PROPAGATION OF A LASER BEAM THROUGH THE STRATOSPHERE

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The variance of random wandering and the mean intensity of a focused laser beam, propagating along horizontal and slightly elevated paths in the stratosphere are analyzed in the paper. Features are shown and assessments of influence of discoid inhomogeneities of the refractive index inherent to the stratosphere on the parameters under study are presented.

INTRODUCTION

The laser beam propagation in the troposphere where turbulent fluctuations of the refractive index adhere to the Kolmogorov––Obukhov laws by now is known in sufficient detail.[1–5] The spectrum and effective scales of turbulent fluctuations of refractive index in the stratosphere differ essentially from those in the lower layer of the atmosphere. The experimental data on spectra of refractive index fluctuations in the stratosphere (Refs. 6–10) accumulated in the recent years allow us to study the variance of random wandering and the mean intensity of a laser beam propagating along horizontal and slightly elevated paths in the stratosphere.

MODEL OF THE REFRACTIVE INDEX FLUCTUATIONS SPECTRUM

The refractive index of the air for the optical wavelength range is determined by the expression[1]

$$n = 1 + 10^{-6}\,(80P\,/\,T)\,, \qquad (1)$$

where P is the pressure in millibars and T is the absolute temperature. The refractive index is a function of coordinates ($n = n(\mathbf{r})$, $\mathbf{r} = \{z, x, y\}$) due to regular and random variations of temperature and pressure. For atmospheric conditions the amplitude of temperature fluctuations $\tilde{T} = T - \langle T \rangle$ is much less than the magnitude of mean of temperature $\langle T \rangle$ ($|\tilde{T}| \ll \langle T\rangle$), and the pressure fluctuations $\tilde{P} = P - \langle P\rangle$ in Eq. (1) can be neglected as compared with the temperature ones. As a result for the refractive index $N = n - 1$ we can write

$$N(\mathbf{r}) = \langle N(\mathbf{r})\rangle\,[1 + \nu(\mathbf{r})]\,, \qquad (2)$$

where $\langle N(\mathbf{r}) \rangle = 10^{-6}(80\langle P(\mathbf{r})\rangle/\langle T(\mathbf{r})\rangle)$ is the mean of the refractive index,

$$\nu(\mathbf{r}) = [N(\mathbf{r}) - \langle N(\mathbf{r})\rangle]\,/\,\langle N(\mathbf{r})\rangle = -\,\tilde{T}(\mathbf{r})\,/\,\langle T(\mathbf{r})\rangle \qquad (3)$$

signifies relative fluctuations of the refractive index determined by temperature ones.

For the stratosphere (height h runs between 11 and 50 km) over a wide range of latitude and longitude the mean of the refractive index $\langle N\rangle$ can be considered as a function of the distance R from the Earth's center $\langle N\rangle = \bar{N}(R)$ only. The refractive index decreases with the altitude increase exponentially,[11] therefore, the following function is a good approximation for $\bar{N}(R)$ in the altitude range of $|R - R_1| \sim H_0$:

$$\bar{N}(R) = \bar{N}(R_1)\,\exp\,[-\,(R - R_1)\,/\,H_0]\,. \qquad (4)$$

The parameter H_0 therewith is from 5 to 8 km.

Fluctuations of the refractive index $\nu(\mathbf{r})$ are caused by turbulence of the atmosphere. The source and the nature of turbulent fluctuations of temperature $\tilde{T}$, determining random variations of the refractive index, differ significantly, for instance, in the case of atmospheric boundary layer (up to 1 km height) and for the stratosphere. As known, the main source of dynamic turbulence in the atmospheric boundary layer is the presence of a sufficiently large vertical gradient of the mean wind velocity due to friction of the air against the Earth's surface. The thermal stratification therewith can essentially affect the intensity and scales of fluctuations of air flow velocity. When vertical gradient of the mean temperature differs from adiabatic gradient (the stable or unstable stratifications take place), then the dynamic disturbances will cause temperature fluctuations. The high–frequency temperature (refractive index) fluctuations are characterized by isotropy, and the inertial subrange of the spatial spectrum of refractive index inhomogeneities is described by the Kolmogorov–Obukhov law.[1,12,13]

Contrary to the boundary layer, the main source of dynamic turbulence in the free atmosphere ($h > 1 - 1.5$ km) is the loss of stability by waves, formed in inversion layers, on the tropopause, and in vicinity of other interfaces (gravity–shear waves), or appearing due to deformation of air flows by mountain obstacles (mountainous waves).[14] In the stratosphere at heights of 11 to 25 km the temperature does not vary practically with the altitude, and above this layer the temperature increases due to ozone absorption of the UV solar radiation. Therefore, the temperature stratification at these altitudes is stable. In the stable stratified atmosphere the waves being excited by different external forces appear. The waves become unstable when interacting with each other, the wave crests are collapsed and, as a result of the collapse, the turbulent regions are formed where intense turbulent mixing of previously stratified air takes place. As a result of mixing inside a collapsed volume the stratification occurs corresponding to neutral equilibrium. The mixed volume spreads inside a

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stratified layer of the atmosphere taking the form of a round relatively thin horizontal disc with a very sharp cutoff of air density. Such a mechanism of shaping of discoid inhomogeneities (discoids) has been considered in Ref. 15.

In parallel with airborne[14] and balloonborne[16,17] investigations into turbulence microstructure in the stratosphere, in recent years the extensive experimental data have been obtained for altitudes $h > 20$ km by means of spaceborne observations of star scintillations.[6–10] An essential difference of the temperature fluctuations spectra from those observed in the troposphere has been discovered. That is the result of the above–described peculiarities of formation of turbulent perturbation of temperature field in the stratosphere.

Let us assume that the Cartesian coordinate system origin is at the Earth's center. We shall consider the correlation of the refractive index relative fluctuations at two points $\mathbf{r}_1 = \{z_1, x_1, y_1\}$ and $\mathbf{r}_2 = \{z_2, x_2, y_2\}$. From the Refs. 6, 18, and 19 it follows that fluctuations of the refractive index are homogeneous in altitude and on sphere, that is,

$$<\nu(\mathbf{r}_1)\,\nu(\mathbf{r}_2)> = B_\nu(R_1 - R_2, \theta)\,, \quad (5)$$

where B_ν is the correlation function, $R_j = |\mathbf{r}_j| = \sqrt{z_j^2 + x_j^2 + y_j^2}$ is the distance from the Earth's center to the observation point $\mathbf{r}_j$, $\theta = \arccos[\,\cos\theta_1\cos\theta_2\cos(\varphi_1 - \varphi_2) + \sin\theta_1\sin\theta_2\,]$ is the angle between the radius–vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, $\varphi_j = \arctan(y_j/x_j)$ is the longitude, and $\theta_j = \arcsin(z_j/R_j)$ is the latitude. Since horizontal scales of correlation are much less than the Earth's radius $R_0 \approx 6\,400$ km, then the correlation function B_ν rapidly vanishes at small values of the angle θ. Therefore, in Eq. (5) we may assume that

$$\theta = \sqrt{(\varphi_1 - \varphi_2)^2 + (\theta_1 - \theta_2)^2}\,. \quad (6)$$

Taking also into account that the altitudes h considered here are much less than R_0, the three–dimensional fluctuation spectrum of the refractive index $\Phi_\nu(\kappa_1, \kappa_2, \kappa_3)$ can be represented as

$$\Phi_\nu(\kappa_1, \kappa_2, \kappa_3) = (2\pi)^{-3}\int_{-\infty}^{\infty} dz'\, dx'\, dy'\, B_\nu \times$$
$$\times \exp\{-i\,[\kappa_1 x' + \kappa_2 y' + \kappa_3 z']\}\,, \quad (7)$$

where $x' = R_1 - R_2$, $z' = R_0(\theta_1 - \theta_2)$, and $y' = R_0(\varphi_1 - \varphi_2)$. The correlation function B_ν is written in the form

$$B_\nu = \int_{-\infty}^{\infty} d\kappa_1\, d\kappa_2\, d\kappa_3\, \Phi_\nu(\kappa_1, \kappa_2, \kappa_3) \times$$
$$\times \exp[-i\,(\kappa_1(R_1 - R_2) + \kappa_2 R_0(\varphi_1 - \varphi_2) + \kappa_3 R_0(\theta_1 - \theta_2))]\,. \quad (8)$$

A major part of spaceborne research of the turbulence microstructure is devoted to measurements of one–dimensional vertical spectrum

$$V_\nu^{(\upsilon)}(\kappa_1) = \int_{-\infty}^{\infty} d\kappa_2\, d\kappa_3\, \Phi_\nu(\kappa_1, \kappa_2, \kappa_3) \quad (9)$$

within spatial frequencies $\kappa_{\min} = 2\pi\cdot 10^{-3}$ m^{-1} $\lesssim \kappa_1 \le \kappa_{\max} = 2\pi\cdot 10^{-1}$ m^{-1} (Refs. 6–10). It is established that this spectrum follows the power dependence on κ_1

$$V_\nu^{(\upsilon)}(\kappa_1) \sim \kappa_1^{\mu}\,, \quad (10)$$

where in the range of low spatial frequencies $\kappa_1 \lesssim \kappa_0$ ($\kappa_0 = 2\pi / l_\nu^*$, $l_\nu^* \sim 50$ m) the index of power $\mu \approx -3$, and at $\kappa_1 > \kappa_0$ $\mu \approx -5$ (Ref. 6). Thus, the stratospheric spectrum of turbulence (10) differs considerably from the Kolmogorov–Obukhov one ($V_\nu^{(\upsilon)}(\kappa_1) \sim \kappa_1^{-5/3}$) which is characteristic for troposphere. Among all spaceborne observations of the temperature field microstructure at altitudes $20 \text{ km} \le h \le 45$ km up to the frequencies corresponding to spatial scales about 10 m, the Kolmogorov's frequency dependence of the spectrum has not been identified.[6–10]

By analogy with Ref. 9 we represent the vertical spectrum of relative fluctuations of the refractive index as

$$V_\nu^{(\upsilon)}(\kappa_1) = \frac{C_\nu^2}{(\kappa_m^2 + \kappa_1^2)^{3/2}\,(1 + \kappa_1^2 / \kappa_0^2)}\,, \quad (11)$$

where in contrast to the spectrum model proposed in Ref. 9 the wave number κ_m, characterizing the maximum vertical scale of discoid inhomogeneities, is introduced. In accordance with Eq. (10) $\kappa_m < \kappa_{\min}$ and $\kappa_m \ll \kappa_0$.

Let us estimate the parameter κ_m and the integral vertical scale of the refractive index correlation

$$L_m = \int_0^{\infty} dx'\, B_\nu(x') / \sigma_\nu^2\,, \quad (12)$$

where

$$B_\nu(x') = \int_{-\infty}^{\infty} d\kappa_1\, V_\nu^{(\upsilon)}(\kappa_1)\, e^{i\kappa_1 x'} \quad (13)$$

is the correlation function normalized to the mean square, and $\sigma_\nu^2 = B_\nu(0)$ is the relative variance of refractive index. Taking into account the condition $\kappa_m \ll \kappa_0$ we can obtain from Eqs. (11)–(13)

$$\sigma_\nu^2 = 2\, C_\nu^2 / \kappa_m^2 \quad (14)$$

and

$$L_m = \pi / (2\kappa_m)\,. \quad (15)$$

It follows from Eq. (3) that the variance $\sigma_\nu^2 = <\nu^2>$ is determined by the expression

$$\sigma_\nu^2 = \sigma_T^2 / <T>^2\,, \quad (16)$$

where σ_T^2 is the temperature variance. Eqs. (14) and (16) allow us to obtain

$$\kappa_m = \sqrt{2}\, C\nu <T> / \sigma_T\,. \quad (17)$$

For $<T> = 240$ K, $\sigma_T \sim 1°$ (Ref. 20) and $C_\nu = 0.84\cdot 10^{-5}$m^{-1} (Ref. 9) the wave number κ_m equals approximately $3\cdot 10^{-3}$m^{-1}.

Whence, in accordance with Eq. (15), we have for the vertical scale of refractive index correlation the estimate $L_m \sim 0.5$ km.

Following Ref. 9, the three–dimensional spectrum Φ_ν can be written in the form:

$$\Phi_\nu(\kappa_1, \kappa_2, \kappa_3) = \Phi_\nu^{(0)}\left(\sqrt{k_1^2 + h^2(k_2^2 + k_3^2)}\right) , \quad (18)$$

where η is the parameter characterizing the spectrum anisotropy. For this spectrum a simple coupling exists between $\Phi_\nu^{(0)}$ and one–dimensional spectra: a vertical one, $V_\nu^{(\upsilon)}(\kappa_1)$,

$$\Phi_\nu^{(0)}(\kappa_1) = -\frac{\eta^2}{2\pi\, k_1}\frac{d\, V_\nu^{(\upsilon)}(k_1)}{d\, k_1} , \quad (19)$$

and a horizontal one, for example,

$$V_\nu^{(h)}(\kappa_2) = \int\limits_{-\infty}^{\infty} d\kappa_1 d\kappa_3 \Phi_\nu(\kappa_1, \kappa_2, \kappa_3),$$

$$\Phi_\nu^{(0)}(\kappa_2\, \eta) = \frac{-1}{2\,\pi\eta k_2}\frac{d\, V_\nu^{(h)}(k_2)}{d\, k_2} . \quad (20)$$

On substitution of Eq. (11) into Eq. (19) we can obtain, taking into account (18), the formula for a three–dimensional spectrum. With the use of formulas (11) and (18)–(20) one can find also an explicit form of the dependence of horizontal spectrum $V_\nu^{(h)}$ on κ_2. The experiment on the measurement of the horizontal spectrum of star scintillations has made it possible, using the fitting of theoretical and experimental dependences of the spectrum measured on κ_2, to determine the numerical value of the anisotropy parameter $\eta \approx 160$ (Ref. 9).

Thus, the characteristic horizontal scales $l_h^* \sim \eta\kappa_0^{-1} \sim 8$ km and $L_h^* \sim \eta\kappa_m^{-1} \sim 160$ km correspond to the characteristic vertical scales of refractive index inhomogeneities $l_m^* \sim \kappa_0^{-1} \sim 50$ m and $L_m^* \sim \kappa_m^{-1} \sim 1$ km.

RANDOM WANDERING OF LASER BEAMS IN THE STRATOSPHERE

The main distortions of laser beams in the stratosphere are made by the large–scale inhomogeneities of the refractive index, whose size far exceed the radius of a propagating beam. Along with a regular refraction, these inhomogeneities cause the random deviations of a beam axis from the line–of–sight propagation as well as the random defocusing of a laser beam. The diffraction of a beam on these inhomogeneities can be neglected. Consider the random wandering of a beam in this section.

Let the laser source be located in the stratosphere at an altitude h_1 above the Earth's surface. The radiation, propagating along the path length L, is received in the plane perpendicular to the beam optical axis. Then we shall go from the coordinate system $\{z, x, y\}$ to the coordinate system $\{z', x', y'\}$ with the center, coinciding with the point of laser source location, and with the axis z' directed along the beam optical axis. The relation between the coordinates of these systems is of the form:

$$\begin{cases} z = z'\sin a - x'\cos a , \\ x = R_0 + h_1 + x'\sin a + z'\cos a , \\ y = y' , \end{cases} \quad (21)$$

where α is the zenith angle.

In view of the fact that the refractive index inhomogeneities are large–scale, the expression for the vector of laser beam centroid ρ_d in the observation plane $\{x', y'\}$ can be written in the form (approximation of geometric optics)[21]

$$\rho_d(L) = \int\limits_0^L d\, z'(L - z')\, [\nabla_{\rho'}\, n(z', \rho')] \Big|_{\rho'=\rho_d(z')} , \quad (22)$$

where $\rho' = \{x', y'\}$, $\nabla_\rho = \{\partial/\partial x', \partial/\partial y'\}$, $\rho_d = \{x_d, y_d\}$. The vector of laser beam centroid can be represented as a sum of regular drift of a beam from its geometric axis $\langle\rho_d\rangle = \{\langle x_d\rangle, 0\}$, caused by the altitude variation of $\bar{N}$ throughout the atmosphere, and the fluctuation component $\tilde{\rho}_d = \{\tilde{x}_d, \tilde{y}_d\}$ $(\langle\tilde{\rho}_d\rangle = 0)$ due to the presence of turbulent inhomogeneities on the propagation path:

$$\rho_d(L) = \langle\rho_d(L)\rangle + \tilde{\rho}_d(L) . \quad (23)$$

Consider at first the vertical coordinate of a beam centroid which is defined according to Eq. (22) by the expression

$$\tilde{x}_d(L) = \int\limits_0^L dz'(L - z') \left[\frac{\partial}{\partial x'}\tilde{n}(z', x', y_d(z'))\right]\Big|_{x'=x_d(z')} , \quad (24)$$

where $\tilde{n} = n - \langle n\rangle$ are the refractive index fluctuations. Equation (24) can be simplified. Actually, even in the portion of a beam propagation path, where the main regular beam displacement $\langle x_d\rangle$ occurs, the magnitude of $\langle x_d\rangle$ is far less than the changes in height of beam propagation direction due to either the inclination of beam trajectory to the horizon or the atmospheric sphericity. It allows us to take $x' = 0$ in Eq. (24) as well as $y_d = 0$.

In accordance with Eq. (2) $\tilde{n} = \langle N\rangle\nu$ and $\frac{\partial}{\partial x'}\tilde{n} = \nu\frac{\partial\langle N\rangle}{\partial x'} + \langle N\rangle\frac{\partial\nu}{\partial x'}$. Since the regular variations of the refractive index are smoother as compared to the random ones ($H_0\kappa_m \gg 1$), then $\nu\frac{\partial\langle N\rangle}{\partial x'} \ll \langle N\rangle\frac{\partial\nu}{\partial x'}$. Thus, for the vertical coordinate of the beam centroid we can write the following approximate formula:

$$\tilde{x}_d(L) = \int\limits_0^L d\, z'(L - z')\, \langle N(z', 0, 0)\rangle \frac{\partial\nu(z', 0, 0)}{\partial x'} . \quad (25)$$

Using Eqs. (25) and (5) the variance of vertical beam wandering $\sigma_x^2 = \langle\, \tilde{x}_d^2(L)\, \rangle$ can be expressed in the form

$$\sigma_x^2 = \int\int\limits_0^L d\, z_1'\, d\, z_2'\, (L - z_1')\, (L - z_2')\, [\bar{N}(R_1) \times \times\bar{N}(R_2)\frac{\partial^2}{\partial x_1'\, \partial x_2'}\, B_\nu(R_1 - R_2, \varphi_1 - \varphi_2, \theta_1 - \theta_2)] \Bigg|_{\substack{x_1'=x_2'=0,\\ y_1'=y_2'=0}} , \quad (26)$$

where, according to Eq. (21)

$R_j = [(R_0 + h_1 + x'_j \sin\alpha + z'_j \cos\alpha)^2 + (z'_j \sin\alpha - x'_j \cos\alpha)^2]^{1/2}$;
$\theta_j = \arcsin [(z'_j \sin\alpha - x'_j \cos\alpha) / R_j]$;
$\varphi_j = \arctan [y'_j / (R_0 + h_1 + x'_j \sin\alpha + z'_j \cos\alpha)], \ (j = 1, 2)$.

The expression for the correlation function B_ν follows from Eqs. (8) and (18) after passing in Eq. (8) to new integration variables ($\kappa_1 = \kappa\cos\theta$, $\kappa_2 = \kappa\sin\theta\cos\varphi/\eta$, $\kappa_3 = \kappa\sin\theta\sin\varphi/\eta$) and integration over angular variables θ and φ in view of Eq. (19):

$$B_\nu(R_1 - R_2, 0, \theta_1 - \theta_2) = 2 \int\limits_0^\infty d\kappa\, V_\nu^{(\upsilon)}(k) \cos [\kappa\, |\mathbf{S}|] , \quad (27)$$

where $\mathbf{S} = \{R_1 - R_2, (\theta_1 - \theta_2)R_0/\eta\}$. On substitution of Eq. (27) into Eq. (26) we obtain

$$\sigma_x^2 = -2\int\limits_0^\infty d\kappa\, \kappa^2\, V_\nu^{(\upsilon)}(k) \times$$

$$\times\int\limits_0^L\int dz'_1\, dz'_2\, (L - z'_1)\, (L - z'_2)\, \bar{N}(R_1)\, \bar{N}(R_2) \times$$

$$\times\left[\frac{\partial|\mathbf{S}|}{\partial x'_1}\frac{\partial|\mathbf{S}|}{\partial x'_2}\cos(k|\mathbf{S}|) + \frac{\partial^2|\mathbf{S}|}{\partial x'_1\,\partial x'_2}\frac{\sin(k|\mathbf{S}|)}{k}\right]\Bigg|_{x'_1=x'_2=0} . (28)$$

Taking into account the condition $\sqrt{2R_0H_0} \ll R_0$ the following approximations may be used in Eq. (28):

$R_j \approx R_0 + z'_j \cos\alpha + \sin^2\alpha\, z'^2_j/2R_0$,
$\theta_j \approx z'_j \sin\alpha/R_0$,
$\partial R_j/\partial x_j \approx \sin\alpha$,
$\partial\theta_j/\partial x_j \approx -\cos\alpha/R_0$.

Thus, if we use in Eq. (28) the formula (11) for the vertical spectrum $V_\nu^{(\upsilon)}(\kappa)$ and the formula (4) for $\bar{N}(R_j)$, the problem on estimating the variance of vertical random wandering σ_x^2 amounts to the calculation of a three–fold integral.

When the path length considerably exceeds the longitudinal (along the beam axis) scale of correlation of refractive index fluctuations, we can use an approximation of delta–correlation of these fluctuations along the propagation axis z'.[1–4,21] At horizontal propagation of the beam in the stratosphere, as it was noted above, the correlation scale is about 160 km that is comparable with the path length L (or with the effective thickness of a homogeneous atmosphere in horizontal direction $\sqrt{2R_0H_0} \sim 280$ km). Therefore, the approximation of delta–correlation in Eq. (28) are inapplicable. However, as the analysis of Eq. (28) has shown, due to large values of the anisotropy coefficient ($\eta \sim 160$), for horizontal or slightly elevated paths we can take in Eq. (28) $\eta \to \infty$, i.e., we can use the approximation of a plane–layered (on a sphere) medium that simplifies essentially the calculations of σ_x^2. This approximation will slightly overestimate the values of σ_x^2. But the relative error of the use of the above approximation does not exceed 10%.

As a result, for the case of horizontal and slightly elevated paths ($|\cos\alpha| \ll 1$) from Eq. (28) a more simple formula is derived:

$$\sigma_x^2 = 2\, \bar{N}^2(h_1)\, L^4 \int\limits_0^\infty d\kappa\, \kappa^2\, V_\nu^{(\upsilon)}(\kappa)\, |J(\kappa)|^2 , \quad (29)$$

where

$$J(\kappa) = \int\limits_0^1 d\xi\, (1 - \xi) \exp \{-(A\, \xi^2 + B\, \xi)\, (1 + i\, \kappa\, H_0)\} ; \quad (30)$$

$A = L^2/(2R_0H_0)$; $B = \cos\alpha\, L/H_0$.

From Eq. (29) it follows that for the paths, whose length satisfies one of the conditions $L \gg 1 / (\kappa_0 |\cos\alpha|)$ or $L \gg \sqrt{2R_0 / k_0} \sim 10$ km, a high–frequency range of the spectrum $\kappa > \kappa_0$ does not affect practically the value of variance of beam random wandering σ_x^2. Therefore, after substituting Eq. (11) into Eq. (29) we can assume $\kappa_0^{-1} = 0$. As a result when substituting the integration variable $\kappa = \kappa_m\zeta$ we have:

$$\sigma_x^2 = 2\, C_\nu^2\, \bar{N}^2(h_1)\, L^4 \int\limits_0^\infty \frac{d\zeta\, \zeta^2}{(1 + \zeta^2)^{3/2}} |J(\kappa_m\, \zeta)|^2 . \quad (31)$$

Let us consider the particular cases. When the beam perigee is in the source plane ($\alpha = 90°$) or the condition $|\cos\alpha| \ll \sqrt{H_0 / R_0}$ is fulfilled, then for the path $L \gg \sqrt{2R_0 / k_m}$ from Eq. (31) we obtain the formula

$$\sigma_x^2 = \pi\, C_\nu^2\, \bar{N}^2(h_1)\, L^2\, R_0 / \kappa_m . \quad (32)$$

Assuming $C_\nu = 0.84 \cdot 10^{-5}$, $\bar{N}(h_1 = 20 \text{ km}) = 2 \cdot 10^{-5}$, $L = 500$ km, $R_0 = 6\,400$ km, and $\kappa_m = 2\pi \cdot 10^{-3}$ m^{-1}, from Eq. (32) we estimate the value of $\sigma_x = 4.74$ m.

It is of interest to compare the amplitude of random beam wandering with the value of regular beam drift from the line–of–sight propagation. Within the first approximation of geometric optics the average beam deflection $\langle x_d \rangle$ for the path length L is determined by the formula obtained from Eq. (22)

$$|\langle x_d \rangle| = \bar{N}(h_1) \sqrt{\frac{\pi\, R_0}{2\, H_0}}\, L . \quad (33)$$

Taking into account Eqs. (32) and (33) we obtain:

$$\sigma_x / |\langle x_d \rangle| = C_\nu \sqrt{2\, H_0 / k_m} . \quad (34)$$

For $C_\nu = 0.84 \cdot 10^{-5}$, $H_0 = 6$ km, and $\kappa_m = 2\pi \cdot 10^{-3}$ m^{-1} the ratio $\sigma_x / |\langle x_d \rangle| \sim 10^{-2}$. Thus, the regular beam deflections are by two orders of magnitude more than the random ones.

We consider the case of elevated paths when the condition $\sqrt{H_0 / R_0} \ll |\cos\alpha| \ll 1$ is fulfilled, and in Eq. (30) the atmospheric sphericity can be neglected ($A\xi^2 \approx 0$). In this case the internal integral in Eq. (30) can be calculated analytically.

It is evident that in two cases only we can neglect the atmospheric sphericity, when zenith angle $\alpha < 90°$ (the beam perigee is lacking on the propagation path) at an arbitrary path length L, and for short paths, when $L \ll \sqrt{R_0H_0}$ at an arbitrary angle α. At $\alpha < 90°$ and $L \gg (\kappa_m\cos\alpha)^{-1}$ from Eq. (31) we can obtain the asymptotical formula

$$\sigma_x^2 = 2\, C_\nu^2\, \bar{N}^2(h_1)\, L^2 / (\cos\alpha\kappa_m)^2 . \quad (35)$$

Comparing the magnitude of variance σ_x^2, when the condition $|\cos\alpha| \ll \sqrt{H_0 / R_0}$ (Eq. (32)) is realized, with σ_x^2 in the case when $|\cos\alpha| \gg \sqrt{H_0 / R_0}$ (Eq. (35)), one can see that in the first case the variance of beam wandering is larger by $\frac{\pi}{2} R_0\kappa_m\cos^2\alpha$ times than in the second case. This quantity far exceeds the parameter $\kappa_m H_0 \gg 1$. Such a large difference is explained by the fact that during beam propagation through a distorting layer of the atmosphere in the case of $|\cos\alpha| \ll \sqrt{H_0 / R_0}$ the turbulent inhomogeneities of refractive index are more extended along a beam axis than in the case of $|\cos\alpha| \gg \sqrt{H_0 / R_0}$ due to their anisotropy. Note that formula (35) is valid also at $\alpha > 90°$ for the short path length $L < H_0 / |\cos\alpha|$ if the condition $|\cos\alpha|\kappa_m L \gg 1$ is fulfilled.

Fig. 1 shows the results of calculating the path length dependence of ratio σ_x/σ_{x0} at different zenith angles α. The parameter σ_{x0} is determined by asymptotical formula (32). For $\alpha = 90°$ (curve *3*) with increasing L the ratio σ_x/σ_{x0} is saturated to the level being equal to unity. The curve *1* demonstrates that in the case of $\alpha = 87°$ the ratio σ_x/σ_{x0} rapidly tends to the level determined by the formula (35). Approximately up to the $L = 120$ km the curve *5* ($\alpha = 93°$) coincides with the curve *1* ($\alpha = 87°$) in view of fulfilment of the condition $L < H_0 / |\cos\alpha|$. With further increasing the path length L in the case of $\alpha = 93°$ the dipping a beam in a more dense layers of the atmosphere and increasing the longitudinal (along a beam axis) size of the turbulent inhomogeneities, crossed by a beam, as the path length approaches to the perigee point $L_p = (h_1 + R_0)\cos\alpha$, cause the significant increase of the amplitude of random wandering of a beam. Then , after passing the perigee point when the condition $L \gg L_p$ is realized, the saturation of the ratio σ_x/σ_{x0} to the level $\sigma_x/\sigma_{x0} = \sqrt{2}\exp[R_0\cos^2\alpha/(4H_0)]$ takes place.

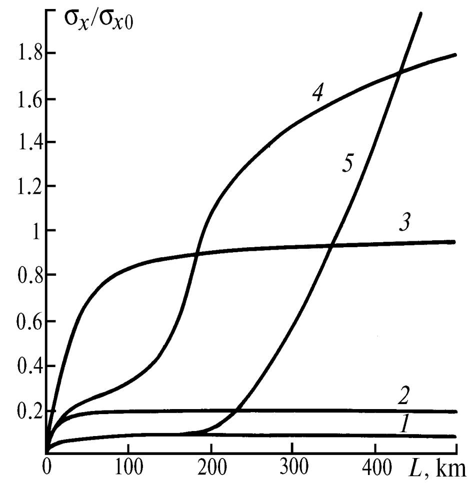

FIG. 1. The path–length dependence of ratio σ_x/σ_{x0} *for* $\kappa_m H_0 = 30$, $\sqrt{2R_0H_0} = 280$ km. *Curves 1–5 correspond to the zenith angles* α *being equal to* 87°, 89°, 90°, 91°, *and* 93°, *respectively.*

To calculate the variance of horizontal beam wandering $\sigma_y^2 = \langle \tilde{y}_d^2 \rangle$, one can use Eq. (26), substituting the derivatives with respect to y_j for the derivatives with respect to x_j. Using the same approach when deriving the formula for σ_x^2, one can obtain:

$$\sigma_y^2 = 2\eta^{-2} L^4 \bar{N}^2(h_1) \int_0^\infty d\kappa\, V_\nu^{(\upsilon)}(\kappa)\, \kappa \int_0^j d\kappa'\, |J(\kappa')|^2 . \quad (36)$$

From Eq. (36) taking into account the condition $\kappa_m H_0 \gg 1$, in particular cases, we have:

$$\sigma_y^2 = 2\eta^{-2} C_\nu^2 \bar{N}^2(h_1) L^2 R_0 k_m^{-1} \ln(2\kappa_m H_0) , \quad (37)$$

when $|\cos\alpha| \ll \sqrt{H_0 / R_0}$ and $L \gg \sqrt{2R_0 / k_m}$,

and

$$\sigma_y^2 = \pi\eta^{-2} C_\nu^2 \bar{N}^2(h_1) L^2 H_0 (\kappa_m \cos^2\alpha)^{-1} , \quad (38)$$

when $|\cos\alpha| \gg \sqrt{H_0 / R_0}$. It follows from the comparison of Eq. (37) with (32) and (38) with (35), that in the first case the vertical beam displacements σ_x are larger than the horizontal ones σ_y by $\eta / \sqrt{(2/\pi)\ln(2k_m H_0)}$ times, and in the second case σ_x is larger than σ_y by $\eta / \sqrt{(\pi/2) H_0 k_m}$ times. The obtained above estimate $\sigma_x = 4.75$ m corresponds to the value of $\sigma_y \approx 0.05$ m, if $\eta = 160$.

MEAN INTENSITY

In the case of extended paths the main distortions of laser beams in the stratosphere will occur in the layer, whose thickness, depending on the geometry of propagation, is far less than path length L. Therefore, for analysis of the beam intensity $I(L, \rho)$ in the stratosphere we can use the phase–screen approximation.[5] In this approximation the expression for intensity of the Gaussian beam $I(L, \rho)$ can be written in a form

$$I(L, \rho) = I_0 \left(\frac{k}{2\pi L}\right)^2 \Bigg| \int_{-\infty}^{\infty} d^2\rho' \exp\{- 1/(2a^2)[1 - i\Omega(1 - L/F)\rho'^2] - i(k/L)\rho\rho' + i\kappa \int_0^L dz'\, n(z', (1 - z'/L)\rho')\} \Bigg|^2 , \quad (39)$$

where I_0 is the intensity at the beam axis in the initial plane ($z' = 0$), a is the effective beam radius, and F is the radius of phase–front curvature at the plane $z' = 0$, $\Omega = ka^2/L$, $k = 2\pi/\lambda$, and λ is the wavelength.

Consider the case of focused beam ($L = F$) that hold the greatest interest. Having regard to that refractive index inhomogeneities are large–scale, we can use in Eq. (39) the series expansion of n being limited to three terms of this series: $n(z', \rho) \approx n(z', 0) + \rho\Delta_\rho n(z', 0) + 1/2\, (\rho\Delta_\rho)^2 n(z', 0)$. This results in

$$I(L, \rho) = I_0 \left(\frac{\kappa}{2\pi L}\right)^2 \int\int_{-\infty}^{+\infty} d^2\rho_1\, d^2\rho_2 \exp\left\{-\frac{1}{2a^2}(\rho_1^2 + \rho_2^2)\right\}$$

×

$$\times \exp\left\{ -\frac{i\kappa}{L}(\rho_1-\rho_2)\left[\rho-\int_0^L dz'\,(L-z')\nabla_\rho n(z',0)\right]\right\}\times$$
$$\times \exp\left\{ \frac{i\kappa}{2}\int_0^L dz'\left(1-\frac{z'}{L}\right)^2[(\rho_1\nabla_\rho)^2-(\rho_2\nabla_\rho)^2]\,n(z',0)\right\}. \quad (40)$$

In Eq. (40) the second term in square brackets of the second exponent is the radius–vector of displacements of a beam centroid which are caused by regular and random refraction of a beam. The last exponent in (40) describes the defocusing of a beam.

Based on Eq. (40) we calculate the mean intensity $< I > = < I(L, \rho_d) >$ which is a result of ensemble–averaging of instantaneous intensity at a point of the beam centroid ρ_d. Assuming that refractive index fluctuations in the last exponent of (40) adhere to the Gaussian law of probability and neglecting the horizontal beam defocusing (along y' axis), we obtain

$$<I> = I_0\,\Omega\,\frac{k}{2\pi L}\int\int_{-\infty}^{+\infty} dx_1\,dx_2 \times$$
$$\times\exp\left\{-\frac{x_1^2+x_2^2}{2a^2}+i\frac{\kappa}{2}<G>(x_1^2-x_2^2)-\frac{\kappa^2}{8}\sigma_G^2(x_1^2-x_2^2)^2\right\}, \quad (41)$$

where

$$G=\int_0^L d\,z'\left(1-\frac{z'}{L}\right)^2\frac{\partial^2 n(z',0)}{\partial x^2} \quad (42)$$

is the parameter characterizing the defocusing of a beam along the x' axis (phase–front curvature of a plane wave in its passage through the equivalent phase screen), $\sigma_G^2 = < G^2 > - < G >^2$. After passing in Eq. (41) to the new integration variables $X = (x_1 + x_2)/2$ and $x = x_1 - x_2$ and integration over X we obtain

$$<I> = I_D\,K\,, \quad (43)$$

where $I_D = I_0\Omega^2$ is the intensity of focused beam when propagated in homogeneous medium;

$$K=\frac{2}{\sqrt{\pi}}\int_0^\infty\frac{dx\,\exp\left\{-\xi^2\left[1+\frac{\gamma^2}{1+2\beta^2\xi^2}\right]\right\}}{\sqrt{1+2\beta^2\xi^2}} \quad (44)$$

is the factor of turbulent decrease of intensity, $\gamma = \Omega<G>L$ and $\beta = \Omega\sigma_G L$ are the parameters characterizing the laser beam regular defocusing and random one, respectively. At $\beta \approx 0$ the factor K is equal to $(1 + \gamma^2)^{-1/2}$. If $\gamma \approx 0$, then $K = 1 - \beta^2/2$ at $\beta \ll 1$ and $K=\sqrt{\frac{2}{\pi}\frac{\ln(2\beta)}{\beta}}$ at $\beta \gg 1$.

For the variance σ_G^2, using Eqs. (42), (2), (5), and (6) one can write the expression

$$\sigma_G^2=\int\int_0^L d\,z_1\,d\,z_2\left(1-\frac{z_1}{L}\right)^2\left(1-\frac{z_2}{L}\right)^2\bar N(R_1)\,\bar N(R_2)\times$$
$$\times\frac{\partial^4}{\partial x_1^2\,\partial x_2^2}B_\nu(R_1-R_2,\,\varphi_1-\varphi_2,\,\theta_1-\theta_2)\Bigg|_{\substack{x_1=x_2=0,\\ y_1=y_2=0}}. \quad (45)$$

Performing the calculations analogous to those made when deriving the formula for σ_x^2, for σ_G^2 from Eq. (45) we obtain

$$\sigma_G^2 = 2\,\bar N^2(h_1)\int_0^\infty d\kappa\,\kappa^4\,V_\nu^{(\upsilon)}(\kappa)\,|\tilde J(\kappa)|^2\,, \quad (46)$$

where

$$\tilde J(\kappa)=\int_0^1 d\xi\,(1-\xi)^2\exp\{-(A\,\xi^2+B\,\xi)\,(1+i\,\kappa\,H_0)\}\,. \quad (47)$$

The analysis of Eq. (46) with the use of Eq. (11) for the spectrum $V_\nu^{(\upsilon)}(\kappa)$ has shown that in contrast to random displacements of a beam, where the main contribution is made by inhomogeneities of the refractive index with scales about κ_m^{-1}, the random beam defocusing is primarily determined by the inhomogeneities whose dimensions are about κ_0^{-1}. In this case for the estimates σ_G^2 in the formula for $V_\nu^{(\upsilon)}(\kappa)$ one can assume that $\kappa_m = 0$. As a result, from Eqs. (46) and (11) we obtain

$$\sigma_G^2 = 2\,\bar N^2(h_1)\,L^2\,C_\nu^2\,k_0^2\int_0^\infty d\zeta\,\zeta\,(1+\zeta^2)^{-1}\,|\tilde J(\kappa\zeta)|^2\,. \quad (48)$$

Provided that $|\cos\alpha| \ll \sqrt{H_0/R_0}$ and $L \gg \sqrt{2R_0 k_0}$ from Eq. (48) we have

$$\sigma_G^2=\frac{1}{2}\pi^2\,\bar N^2(h_1)\,C_\nu^2\,R_0\,\kappa_0\,. \quad (49)$$

At zenith angles $\alpha < 90°°$ and if $|\cos\alpha| \gg \sqrt{H_0/R_0}$ and $L \gg (|\cos\alpha|\kappa_0)^{-1}$ Eq. (48) transforms to the following expression:

$$\sigma_G^2 = 2\,\bar N^2(h_1)\,C_\nu^2\,\ln(\kappa_0\,H_0)\,/\,\cos^2\alpha\,. \quad (50)$$

This formula is also valid at $\alpha > 90°$ if the path lengths are relatively short: $L < H_0/|\cos\alpha|$.

Let us estimate the ratio $\gamma^2/\beta^2 \equiv < G >^2/\sigma_G^2$ for extended ($L > \sqrt{2R_0/H_0}$) horizontal ($|\cos\alpha| \ll \sqrt{H_0/R_0}$) propagation paths, using the approximation

$$<G>=\frac{\sqrt{\pi}}{2}\frac{\bar N(h_1)}{H_0}\sqrt{2\,R_0\,/\,H_0}\,, \quad (51)$$

which is obtained as a result of averaging Eq. (42) with the use of Eq. (4). For $H_0 = 6$ km, $C_\nu = 0.84\cdot 10^{-5}$ m^{-1}, and $\kappa_0 = 2\pi/50$ m^{-1} from Eqs. (49) and (51) we have estimate $\gamma^2/\beta^2 \approx 0.16$. Therefore, in this case the decrease of intensity is caused mainly by a beam defocusing due to random inhomogeneities of the refractive index, so that when calculating the mean intensity we can neglect regular refraction putting $\gamma \approx 0$ in Eq. (44).

In Fig. 2 the results of calculating the on–axis intensity of the focused beam as a function of the zenith angle at different values of the Fresnel ratio Ω are plotted. It is seen that the main variations of mean intensity due to stratospheric turbulence occur in the range $\alpha \sim 90°$. For $\alpha < 89°$ we can neglect these variations. The more is the parameter Ω the more is the decrease of beam on–axis intensity as compared to that in a homogeneous medium. For $\Omega \lesssim 1$ the discoid inhomogeneities of the refractive index do not practically affect the beam intensity.

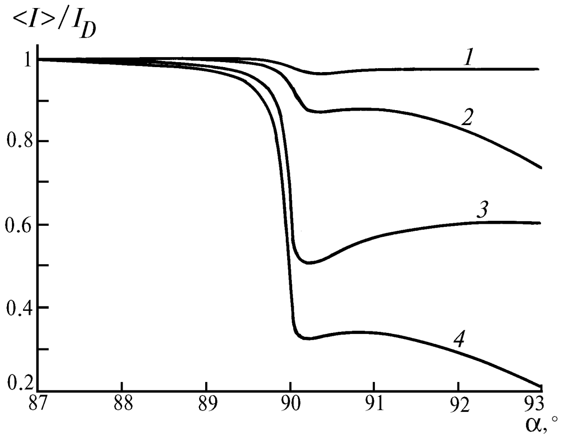

FIG. 2. The factor of intensity decrease $<I>/I_D$ *as a function of zenith angle* α *for* $h_1 = 20$ km, $\overline{N} = 2{\cdot}10^{-5}$, $C_v = 0.84{\cdot}10^{-5}$ m^{-1}, *and* $\kappa_0 = 2\pi/50$ m^{-1}. *Curves 1 and 2 correspond to the parameter* $\Omega = 1$. *Curves 3 and 4 correspond to* $\Omega = 10$. *Curves 1 and 3 are calculated for* $L = 500$ km, *curves 2 and 4 are calculated for* $L = 1000$ km.

At $h_1 = 20$ km, $L > \sqrt{2R_0H_0} \sim 280$ km, and $\alpha = 93°$ the point of perigee of a beam trajectory will be located at the lower line of the stratosphere ($L_p \approx 11$ km), where along with discoids the small–scale Kolmogorov turbulence exists. The estimates of mean intensity, based on the model for C_n^2 (Ref. 22) at the altitudes from 11 to 20 km, show that the influence of the small–scale Kolmogorov turbulence for such a path ($h_1 = 20$ km, $\alpha = 93°$) is greater than the discoid influence.

CONCLUSION

1. Based on the vertical spectrum of refractive index fluctuations generalized to the entire range of spatial frequencies ($0 \le \kappa_1 \le \infty$) we have obtained the formulas for variances of the transverse random displacements σ_x^2 and σ_y^2 and for variance of the phase–front curvature σ_G^2 of a beam. It is shown that the variance of the vertical wandering σ_x^2 is determined mainly by the large–scale refractive index inhomogeneities with the size of the order $\sim \kappa_m^{-1}$ and the variance σ_G^2 is determined by inhomogeneities of the size of $\sim \kappa_0^{-1}$. Both σ_x^2 and σ_G^2 significantly depend on the zenith angle of the propagation direction due to strong anisotropy of the refractive index fluctuations as well as the sphericity of the atmosphere.

2. The amplitude of vertical random wandering of a beam σ_x on the one hand is by two orders of magnitude less than that caused by regular refraction $< x_d >$, and on the other hand it is approximately by two orders of magnitude more than the amplitude of horizontal random displacements of a beam σ_y.

3. Along the extended paths ($L > 500$ km) the broadening of a focused beam, caused by random inhomogeneities, is more essential than that induced by regular variations of refractive index in the stratosphere. The on–axis intensity of a focused beam therewith is at least half as that in a homogeneous medium, if the conditions $\alpha \gtrsim 90°°$ and $\Omega \gtrsim 10$ are fulfilled.

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