RECONSTRUCTION OF LASER BEAM INTENSITY FROM A NOISY TEMPERATURE FIELD OF A TARGET. I. CONSTRUCTION OF A REGULARIZATION ALGORITHM BASED ON FFT

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In this paper we present algorithm for reconstruction of intensity distribution over a laser beam cross section from the noisy temperature field of a thin target. The regularizing FFT algorithm consistent with the noise in the initial data and allowing the calculation time to be significantly reduced when processing great bulk of the initial data is used for suppressing the noise of temperature measurements.

High-power laser beam undergoes distortions due to heating the air owing to molecular absorption what results in formation of a diverging lens. In that case, use of a metal target is the only way to measure intensity of a beam close to destruction and nonlinear distortion thresholds. Interaction of radiation with the target forms its surface temperature field. Given the relationship between intensity and temperature of a target surface one can follow the dynamics of the intensity of incident laser beam in its cross-section (see Fig. 1).

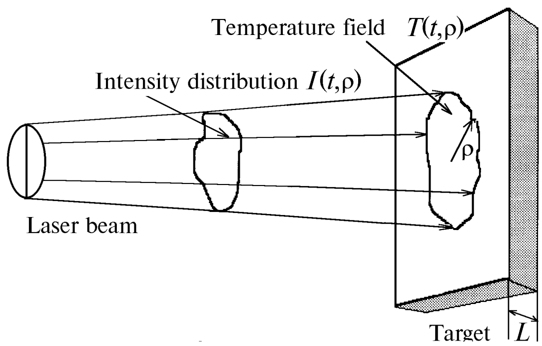

FIG. 1. Geometry of the problem

The relationships between the intensity $I(\rho, t)$ and temperature on the target surface $T(\rho, t)$ have been derived[1–3] in terms of heat flux $q(\rho, t)$ reconstructed using the temperature (ignoring heat losses, $q(\rho, t) = (1 - R)\, I(\rho, t)$, here R is the reflection coefficient). The target was considered to be limited in the transverse direction and infinite in its longitudinal dimension. The expressions were derived for different boundary conditions on the opposite target surface.

Let us write expressions assuming that the initial temperature at the plate surface is equal to zero ($T(0, \rho, 0) = T_{\rm H} = 0$):

a) The back surface of the plate is kept at a constant temperature (cooled target, $T(L, \rho, t)=0$). According to Ref. 2, we can write the following equations:

$$q(\rho, t) = -\frac{\pi k}{a^2 L}\int\limits_0^t {\rm d}\tau \frac{\rm d}{{\rm d}\tau}\,\vartheta_3\left(1, \frac{t-\tau}{L^2}\,a^2\right)\times$$

$$\times\int\limits_{-\infty}^{\infty} {\rm d}^2\rho'\,\frac{T(\rho', \tau)}{(t-\tau)}\exp\left(-\frac{(\rho-\rho')^2}{4a^2(t-\tau)}\right). \qquad (1)$$

Here k and a^2 are the coefficients of heat and temperature conductivity, respectively; L is the plate thickness; t is time; $\rho = \{x, y\}$ is the transverse coordinate;

$$\vartheta_3\left(1, \frac{t-\tau}{L^2}\,a^2\right) = 1 + 2\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2 n^2}{L^2}\,(t-\tau)\right\}$$

is the Jacobi ϑ-function (see Ref.4).

b) If the back surface is thermally insulated $\left(\frac{\partial T}{\partial z}(L,\rho,t)=0\right)$, the relationship between the temperature of the target surface and the light flux takes the following form:

$$q(\rho, t) = -\frac{\pi k}{a^2 L}\int\limits_0^t {\rm d}t \frac{\rm d}{{\rm d}t}\,\vartheta_1\left(\frac{1}{2}, \frac{t-\tau}{L^2}a^2\right)\times$$

$$\times\int\limits_{-\infty}^{\infty} {\rm d}^2\rho'\,\frac{T(\rho',t)}{(t-t)}\exp\left(-\frac{(\rho-\rho')^2}{4a^2(t-{\rm t})}\right). \qquad (2)$$

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Here

$$\vartheta_1\left(\frac{1}{2}, \frac{t-\tau}{L^2}a^2\right) = \sum_{n=1}^{\infty}(-1)\exp\left\{-\frac{a^2(2n-1)^2\pi^2}{4L^2}(t-\tau)\right\}$$

is the Jacobi ϑ-function (see Ref. 4).

Note that $\vartheta_3\left(1, \frac{t-\tau}{L^2}a^2\right)$ and $\vartheta_1\left(\frac{1}{2}, \frac{t-\tau}{L^2}a^2\right)$ are the generalized functions and it can be shown (see Refs.5 and 6) that if thermal physics Fourier parameter $Fo = \frac{a^2}{L^2}t > 1$, these functions take the following form:

$$\vartheta_3\left(1, \frac{t-\tau}{L^2}a^2\right) = \theta(t-\tau) + \delta(t-\tau)\frac{L^2}{3a^2},$$

$$\vartheta_1\left(\frac{1}{2}, \frac{t-\tau}{L^2}a^2\right) = \delta(t-\tau)\frac{L^2}{2a^2},$$

where $\theta(t)$ is the generalized Heaviside step-wise unit function, $\delta(t)$ is the generalized Dirac function. Substituting these functions into Eqs. (1) and (2) and having in mind the following expressions:

$$\lim_{\tau\to t}\frac{\exp\left\{-\frac{(\rho-\rho')^2}{4a^2(t-\tau)}\right\}}{4\pi a^2(t-\tau)} = \delta(\rho-\rho'),$$

$$\lim_{\tau\to t}\frac{\partial}{\partial t}\frac{\exp\left\{-\frac{(\rho-\rho')^2}{4a^2(t-\tau)}\right\}}{4\pi a^2(t-\tau)} = \Delta\delta(\rho-\rho'),$$

we obtain the relationship between the intensity and the flux at the surface of a thin target in the case of thermally insulated and cooled back surface to be written as:

$$q(\rho, t) = \frac{kL}{a^2}\left[\frac{\partial}{\partial t}T(\rho, t) - a^2\Delta_\perp T(\rho, t)\right], \qquad (3)$$

$$q(\rho, t) = \frac{k}{L}T(\rho, t) + \frac{kL}{3a^2}\left[\frac{\partial}{\partial t}T(\rho, t) - a^2\Delta_\perp T(\rho, t)\right], \qquad (4)$$

respectively. Here $\Delta_\perp = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$ is the transverse Laplacian.

From the temperature field $T(\rho, t_i)$ recorded within a time interval $t = \{0, t_{\max}\}$ in time steps $\Delta t = t_{\max}/M$ (see Fig. 2) and using Eqs. (3), (4) one can reconstruct the intensity $I^l(\rho) = I(\rho, t_l)$ at the same moments in time ($t_l = l\,\Delta t,\ l = 1, M$). In this manner a set of M frames of the temperature field $T^l(\rho) = T(\rho, t_l)$ is obtained, each frame being the temperature distribution $T^l_{mj} = T(x_m, y_j, t_l)$ ($x_m = mh$, $y_j = jh$, $h = x_0/N = y_0/N$, $m, j = 1, N$, x_0, and y_0 are the transverse dimensions of the target) over the target surface. Here N is determined by the recording device resolution. For instance, if the device that records temperature field has transverse resolution of 100×100 and records 24 frames per second, the dimension of the data set on temperature is equal to $N^2M = 100 \times 100 \times 24 = 240\ 000$. This dimension provides the initial data for processing to be very laborious. Therefore, we choose the algorithm of fast Fourier transform (FFT) to solve Eqs. (3) and (4) which provides a reduction of the number of manipulations in the data processing. Thus, using discrete Fourier transform over spatial coordinate (see Refs. 7, 8) and differential analog instead of differentiation with respect to time, one can write Eqs. (3) and (4) in the following form:

$$\tilde{q}^l_{ps} = \frac{kL}{a^2}\left[\frac{\tilde{T}^l_{ps} - \tilde{T}^{l-1}_{ps}}{\Delta t} + a^2\tilde{T}^l_{ps}(p^2+s^2)\frac{4\pi^2}{N^2}\right]; \qquad (5)$$

$$\tilde{q}^l_{ps} = \frac{k}{L}\tilde{T}^l_{ps} + \frac{kL}{3a^2}\left[\frac{\tilde{T}^{l-1}_{ps} - \tilde{T}^l_{ps}}{\Delta t} + a^2\tilde{T}^l_{ps}(p^2+s^2)\frac{4\pi^2}{N^2}\right], \qquad (6)$$

where

$$\tilde{q}^l_{ps} = \frac{1}{N^2}\sum_{m,j=0}^{N-1} q^l_{mj}\exp\left(-\frac{2\pi i(mp+js)}{N}\right) \qquad (7)$$

and

$$\tilde{T}^l_{ps} = \frac{1}{N^2}\sum_{m,j=0}^{N-1} T^l_{mj}\exp\left(-\frac{2\pi i(mp+js)}{N}\right) \qquad (8)$$

are discrete Fourier transforms of the flux and the temperature, respectively; $q^l_{mj} = q(x_m, y_j, t_l)$.

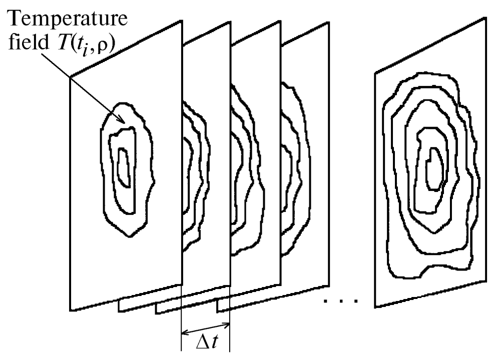

FIG. 2. The temperature field variation versus time.

It should be pointed out that temperature is a measured parameter and, hence, measurement error inevitably introduces distortion into the intensity to be reconstructed due to the use of differentiation in Eqs. (3) and (4). In our case, this distortion manifests itself in unstable summation of Fourier series because the error in the initial data enhances the error in definition of the expansion coefficients. Therefore, when processing experimental data (see Refs. 9, 10) it is advisable to apply filtering of the initial data by their convolution over every coordinate with a stabilizing function, for instance, with the sinc-function:

$$\mathrm{sinc}\,(x) = [\sin(\varkappa_{\max}\, x)]/(\pi x).$$

Here $\varkappa_{\max}$ is a maximum spatial frequency.

The factors of this sort set a limit on the spectrum thus cutting off high-frequency noise component. The use of these filters being well founded owing to simple algorithm structure is possible only in special case when the absence of pulsed objects with the separation less than $2h$ (here h is the discretization step) is *a priori* known. Otherwise, "oversmoothing" of the image is evident and its processing is performed in a dialogue regime. This regime makes it possible to choose the best factor, but significantly moderates processing of great bulk of the initial data.

To automate the selection of optimal filter correlated with errors in the initial data, we use variation principle of selection of possible solutions based on minimizing of smoothing Tikhonov functional (see Refs. 11, 12) in the following form:

$$\mathrm{M}_\alpha = \delta^2 + \alpha\,\Omega, \tag{9}$$

where the first term requires the deviation of desired solution from exact one, $\delta^2 = \int_{-\infty}^{\infty} (T(\rho, t) - T_\alpha(\rho, t))^2\, \mathrm{d}^2\rho$, to be minimal; the second term introduces *a priori* information about the desired solution, in other words, limits the multitude of possible solutions. As *a priori* information we take that the function and its second derivative are finite which is equivalent to omitting of terms with extremely high values of their second derivatives from the multitude of solutions:

$$\Omega = \int_{-\infty}^{\infty} (T_\alpha(\rho, t)^2 + (\Delta T_\alpha\,(\rho, t))^2)\, \mathrm{d}^2\rho.$$

Substitution of the expression for temperature in the FFT form (8) into Eq. (9) and use of Plansherel theorem give:

$$\mathrm{M}_\alpha = \frac{1}{N^2} \times$$

$$\times \left[\sum_{s,p=0}^{N-1} (\tilde{S}^l_{ps} - \tilde{T}^l_{ps})^2 + \alpha \sum_{s,p=0}^{N-1} (\tilde{S}^l_{ps})^2 \left\{ 1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right\} \right].$$

Here $\tilde{T}^l_{ps}$ and $\tilde{S}^l_{ps}$ are coefficients of a noisy solution $T(\rho, t)$ and the smoothed one $T_\alpha(\rho, t)$, respectively. Then, we minimize M_α what is equivalent to equalizing the partial derivatives of the functional with respect to the variables $\tilde{S}^l_{ps}$ to zero:

$$(\tilde{S}^l_{ps} - \tilde{T}^l_{ps}) + \alpha\, \tilde{S}^l_{ps} \left\{ 1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right\} = 0.$$

Thus we obtain the following FFT coefficients stable to the noise :

$$\tilde{S}^l_{ps} = \frac{\tilde{T}^l_{ps}}{1 + \alpha \left[1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right]}.$$

Note that the value of parameter α should correlate with the error in the initial data. Therefore, that α_n value is taken as required for the following equality holds to a preset accuracy:

$$\delta^2 - \sigma^2 = \frac{1}{N^2} \sum_{s,p=0}^{N-1} (\tilde{T}^l_{ps})^2 \times$$

$$\times \frac{\alpha^2 \left\{ 1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right\}^2}{\left\{ 1 + \alpha \left[1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right] \right\}^2} - \sigma^2 = 0,$$

where σ^2 is the variance of the noise in the initial data.

Estimation of the parameter α is a simple task since it can be shown (see Refs. 13, 14) that the function $\delta^2(\alpha)=\gamma(\beta)$ is the falling off and down convex function of β, where $\beta=1/\alpha$. Therefore, the method of Newton tangents can be used to find the root of the following nonlinear equality:

$$\beta_n = \beta_{n-1} - \gamma(\beta_{n-1})/\gamma'(\beta_{n-1}),$$

where

$$\gamma\,(\beta) = \frac{1}{N^2} \sum_{s,p=0}^{N-1} (\tilde{T}^{\,l}_{ps})^2 \frac{\left\{ 1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right\}^2}{\left\{ \beta + \left[1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right] \right\}^2},$$

$$\gamma'(\beta) = \frac{-2}{N^2} \sum_{s,p=0}^{N-1} (\tilde{T}^{\,l}_{ps})^2 \frac{\left\{ 1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right\}^2}{\left\{ \beta + \left[1 + (s^2 + p^2)^2 \left(\frac{2\pi}{N}\right)^4 \right] \right\}^3}.$$

In this paper, the algorithm of reconstruction of laser beam intensity from a noisy temperature field of a thin target on the base of fast Fourier transform is presented. This algorithm provides for reduction of manipulations and calculation time when processing large bulk of the initial data. We succeeded in finding the regularization factors for every term of the series using *a priori* information on limitation of the second derivative and thereby in regularizing FFT series which are stable to the initial data noise and automating the search of regularization parameter correlating with the noise. In the second part of the paper, we are planning to apply the algorithm to process the results of numerical calculations and bench experiments.

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