ON THE DETERMINATION OF PARAMETERS OF SMOKE PLUMES BASED ON BACKSCATTERING OF OPTICAL RADIATION

V.A. Banakh and I.N. Smalikho

Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk

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Backscattering of a cw optical beam when sensing smoke plumes has been theoretically analyzed. The possibility of determining the concentration of plume particles and the power of local industrial emissions is demonstrated based on measurements of the backscattered light intensity distribution over the telescope focal plane.

The lidar methods can be employed in monitoring of the atmosphere over industrial areas for sensing such local emissions of pollutants as smoke plumes. A large number of publications can be found in the literature (see, e.g., Refs. 1–5) where different approaches to this problem and results of sensing are described.

However, the problems of employing continuous optical radiation for these purposes have not yet been studied though some alternative methods can be suggested here. The present paper deals with the analysis of backscattering of directed optical radiation by a smoke plume and with the study of possibility of determining particle concentration and power of effluents from plant stacks from the distribution of intensity of light scattered by a plume over the plane of its image behind the collecting lens of a receiving telescope.

Let the optical radiation source be in the plane $z = -z_0$, where z_0 is the distance from the smoke plume of the cross sectional size a $\ll z_0$. Let also the radiation propagating along the z axis, after it is scattered by the smoke plume particles, be received with a telescope in the plane of sounding radiation source.

The scattering of an optical beam is described by the following equation:[6]

$$\frac{\mathrm{d}\, I\mathrm{d}(\tilde{\mathbf{r}}(z'),\, \mathbf{S})}{\mathrm{d}z'} + \sigma_t\, \rho(\tilde{\mathbf{r}}(z')) \times$$

$$\times \Big[I_d(\tilde{\mathbf{r}}(z'),\, \mathbf{S}) - B(\mathbf{S},\, \mathbf{S}_0)\, U_0(\tilde{\mathbf{r}}(z')) \Big] =$$

$$= \frac{\sigma_t}{4\pi}\, \rho(\tilde{\mathbf{r}}(z')) \int\limits_{4\pi} \mathrm{d}\omega' B(\mathbf{S},\, \mathbf{S}') I_d(\tilde{\mathbf{r}}(z'),\, \mathbf{S}')\ , \qquad (1)$$

where $I_d(\mathbf{r},\, \mathbf{S})$ is the diffuse brightness of radiation at a point $\mathbf{r} = \{z,\, x,\, y\}$ in the direction $\mathbf{S} = \{\cos\theta,\, \sin\theta\, \cos\varphi,\, \sin\theta\, \sin\varphi\}$, $\tilde{\mathbf{r}}(z') = \{z'\cos\theta,\, x - (z/\cos\theta - z')\sin\theta\, \cos\varphi,$ $y - (z/\cos\theta - z')\sin\theta\, \sin\varphi\}$, $z' \in [-\infty,\, z/\cos\theta]$, σ_t is the cross section of the total scattering, ρ is the concentration of smoke particles, B is the scattering phase function, $\mathbf{S}_0 = \{1,\, 0,\, 0\}$ is the unit vector coinciding with direction of the light beam incidence on the plume,

$$U_0(\mathbf{r}) = \frac{1}{4\pi} \int\limits_{4\pi} \mathrm{d}\omega' I_0(\mathbf{r},\, \mathbf{S}) \qquad (2)$$

is the coherent component of intensity,

$$\int\limits_{4\pi} \mathrm{d}\omega = \int\limits_0^{2\pi} \mathrm{d}\varphi \int\limits_0^{\pi} \mathrm{d}\theta\, \sin\theta\ , \text{ and}$$

$$I_0(\mathbf{r},\, \mathbf{S}) = I_0\left(\tilde{\mathbf{r}}\left(-\frac{z_0}{\cos\theta}\right),\, \mathbf{S}\right) \exp\left[-\sigma_t \int\limits_{-\infty}^{z/\cos\theta} \mathrm{d}z' \mathrm{r}(\tilde{\mathbf{r}}(z'))\right] \qquad (3)$$

is the attenuated brightness of incident radiation (total brightness $I = I_0 + I_d$).

The boundary conditions for Eq. (1) have the form

$$I_d(\tilde{\mathbf{r}}(z'),\, \mathbf{S}) \Big|_{|z'| = \infty} = 0\ . \qquad (4)$$

Let the solution of Eq. (1) be represented by a series of approximations of the theory of multiple scattering of waves (TMSW)

$$I_d(\mathbf{r},\, \mathbf{S}) = \sum_{j=1}^{\infty} I_j\, (\mathbf{r},\, \mathbf{S})\ . \qquad (5)$$

The first approximation of the TMSW or the single scattering approximation has the form

$$I_1(\mathbf{r},\, \mathbf{S}) = \sigma_t\, B(\mathbf{S},\, \mathbf{S}_0) \int\limits_{-\infty}^{z/\cos\eta} \mathrm{d}z' \rho(\tilde{\mathbf{r}}(z')) \mathrm{U}_0(\tilde{\mathbf{r}}(z')) \times$$

$$\times \exp\left\{-\sigma_t \int\limits_{z'}^{z/\cos\theta} \mathrm{d}z'' \rho(\tilde{\mathbf{r}}(z''))\right\}\ . \qquad (6)$$

For the subsequent terms of the series it is possible to write a recurrent relation

$$I_j(\mathbf{r},\, \mathbf{S}) = \frac{\sigma_t}{4\pi} \int\limits_{-\infty}^{z/\cos\theta} \mathrm{d}z' \rho(\tilde{\mathbf{r}}(z')) \exp\left\{-\sigma_t \int\limits_{z'}^{z/\cos\theta} \mathrm{d}z'' \rho(\tilde{\mathbf{r}}(z''))\right\} \times$$

$$\times \int\limits_{4\pi} \mathrm{d}\omega' B(\mathbf{S},\, \mathbf{S}')\, I_{j-1}(\tilde{\mathbf{r}}(z'),\, \mathbf{S}')\ . \qquad (7)$$

Let the scattered radiation be collected with a receiving optical system in the plane of a smoke plume image. We assume that diffraction on transmitting and receiving apertures can be neglected since $ka_0^2 \gg z_0$, $ka_t^2 \gg z_0$, where $k = 2\pi/\lambda$, λ is the wavelength, a_0 is the beam radius, and a_t is the radius of the receiving telescope lens. Let also the typical size of a smoke plume cross section be $a \gg a_0, a_t$. Then the intensity of scattered light in the plane of the plume image for a collimated Gaussian sounding beam directly related to its brightness

$$U(\mathbf{R}) = \frac{\pi\, a_0^2\, a_t^2}{(a_0^2 + a_t^2)\, F_t^2}\, I_d\!\left(-\, z_0,\ \mathbf{R}\,\frac{z_0}{F_t},\ \mathbf{S}_\pi\right), \tag{8}$$

where $\mathbf{R} = \{x, y\}$ is the radius vector in the plane perpendicular to the z axis, F_t is the focal length of the receiving telescope lens, and $\mathbf{S}_\pi = \{-1, 0, 0\}$. In this case a coherent component of the beam intensity entering into Eq. (6) can be written by a simple formula

$$U_0(\mathbf{r}) = \mathrm{U}^0 \exp\left\{-\frac{\mathbf{R}^2}{a_0^2} - \sigma_t \int_{-\infty}^{z} \mathrm{d}z' \ \ (z', \mathbf{R})\right\}, \tag{9}$$

where U^0 is the beam intensity on its axis in the plane of the radiation source output.

The intensity U, as in the case with Eq. (5), is represented by a series

$$U(\mathbf{R}) = \sum_{j=1}^{\infty} U_j(\mathbf{R})\,. \tag{10}$$

The first approximation can easily be found from Eqs. (6) and (8). At the point $\mathbf{R} = 0$ the relation for U_1 has the form

$$U_1 = q\,[1 - \exp(-2\tau)]\,, \tag{11}$$

where

$$q = \frac{\pi\, a_0^2\, a_t^2\, U^0}{2(a_0^2 + a_t^2)\, F_t^2}\, B(\mathbf{S}_\pi, \mathbf{S}_0) \ \text{and}\ \tau = \sigma_t \int_{-\infty}^{+\infty} \mathrm{d}z' \ \ (z', 0)$$

is the optical thickness. Similar result is given in Ref. 6. It should be noted that the expression for U_1 has been derived irregardless of the ratio of a particle size to the radiation wavelength. However, for the consequent approximations U_j this ratio plays an important role.

For large particles $(kr_0)^2 \gg a/\mathrm{a}_0$ (r_0 is the characteristic radius of particles), the consequent terms of series (10) can be found from Eqs. (6) – (8):

$$U_j = q\, W^{j-1}\left[1 - \mathrm{e}^{-2s} \sum_{m=0}^{j-1} \frac{(2\tau)^m}{m!}\right], \tag{12}$$

where $W = \sigma_s/\sigma_t$ is the albedo of the smoke plume particles, $\sigma_t = \sigma_s + \sigma_a$, σ_s and σ_a are the cross sections of scattering and absorption, respectively. In so doing, we can sum up series (10) and, as a result, one can write for intensity the equation

$$U = q\, \frac{1 - \exp[-(1 - W)\, 2\tau}{1 - W}\,. \tag{13}$$

A solution, similar to Eq. (13), for the case of a plane wave incident on an unbounded scattering layer of some thickness has been found earlier[6] from the radiation transfer equation in a small–angle approximation. As was noted in Ref. 6 it is necessary to take into account the fact that the angle between the directions of radiation incidence and observation must be larger than $1/(kr_0)$.

If the condition of large particles is invalid it is impossible to find an asymptotical expression for the intensity U. Therefore, we shall estimate only the second term in series (10). To do this, we shall assume the optical beam to be incident perpendicularly on a smoke plume. In addition, the averaged distribution of particle concentration over the plume cross section is approximated by a Gaussian model[7,8]

$$\rho(z, x, y) = \rho_0(y) \exp\left\{-\frac{z^2}{a_z^2(y)} - \frac{x^2}{a_x^2(y)}\right\}, \tag{14}$$

where a_z and a_x are the effective size of the plume along the z and x axes, respectively.

An approximate expression for U_2 can be obtained when the conditions kr_0 3 and $P \gg \max(1, \tau)$ are satisfied

$$U_2 \approx U_1\, \frac{2}{\pi}\, B(\mathbf{S}_0, \mathbf{S}_0)\, \frac{\ln(2.94\, P)}{P}\, F(\tau)\,, \tag{15}$$

where $P = a_z/\sqrt{\mathrm{a}_0^2 + \mathrm{a}_t^2}$, $\tau = \sqrt{\pi}\, \sigma_t\, \rho_0 a_z$. The view of the function

$$F(\tau) = \frac{\tau \mathrm{e}^{-\tau}}{1 - \mathrm{e}^{-2\tau}} \int_{-\infty}^{+\infty} \mathrm{d}z\ z\ \mathrm{e}^{-z^2} \exp[\tau\, \mathrm{erf}(z)]$$

is represented in Fig. 1.

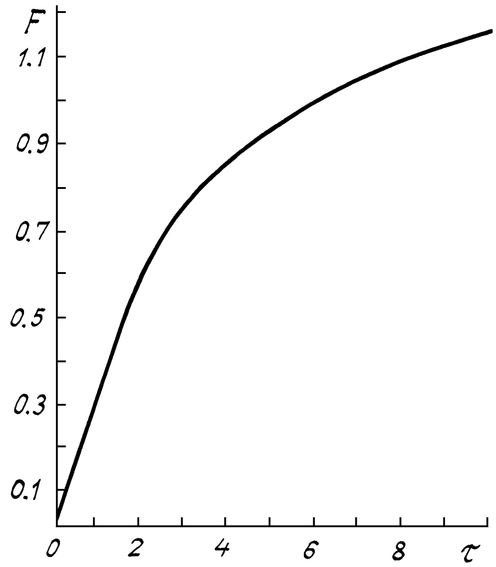

FIG. 1. Function F(τ).

Let us estimate the ratio U_2/U_1 for $kr_0 \ll 1$ when the scattering phase function in the forward direction $B \approx \frac{3}{2} W$. Let us now assume that $\tau = 5$ and $P = 50$. Then it follows from Eq. (15) that $U_2/U_1 \approx 0.09\,W$. Therefore, the contribution of the second approximation to the intensity of the scattered radiation U is at least an order of magnitude lower than that from the first one U_1.

Taking into account the above mentioned example as well as other estimates of U_2 made for different B, τ, and P, one arrives at a conclusion that for small particles $(kr_0\ \ 3)$ and optical beams with radii much smaller than the size of a smoke plume cross section $(P \gg \max(1, \tau))$ it is possible to employ only the first approximation of the TMSW for calculating U.

DETERMINATION OF PARTICLE CONCENTRATION

Consider now the case of large particles $((kr_0)^2 \gg P)$. Then to extract information about the concentration ρ_0 for *a priori* known values of the absorption cross section σ_a, a_z, and a_x one should measure $U^{(1)}$ and $U^{(2)}$ where $U^{(1)}$ is the mean intensity measured using the above scheme of sounding, and $U^{(2)}$ is the corresponding value of the intensity measured when the beam is directed to the smoke plume at an angle α with respect to the z axis in the zx plane.

As follows from Eqs. (13) and (14)

$$\frac{U^{(1)}}{U^{(2)}} = \frac{1 - \exp[-(1 - W)\,2\tau]}{1 - \exp[-(1 - W)\,2\tau\beta]}\,, \tag{16}$$

where $\beta = \exp[-(\sin\alpha\, z_0/a_x)^2]$. Assuming the angle $\alpha = \arcsin(\sqrt{\ln 2}\, a_x/z_0) \approx \sqrt{\ln 2}\, a_x/z_0$ we can derive from Eq. (16) a simple formula for particle concentration

$$\rho_0 = \frac{1}{\sqrt{\pi}\,\sigma_a\, a_z} \ln\left[\frac{U^{(2)}}{U^{(1)} - U^{(2)}}\right]. \tag{17}$$

MEASUREMENTS OF THE STACK EMISSION POWER

Let us define the stack emission power Q as a number of particles N transported through the zx plane per unit time[1]

$$Q = \frac{\mathrm{d}N}{\mathrm{d}t} = \upsilon_y \frac{\mathrm{d}N}{\mathrm{d}y}\,, \tag{18}$$

where υ_y is the component of wind velocity along the y axis which satisfies the condition $\upsilon_y \gg \tilde{\upsilon}_y$ ($\tilde{\upsilon}_y$ is the deviation of the velocity from the value υ_y which takes place in the region of smoke plume localization in the zx plane).

Taking into account that

$$\frac{\mathrm{d}N}{\mathrm{d}y} = \iint\limits_{-\infty}^{+\infty} \mathrm{d}z\, \mathrm{d}x\, \rho(z, x, y)\,, \tag{19}$$

from Eqs. (14) and (18) we have for the stack emission power

$$Q = \upsilon_y\, \pi\, a_z\, a_x\, \rho_0\,.$$

The use of Gaussian model (14) assumes here that fluctuations of the particle concentration can be neglected. The analysis of Eq. (18) shows that the latter can be justified only in conditions of weak fluctuations of the scattered radiation intensity, i.e., when $\sigma_U^2/\langle U\rangle^2 \ll 1$, where $\langle U\rangle$ and σ_U^2 are the mean value and the variance of radiation intensity, respectively. In practice, high–power large–scale wind eddies and strong turbulent mixing cause large variations of the concentrations ρ. This is means that in this case Gaussian model (14) is inaplicable for calculating U. However, even without direct averaging of U it is possible to determine instantaneous quantities of the emission power Q. In order to do this one needs to forming the incident beam so that it overlaps with the smoke plume cross section. This can be performed with a diverging beam with the initial wavefront curvature $F_0^{-1} > a/(a_0 z_0)$. The scattered radiation intensity distribution, with the account for the beam divergence, has the form

$$U(\mathbf{R}) = T(\mathbf{R}) \times \left[1 - \exp\left\{-2\sigma_a \int\limits_{-\infty}^{+\infty} \mathrm{d}z\, \rho\left(z, \frac{z_0}{F_t}\mathbf{R}\right)\right\}\right], \tag{20}$$

where

$$T(\mathbf{R}) = \frac{\pi\, a_t^2}{F_t^2} \cdot \frac{U^0 B(\mathbf{S}_\pi, \mathbf{S}_0)\, \sigma_t}{2\,(1 + z_0/F_0)^2\, \sigma_a} \exp\left\{-\left(\frac{z_0}{F_t}\right)^2 \frac{\mathbf{R}^2}{a_0^2(1 + z_0/F_0)^2}\right\}.$$

Finally, from Eqs. (18) – (20) we find

$$Q = \frac{z_0}{F_t}\frac{\upsilon_y}{2\sigma_a} \int\limits_{-\infty}^{+\infty} \mathrm{d}x \ln\left[\frac{1}{1 - U(x, y)/T(x, y)}\right]. \tag{21}$$

If *a priori* information about optical parameters of the smoke plume particles $B(\mathbf{S}_\pi, \mathbf{S}_0)$, σ_a, and σ_s is available then it is possible, using measurement data on the distribution $U(x)$, to determine the instantaneous emission power in the plane y = const from Eq. (21).

Thus, using measurement data on the scattered optical radiation intensity distribution over the telescope focal plane makes it possible to determine such parameters of a smoke plume as the particle concentration and the emission power of local sources of pollutions.

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