 1.3 cm (*2–4*).

At small fields of view, when the photodetector intercepts only the axial part of the diffraction pattern in the focal plane, the mean received power begins to oscillate at z variation in accordance with intensity variations of the received scattered radiation at the photodetector center (see Fig. 3). Figure 5 illustrates the variation of the mean received power as a function of the distance to the scattering layer in the case of small photodetector's fields of view.

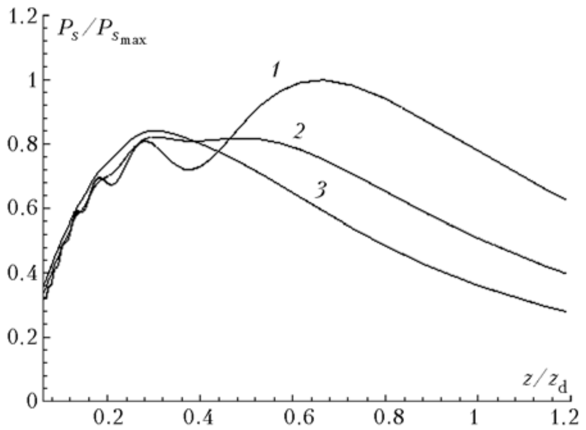

Fig. 5. Received power of the backscattered radiation of the collimated beam as a function of the distance z to the scattering layer at the small detector's field of view; $\alpha_0 = 2\cdot10^{-7}$ rad: coherent source (curve *1*); ρ_c = 4.8 cm (*2*); ρ_c = 2.7 cm (*3*); $P_{s_{max}}$ is the maximal received power for the coherent source.

Figure 6 demonstrates the variation of the mean received power P_s as a function of the opening angle of the exit aperture $\alpha = a/F$ at various spatial coherences of the initial field.

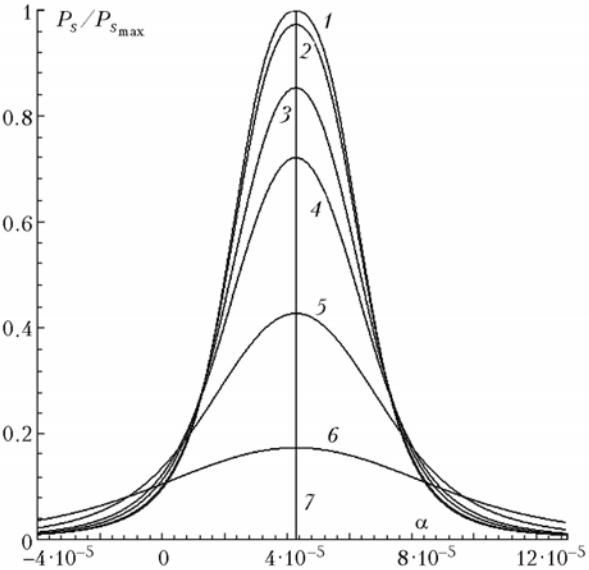

Fig. 6. Received power $P_s(\alpha)$ of the backscattered radiation as a function of the opening angle α of the exit aperture and the coherence radius ρ_c of the field at the exit aperture: coherent source (curve *1*); ρ_c = 4.8 cm (*2*); 2.7 (*3*); 1.8 (*4*); 1 cm (*5*); 5.7 mm (*6*); beam focus point on the scattering layer (*7*).

It follows from Fig. 6 that independently of the field spatial coherence at the exit aperture the mean received power is maximal if the irradiating beam is focused on the layer at $F = z$. The influence of the field initial spatial coherence on the mean received power is also more significant in a narrow range of the opening angles of the transmitting aperture near $\alpha = a/z$ determined by the distance to the scattering layer.

Conclusions

It follows from the results shown in Figs. 4–6 that the mean power of the scattered radiation depends on the spatial coherence of the initial field and changes due to the diffraction variation in the intensity of the irradiating beam when varying the distance to the scattering layer. Independently of the field initial coherence, the mean received power is maximal providing the opening angle of the transmitting aperture corresponds to the focusing of the irradiating beam on the scattering layer and, consequently, the laser beam energy concentration at the given distance is maximal. The dependence of the mean received power of the scattered radiation on the initial field spatial coherence allows the scattered radiation to be used to close the feedback loop when correcting laser beam distortions.

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