DESCRIPTION OF NONLINEAR ABERRATIONS OF WAVE BEAMS WITH NONCIRCULAR CROSS SECTIONS DUE TO THERMAL BLOOMING

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An analytical solution of the radiation transfer equation is constructed in the space of Lagrangian variables spatially related to three–dimensional trajectories of optical rays. Using this solution the parabolic equation for the wave eikonal is reduced to a system of five ordinary differential equations. This system of equations describes the lowest–order aberrational distortions of noncircular beams with axial symmetry in media with an arbitrary mechanism of nonlinearity.

To illustrate the efficiency of the obtained system the nonlinear part of the dielectric constant of media with the Kerr and thermal types of nonlinearity has been calculated. Evolutions of wave aberrations of axially symmetric beams in both types of nonlinear media are compared.

Propagation of light beams through nonlinear media is accompanied by their distortions of aberration type. In the so–called aberration–free approximation such nonlinear distortions are reduced to self–action of beams, during which the parabolic shape of their wavefront is conserved.[1-3] Aberrations of the fourth order, in particular, spherical aberrations, as well as aberrations of higher orders, can significantly alter the character of the beam propagation, introducing such specific features into it as generation of aberration rings, limitation of the transverse size of Gaussian–type beams, redistribution of the field at the near focal zone, etc.[4-8]

An even wider spectrum of nonlinear aberrations is associated with self–action of noncircular beams, in particular, of elliptic ones.[4-5] No mathematical apparatus has been developed so far, which would make it possible to study aberrations of such beams that made the investigation of aberrations of even the lowest orders very difficult. Moreover, it was impossible to study the suppression or induced development in nonlinear media of aberrations, which were initially introduced into the beam wavefront. All that hampered solving a wide range of problems in optics of nonlinear media.

This paper develops the aberrational theory of thermal blooming of light beams with degenerated central symmetry of their cross section for the case of the axially symmetric (elliptic) beams. Such a self–action takes place in media with arbitrary mechanisms of nonlinearity. Aberration distortions of the fourth order are self–consistently described by a system of ordinary differential equations, using three additionally introduced aberration functions. The theory agrees with its earlier developed simplified modifications,[6,7] and naturally generalizes them. The methodology used may be applied to description of aberrational distortions of other even orders, which are characteristic of noncircular beams.

Prior to description of nonlinear aberrations of beam with axial symmetry, whose field remain invariable at either simultaneous or independent change of signs of transverse coordinates x and y, let us first determine a wavefront profile characteristic of such beams. For this purpose we consider the characteristic equation of the ray trajectories

$$d\rho/dz = \nabla_\perp s \, , \tag{1}$$

which relates the change of the transverse vector $\rho = \{x, y\}$ of a running point of the ray trajectory along the longitudinal coordinate z to the addition to the eikonal of the plane wave $s(\mathbf{r})$, where $\mathbf{r} = \{x, y, z\}$ and $\nabla_\perp = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ is the transverse Hamiltonian.

Within the so–called aberration–free approximation, the ray trajectories (1) of an elliptic beam are completely defined by the vector function $\xi = \{\xi_1, \xi_2\}$,

$$\xi_1 = x/a_{10} f_1(z), \;\; \xi_2 = y/a_{20} f_2(z) \, , \tag{2}$$

which is conserved along each trajectory; a_{10} and a_{20} are the characteristic initial and $f_{1,2}(z)$ the running dimensionless semi–axes of the cross section of an elliptic beam.[4,5] In accordance with Eqs. (1) and (2) we have

$$\partial s/\partial \xi_i = a_{i0} f_i (a_{i0} f_i' \xi_i + a_{i0} f_i \, d\xi_i/dz) \;\; (i = 1, 2) \, , \tag{3}$$

and, therefore, integrating Eq. (3) we obtain

$$s(\mathbf{r}) = s_0(z) + \sum_{i=1}^{2} a_{i0}^2 f_i^2 \left(\frac{f_i' \xi_i^2}{2 f_i} + \int_{(\xi=0)}^{(\xi)} \frac{d\xi_i}{dz} \, d\xi_i \right) , \tag{4}$$

where $s_0(z)$ is the additional phase shift due to the change in the velocity of wave propagation.

If aberrations are present, the vector $\xi(\mathbf{r})$ is no longer conserved, and it is characterized (as may be easily calculated) by the $n = N(N + 5)/2$ dimensionless aberration functions in accounting for aberrations described in the eikonal by products of various even powers of transverse coordinates x and y, right up to the terms of the cumulative power $2(N + 1)$ ($N = 1, 2, \ldots$). In the particular case of aberrations of the fourth order ($N = 1$), being considered below, the number of the needed aberration functions is $n = 3$.

Denoting the aberration functions as $A_{1,2}(z)$ and $A_{12}(z)$ we may express the integrands in Eq. (4), which satisfy the condition of integration

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$$\frac{a_{i0} f_i}{a_{3-i,0} f_{3-i}} \frac{\partial}{\partial \xi_{3-i}} \left(\frac{d\xi_i}{dz}\right) = \frac{a_{3-i,0} f_{3-i}}{a_{i0} f_i} \frac{\partial}{\partial \xi_i} \left(\frac{d\xi_{3-i}}{dz}\right) \quad (i = 1, 2) ,$$

as follows:

$$\frac{d\xi_i}{dz} = \frac{\xi_i^3}{2} \frac{dA_i}{dz} + \frac{a_{3-i,0} f_{3-i}}{2a_{i0} f_i} \xi_i \xi_{3-i}^2 \frac{dA_{12}}{dz} \qquad (i = 1, 2) , \qquad (5)$$

so that the eikonal of an axially symmetric beam itself is represented in the form

$$s(\xi, z) = s_0(z) + \frac{1}{2} \sum_{i=1}^{2} a_{i0}^2 f_i^2 \left(\frac{f_i'}{f_i} + \frac{1}{4} A_i' \xi_i^2\right) \xi_i^2 + $$
$$+ \frac{1}{4} a_{10} a_{20} f_1 f_2 A_{12}' \xi_1^2 \xi_2^2 . \qquad (6)$$

The law of intensity distribution of a beam distorted by aberrations can be found from the parabolic equation of the radiation transfer

$$\partial I/\partial z + \nabla_\perp (I \nabla_\perp s) + dI = 0 , \qquad (7)$$

where I is the radiation intensity and δ is the absorption coefficient of the medium.

The solution of Eq. (7) satisfying the condition

$$I(\mathbf{r}, 0) = I_0 (x_0) , \qquad (8)$$

at the boundary $z = 0$ of the nonlinear medium, where $\xi(\rho, 0) = \xi_0(\rho)$ can be expected to have the following form:

$$I(\mathbf{r}) = I_0(\mathbf{N}) \, e^{-\delta z} |J(\Xi)| , \qquad (9)$$

where $\Xi = \{\Xi_1, \Xi_2\}$ is a continuous transverse vector, coinciding with ξ_0 at z=0, its transformation from the variables $\Xi_{1,2}(\xi, z)$ to the variables $\xi_{10} = x/a_{10}$, $\xi_{20} = y/a_{20}$ ($f_{1,2}(0) = 1$) or to the variables $\xi_{1,2}$ is performed with the help of the Jacobian

$$J(\Xi) = \partial(\Xi_1, \Xi_2)/\partial(\xi_{10}, \xi_{20}) = (f_1 f_2)^{-1} \partial(\Xi_1, \Xi_2)/\partial(\xi_1, \xi_2). \quad (10)$$

It can easily be seen that representations (9) and (10) satisfy the law of conservation of the total radiation power of the intensity $I(\mathbf{r}) \exp(\delta z)$, which follows from Eq. (7), while substitution of Eqs. (9) and (10) into equation Eq. (7) reduces it to an identity provided that equations

$$\partial \Xi_i/\partial z + \nabla_\perp \Xi_i \nabla_\perp s = 0 \; (i = 1, 2) , \qquad (11)$$

$$\partial J/\partial z + \nabla_\perp (J \nabla_\perp s) = 0 . \qquad (12)$$

are satisfied.

If we take Eq. (1) account into from Eq. (11) it follows that the functions $\Xi_{1,2}(\xi, z)$ are conserved along the ray trajectories, being thus the Lagrangian variables determining the running point (the coordinate z plays the role of time). Such a point belongs to the ray trajectory of the trajectory in the space, of the trajectory originating from the point $\Xi(\xi_0, 0) = \xi_0$. As to the integrals of the equation of the ray trajectories (Eq. (1)), from which one finds the variables $\Xi_{1,2}$, they are generally represented by the recurrent relation

$$C_i = M_i(\mathrm{x_i}, \mathrm{F_i}, z) \; (i = 1, 2) , \qquad (13)$$

where $\Phi_i(\mathbf{C}, z)$ are the integral functions of corresponding indices, and the vector $\mathbf{C} = \{C_1, C_2\}$ denotes the set of the same integrals (13). Since the integral function Φ_i always enters into Eq. (13) additively, simple redenoting the constant C_i can make the initial value of $\Phi_i(\mathbf{C}, 0) = 0$.

From integrals (13) one can write the variables

$$\xi_i = M_i^{-1}(C_i, \Phi_i, z) \; (i = 1, 2) \qquad (14)$$

in terms of the functions M_i^{-1} which are inverse to the functions in the right side of integrals (13) and determine the Lagrangian variables at $z = 0$

$$\Xi_i = M_{i0}^{-1} (C_i) \equiv M_i^{-1} (C_i, 0, 0) \; (i = 1, 2) . \qquad (15)$$

After substituting constants (13) into Eq. (15) the Lagrangian variables become related to the variables $\xi_{1,2}$, z (or to the Euler variables x, y, and z) as follows:

$$\Xi_i = M_{i0}^{-1} (M_i(\xi_i, \Phi_i, z)) \; (i = 1, 2) . \qquad (16)$$

After the recursive substitutions of Eq. (13) the value $\mathbf{C}$ in the argument of Φ_i is also expressed in terms of ξ and z.

Integration of the function $J(\Xi)$ from Eq. (12) over trajectories (1) at the initial condition $J(\xi_0) = 1$ yields

$$J(\Xi) = \exp \left\{ - \int_0^z [\nabla_\perp s(\rho, \zeta)]_{\rho=\rho(\xi)} \, d\xi \right\} , \qquad (17)$$

where in accordance with Eqs. (2) and (14)

$$\rho(\zeta) = \{a_{10} f_1 M_1^{-1}(C_1, \Phi_1, \zeta), \; a_{20} f_2 M_2^{-1}(C_2, \Phi_2, \zeta)\} . \quad (18)$$

Note that the functions $f_{1,2}(\zeta)$ and $\Phi_{1,2}(\mathbf{C}, \zeta)$ are taken with the argument ζ and according to Eq. (13),

$$C_i = M_{i0}(\Xi_i) \equiv M_i (\Xi_i, 0, 0) \; (i = 1, 2)$$

at $z = 0$, what finally determines the functional dependence of the Jacobian.

Using expressions (9)–(17) and applying them to noncircular beams with wavefront (6) one can find integrals (13) of Eq. (1) in the form

$$C_i = \xi_i^{-2} + A_i + \Phi_i \quad (i = 1, 2) , \qquad (13')$$

where the integral functions are

$$\Phi_i(\mathbf{C}, z) = \frac{a_{3-i,0}}{a_{i0}} \int_0^z \frac{f_{3-i}(\zeta)}{f_i(\zeta)} \frac{C_i - A_i(\zeta) - \Phi_i(\mathbf{C}, \zeta)}{C_{3-i} - A_{3-i}(\zeta) - \Phi_{3-i}(\mathbf{C}, \zeta)} A_{12}'(\zeta) \, d\zeta$$

(i=1, 2).

In accordance with Eq. (16) the Lagrangian variables are determined from Eq. (13′) by the formulas

$$\Xi_i = \xi_i \{1 + [\alpha_i(z, 0) + \varphi_i(\xi, z, 0)] \, \xi_i^2\}^{-1/2} (i = 1, 2) , \qquad (16')$$

in which $\alpha_i(z, 0) = A_i(z) - A_{i0}$ and $A_{i0} = A_i(0)$. As to the Jacobian of transformation (10), it appears to be represented, in agreement with Eqs. (17) and (18), as a function

$$J = (f_1 f_2)^{-1}\exp\left[-\sum(\xi_1^2, \xi_2^2)\right], \tag{10'}$$

of the Euler coordinates, where

$$\sum(\xi_1^2, \xi_2^2) = \sum_{i=1}^{2} \xi_i^2 \int_0^z \frac{H_i'(\zeta)}{1 + [\alpha_i(z, \zeta) + \varphi_i(\xi, z, \zeta)]\,\xi_i^2}\, d\zeta, \tag{19}$$

$$H_i(z) = \frac{1}{2}[3\alpha_i(z, 0) + F_i(z)],$$

$$F_i(z) = \frac{a_{i0}}{a_{3-i,0}} \int_0^z \frac{f_i(\zeta)}{f_{3-i}(\zeta)} A_{12}'(\zeta)\, d\zeta,$$

$$\alpha_i(z, \zeta) = \alpha_i(z) - \alpha_i(\zeta),\ \varphi_i(\xi, z, \zeta) = \Phi_i(\mathbf{C}, z) - \Phi_i(\mathbf{C}, \zeta)\ (i = 1, 2).$$

Note that the functions $\varphi_{1,2}(\xi, z, \zeta)$ describing the three–dimensional shape of the ray trajectories (16′) can be represented by the recurrent dependence

$$\varphi_i(\xi, z, \zeta) = \int_\xi^z \frac{\xi_i^{-2} + \alpha_i(z, \gamma) + \varphi_i(\xi, z, \gamma)}{\xi_{3-i}^{-2} + \alpha_{3-i}(z, \gamma) + \varphi_{3-i}(\xi, z, \gamma)} \times$$

$$\times F_{3-i}'(\gamma)\, d\gamma\ (i = 1, 2) \tag{20}$$

if one takes Eq. (13′) into account

Therefore the distribution of intensity over the cross section of a noncircular beam with its axial intensity I_0, having a symmetric, with respect to the beam axes, initial profile (8)

$$I_0(\xi_0) = I_0 \mathcal{P}(\xi_{10}^2, \xi_{20}^2)/\mathcal{P}(0, 0),$$

which may be represented by a hyper–Gaussian function

$$\mathcal{P}(\xi_{10}^2, \xi_{20}^2) = \exp\left(-\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn} \xi_{10}^{2m} \xi_{20}^{2n}\right) \tag{21}$$

is also described by a hyper–Gaussian distribution

$$I(\mathbf{r}) = I_0 \exp\left[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} l_{mn}(z)\, \xi_1^{2m} \xi_2^{2n}\right] \tag{22}$$

if Eq. (10′) is substituted into Eq. (9). Here

$$l_{mn}(z) = -\delta_{so}(\delta z + \ln f_1 f_2) - (1 - \delta_{s0})[b_{mn}(z) + h_{mn}(z)] \tag{23}$$

($s = m + n$ and δ_{st} is the Kronecker symbol). The latter coefficients may be represented in terms of the coefficients from the expansion into a double series over variables $\xi_{1,2}^2$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn} \Xi_1^{2m} \Xi_2^{2n} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} b_{mn}(z)\, x_1^{2m} \xi_2^{2n}. \tag{24}$$

This series is formed of expressions (16′) and the expansion of function (19)

$$(\xi_1^2, \xi_2^2) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h_{mn}(z)\, \xi_1^{2m} \xi_2^{2n}. \tag{19'}$$

It should be noted that representation (21) of the initial profile of the beam is universal in the sense that the numbers

$$B_{mn} = -\delta_{s0}\ln\mathcal{P}_{00} - \frac{1-\delta_{s0}}{\mathcal{P}_{00}}\left[\mathcal{P}_{mn} + \frac{1}{s + \delta_{s0}} \times\right.$$

$$\left.\times \sum_{p=0}^{m}\sum_{q=0}^{n} (p + q)(1 - \delta_{mp}\delta_{nq})\mathcal{P}_{m-p,n-q} B_{pq}\right]$$

($0 \le s = m + n < \infty$) can always be recurrently found from the coefficients $\mathcal{P}_{mn}$ ($\mathcal{P}_{00} = \mathcal{P}(0, 0)$) of the expansion of an arbitrary continuous function into a double series

$$\mathcal{P}(\xi_{10}^2, \xi_{20}^2) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \mathcal{P}_{mn} \xi_{10}^{2m} \xi_{20}^{2n}.$$

When calculating the coefficients $b_{mn}(z)$ it appears reasonable to confine oneself to terms up to the sixth cumulative power in the expansion of the squared Lagrangian variables (16′) over the products of powers of squared $\xi_{1,2}$, i. e.,

$$\Xi_i^2 = \xi_i^2 \sum_{m=0}^{2}\sum_{n=0}^{2-m} \gamma_i^{(m,n)}(z)\, \xi_1^{2m} \xi_2^{2n}\ (i = 1, 2),$$

where the functions

$$\eta_i^{(m,n)}(z) = \theta(i) \times$$

$$\times \begin{cases} \delta_{m0} - \delta_{m1} \int_0^z [A_i'(\zeta) - F_i'(\zeta)]\, F_{3-i}(\zeta)\, d\zeta + \\ + 2\delta_{m1}\alpha_i(z, 0)\, F_{3-i}(z),\ m = n, \\ \Theta_i^{(m,n)}[-\alpha_i(z, 0)]^s + \dfrac{\Theta_i^{(m,n)}}{s}[-F_{3-i}(z)]^s + \\ + (\delta_{i1}\delta_{n2} + \delta_{i1}\delta_{m2}) \int_0^z A_{3-i}'(\zeta) F_{3-i}(\zeta)\, d\zeta,\ m \ne n \end{cases} \tag{25}$$

are defined for $0 \le (i, s) \le 2$, $s = m + n$, and $\theta(p)$ is the Heaviside unit function equal to zero for $p \le 0$

$$\Theta_i^{(m,n)} = \delta_{i1}\theta(m) + \delta_{i2}\theta(n)\ (i = 1, 2).$$

Using the designations we find from Eq. (24) that

$$b_{mn}(z) = B_{mn} + \sum_{p=0}^{m}\sum_{q=0}^{n} p!\, q!\, (1 - \delta_{mp}\delta_{nq}) \times$$

$$\times B_{pq}\left[\eta_{\theta(p)}^{(m-p,\, n-q)}(z) + \eta_{2\theta(q)}^{(m-p,\, n-q)}(z)\right] \tag{26}$$

within the interval of indices $0 \le m + n \le 3$.

The coefficients $h_{mn}(z)$ of the expansion of function (19) into series (19′) within the same interval of indices $0 \le m + n \le 3$ appear to be equal to

$$h_{mn}(z) = (-1)^{s+1}(1 - \delta_{s0})\, C_s^m\, H^{(s)}_{1(m),2(n)}(z)\,, \tag{27}$$

where $s = m + n$ and C_s^m are the binomial coefficients. The notation $k(l)$ denotes here the index k ($k = 1, 2$) repeated l times ($l = m, n$) when reproducing the functions

$$H^{(p)}_{i(p)}(z, \zeta) = \int\limits_0^{\xi} \alpha_i^{p-1}(z, \gamma)\, H_i'(\gamma)\, d\gamma \quad (p = 1, 2, 3;\ i = 1, 2)\,,$$

$$H^{(p)}_{12}(z) = \frac{1}{2}\int\limits_0^{z} \left[H_1(\gamma)F_2'(\gamma) + (-1)^p H_2(\gamma)F_1'(\gamma)\right] d\gamma\,,$$

$$H^{(3)}_{1i2}(z) = \frac{2}{3}\int\limits_0^{z} \left[(-1)^i\alpha_i(z, \gamma)\, H^{(1)'}_{12}(\gamma) + \right.$$

$$\left. + H^{(2)}_{12}(\gamma)F_i'(\gamma) + H^{(2)}_{ii}(z, \gamma)F_{3-i}'(\gamma)\ \right] d\gamma\,. \tag{28}$$

It is assumed that $H^{(p)}_{i(p)}(z, z) = H^{(p)}_{i(p)}(z)$ in Eq. (27), so that $H^{(1)}_i(z, z) = H_i(z)$ ($i = 1, 2$), as can be seen from Eq. (28).

It is convenient to ascribe the following form to the profile of the beam intensity (22) within the employed approximation (27)

$$I(\mathbf{r}) = I_0\,\frac{\mathcal{P}(\Xi_1^2, \Xi_2^2)}{\mathcal{P}(0, 0)} \times$$

$$\times \frac{\varepsilon^{-\delta z}}{f_1 f_2}\left(\frac{\Xi_1\Xi_2}{\xi_1\xi_2}\right)^2 \exp\left[-\sum_{m=0}^{3}\sum_{n=0}^{3-m} h^{(-)}_{\mathrm{mn}}(z)\, \xi_1^{2m}\xi_2^{2n}\right]. \tag{22$'$}$$

To do that we have to integrate the argument of the exponential function (19) of the Jacobian (10′). Variables $\Xi_{1,2}$ here are given by Eq. (16′) and the functions $h^{(-)}_{mn}(z)$ are given by formulas (27) and (28) in which $H_{1,2}(z)\ \}\ H^{(+)}_{1,2}(z)$ is replaced by $H^{(-)}_{1,2}(z)$, and

$$H^{(\pm)}_i(z) = \alpha_i(z, 0) \pm \frac{1}{2}\left[\alpha_i(z, 0) + F_i(z)\right] \quad (i = 1, 2)\,.$$

It is evident that the expression for the beam intensity in its last form is a generalization of a similar expression well known from theory, which accounts for spherical aberrations of the axially symmetric beams. If all terms in the series (22) are held at a constant index, while the other index being equal to zero (or if integration is carried out in Eq. (19) at $a_{3-i,0} \leftrightarrows \infty$, $f_{3-i} = 1$ and $A_{3-i} = A_{12} = 0$) it is reduced to the intensity distribution for a one–dimensional beam

$$I(\mathbf{r}) = I_0\,\frac{\delta_{i1}\mathcal{P}(\Xi_i^2, 0) + \delta_{i2}\mathcal{P}(0, \Xi_i^2)}{\mathcal{P}(0, 0)} \times$$

$$\times \frac{\exp(-\delta z)}{f_i[1 + \alpha_i(z, 0)\,\xi_i^2]^{3/2}} \quad (i = 1, 2)\,. \tag{22$''$}$$

The obtained expressions make it possible to find equations describing the behavior of the sought–after aberration functions $A_{1,2}(z)$ and $A_{12}(z)$. To this end, let us use the parabolic equation for the wave eikonal

$$2\frac{\partial s}{\partial z} + (\nabla_\perp s)^2 = \frac{\varepsilon'_{\mathrm{nl}}[I]}{\varepsilon_0} + \frac{1}{2k^2}\left[\nabla_\perp \ln I + \frac{1}{2}(\nabla_\perp \ln I)^2\right], \tag{29}$$

in which a real nonlinear addition $\varepsilon'_{nl}[I]$ to the dielectric constant of the medium (ε_0 being its nonperturbed value) is, generally speaking, some functional of the beam intensity; $k = 2\pi/\lambda$ is the wave number.

Within the approximation of aberrations of the fourth order it appears sufficient to expand this nonlinear addition in a series over the even powers of $\xi_{1,2}$ (or over the powers of x, y), only to the terms which have a cumulative power not exceeding 4, i.e.,

$$\varepsilon'_{\mathrm{nl}}[I] = \sum_{m=0}^{2}\sum_{n=0}^{2-m} \varepsilon'_{2m,2n}(z)\, \xi_1^{2m}\xi_2^{2n}\,. \tag{30}$$

Note that the coefficients of such an expansion $\varepsilon'_{2m,2n}(z)$ are determined taking into account the specific mechanism of the medium nonlinearity. For example, if the medium is of cubic nonlinearity i. e., $\varepsilon'_{nl}[I] = \varepsilon^2[I]$

$$\varepsilon'_{2m,\,2n}(z) = \varepsilon^{(2)}\, I_{mn}(z)\,,$$

where

$$I_{mn}(z) = I_0(f_1 f_2)^{-1}\exp(-\delta z)\, D^{(m,\,n)}(z)$$

characterize the expansion of the beam intensity in a doulbe series and may be expressed in terms of the coefficients (23) using the recurrent formulas

$$D^{(\mathrm{m,n})}(z) = \delta_{s0} +$$

$$+\frac{1-\delta_{s0}}{s+\delta_{s0}}\sum_{p=0}^{m}\sum_{q=0}^{n}(s-p-q)\, l_{m-p,\,n-q}(z)\, D^{(p,\,q)}(z)\ (s = m + n).$$

Within the interval of indices $0 \le s = m + n \le 2$ we have

$$\varepsilon'_{2m,\,2n}(z) = \varepsilon^{(2)} I_0(f_1 f_2)^{-1}\varepsilon^{-\delta z}\Big\{\delta_{s0} + (1 - \delta_{s0}) \times$$

$$\times\left[\frac{1-\theta(m)\theta(n)}{s+\delta_{s0}}\ \ (\theta(m)l_{10} + \theta(n)l_{01})^s + (\theta(m-1) + \right.$$

$$\left. + \theta(n-1)\}l_{mn} + \theta(m)\theta(n)\, l_{10}l_{01}\right]\Big)\,, \tag{30$'$}$$

where $l_{mn}(z)$ are calculated using formula (23) taking Eqs. (26) and (27) into account.

As to the medium with a thermal nonlinearity, the value $\varepsilon'_{nl}[I] = (d\varepsilon/dT)[T(\mathbf{r}) - T(0, z)]$ is determined by the temperature profile $T(\mathbf{r})$ within the beam cross section, and the coefficients $\varepsilon'_{2m,\,2n}(z)$ can be determined by solving the equation of heat conductivity. Assuming that the Gaussian profile of the beam is conserved, i.e., $|l_{mn}| < (|l_{10}|, |l_{01}|)$ for $m + n > 1$, at a moderate ellipticity of its cross section, when $|\mu(z)| < 1$ and $\mu(z) = \left(a_{20}^2 f_2^2\, l_{10} - a_{10}^2 f_1^2\, l_{01}\right)/\left(a_{20}^2 f_2^2\, l_{10} + a_{10}^2 f_1^2\, l_{01}\right)$ the calculations of the terms up to the terms containing $\mu(z)$ and $\mu^2(z)$ yield, within the same interval of indices $0 \le s = m + n \le 2$

$$\varepsilon'_{2m,2n}(z) = -\frac{a_{10}a_{20}\delta I_0 \exp(-\delta z)}{4\text{k}} \frac{d\varepsilon}{dT} \times$$
$$\times \frac{(C_s{}^m + C_s{}^n)\,(a_{20}^2 f_2^2 l_{10} + a_{10}^2 f_1^2 l_{01})^{s-1}}{2^s s!\,(a_{10} f_1)^{2n-1}\,(a_{20} f_2)^{2m-1}}\,\gamma_{mn}(z)\,, \qquad (30'')$$

where

$$\gamma_{mn}(z) = (1-\delta_{s0})\left\{\left[1 + \frac{\theta(m)-\theta(n)}{\theta(m)+\theta(n)}\frac{\mu(z)}{2}\right]^s + \right.$$
$$\left. + \frac{1}{3}[\theta(m-1)-\theta(n-1)]\,\mu(z) - \frac{3}{4}\theta(m)\,\theta(n)\,\mu^2(z)\right\},$$

κ is the coefficient of the medium heat conductivity, and $l_{10}(z)$ and $l_{01}(z)$ are set in accordance with Eqs. (23), (26), and (27).

Substitution of Eqs. (6), (22), and (30) into Eq. (29) yields (after the coefficients at the equal powers of $\xi_{1,2}^{2m}$ ($m = 0, 1, 2$) and at the product $\xi_1^2\xi_2^2$ are equalized) the equations for the sought–after functions S_0, $f_{1,2}$, $A_{1,2}$, and A_{12}

$$s'_0 = \frac{\varepsilon'_{00}}{2\varepsilon_0} + \frac{1}{2}\sum_{i=1}^{2}\frac{a_{i0}^2\, l_{10}^{(i)}}{R_{\text{d}i}^2 f_i^2}\,,$$

$$f_i'' = \frac{\varepsilon_{20}^{(i)}}{\varepsilon_0 a_{i0}^2 f_i} + \frac{1}{R_{\text{d}i} f_i}\left[\frac{(l_{10}^{(i)})^2 + 6\,l_{20}^{(i)}}{R_{\text{d}i} f_i^2} + \frac{l_{11}}{R_{\text{d},3-i} f_{3-i}^2}\right],$$

$$(f_i^2 A_i')' = \frac{4\varepsilon_{40}^{(i)}}{\varepsilon_0 a_{i0}^2} + \frac{4}{R_{\text{d}i}}\left[\frac{4l_{10}^{(i)}\, l_{20}^{(i)} + 15 l_{30}^{(i)}}{R_{\text{d}i} f_i^2} + \frac{l_{21}^{(i)}}{R_{\text{d},3-i} f_{3-i}^2}\right] (i = 1, 2),$$

$$(f_1 f_2 A'_{12})' = \frac{2\varepsilon'_{22}}{\varepsilon_0 a_{10} a_{20}} + \frac{4}{R_{\text{d}1}^{1/2} R_{\text{d}2}^{1/2}}\sum_{i=1}^{2}\frac{l_{10}^{(i)}\, l_{11} + 3\,l_{21}^{(i)}}{R_{\text{d}i} f_i^2}\,, \qquad (31)$$

where $l_{mn}^{(i)} = \delta_{i1}\, l_{mn} + \delta_{i2}\, l_{nm}$, $\varepsilon_{2m,2n}^{(i)} = \delta_{i1}\varepsilon'_{2m,2n} + \delta_{i2}\varepsilon'_{2n,2m}$, the values $l_{mn}(z)$ and $\varepsilon'_{2m}(z)$ are given by expressions (23) and (30′) or (30″), respectively, and $R_{di} = ka_{i0}^2$ ($i = 1, 2$) are the diffractional lengths of the beam. It is easy to find that for the case of a circular beam ($a_{10} = a_{20} = a_0$) $\varepsilon'_{20} = \varepsilon'_{02}$, $\varepsilon'_{40} = \varepsilon'_{04} = \frac{1}{2}\varepsilon'_{22}$, and, similarly,

$$L_{mn} = L_{nm},\ L_{m0} = L_{0m} = \frac{1}{m} L_{m-1,1}\ \}\ L_m (0 \le n < m + n \le 3)\,,$$

where L_{mn}=$(B, b, h, l)_{mn}$. System (31) is then reduced to three equations for the functions s_0, f, and A_1 ($f = f_1 = f_2$, $A_1 = A_2 = A_{12}$) which are transformed, after introducing the relevant nonlinear lengths and taking Eqs. (30′) or (30″) into account, to equations obtained elsewhere.[7]

The analysis of the system of equations (31) shows that nonlinear aberrations inevitably develop in the field of an arbitrary beam of limited geometric size. However, the development is different in media with different mechanisms of nonlinearity. For example, if the medium is cubically nonlinear, the aberrations are predominantly induced along the coordinate, where the size of the beam is minimum, and its refraction is the strongest. As to the medium with thermal blooming, aberrations are, in contrast, the strongest along the major axis of the beam cross section, where, depending on the ratio of the two beam axes, its refraction may be almost three times as low as that in the plane normal to it. Moreover, higher beam ellipticity results in stronger perturbations of both its wavefront and cross section, which become wavy, so that the beam ellipticity is lost.

A significant difference between the two cases is that the direction, in which aberrations develop, depends on the geometric and optical parameters of the beam. As to the medium with cubic nonlinearity of this or that sign, the aberrations always develop along the principal axes of the beam cross section, irregardless of their ratio. The shape of the initial beam intensity profile only determines whether these aberrations are of the same or the opposite signs. During the thermal blooming, however, the signs of these aberrations in the same mutually normal planes are, first of all, determined by the ratio of the principal axes of the beam.

Note also that the wave properties themselves affect the aberrational distortions of the beam, either hampering or stimulating their development depending on the intensity profile. It is interesting to note that the role of diffraction is the stronger, the larger the beam profile deviates from the Gaussian.

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