Possible improvement of an object visibility at its submergence into a turbid medium

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The problem of observation of an opaque object through a surface-bounded layer of a scattering medium is solved based on the small-angle approximation with the allowance made for the formation of a shadow zone. It turns out that the image contrast can have a local extremum of best visibility inside a medium. It is shown that the object visibility can increase monotonically with the increasing depth in a turbid medium. The conditions under which this improvement occurs and the contrast extremum is reached are numerically evaluated.

Can an object be seen better when deep in a turbid medium than on its surface? The first intuitive answer is, most likely, no. The everyday practice teaches us that the thicker is a veiling layer, for example, of fog in front of eyes, the worse is the visibility. However, an elementary example shows that this is not always the case. Let a layer of a scattering medium, *SM*, (Fig. 1) be illuminated with a wide source *S*, and a narrow-angle receiver *R* records the reflected radiation. From the bottom *SM* is bounded by a Lambertian surface *B* with the albedo A_b. Let a Lambertian opaque object *O* with the albedo A_o be on the upper boundary of the layer at the position I. The observation of the object or vision consists in comparison, by a receiver *R*, of two optical signals coming from two neighboring points of the field of view, i.e., signals at the receiver orientation toward I (to the object) and *2* (to the background). For simplicity we assume $A_b = 0$. Let the brightness coefficients of the object and the scattering medium be the same. Then, naturally, the object at the position I is invisible, because the optical signals from *O* (in the direction *1*) and from *SM* (*2*) are equal, or the contrast of the object's image (see below) is close to zero. Let us now move the same object to the position II. In this case, the signals from the medium at the receiver's orientations *1* and *2* are almost the same, and the object can be rather clearly seen, if the thickness z_b of the scattering medium is not very large. Thus, the considered simple example demonstrates qualitatively that the layer of turbid medium can improve visibility under certain conditions.

Let us consider this problem quantitatively. Let an active vision system including a wide illumination source and a narrow-field-of-view receiver observes an object in an arbitrary position z_o in the layer of the scattering medium (right-hand part of Fig. 1). The shadow area *Sh* is formed behind the opaque object *O* in the *SM*. As usual,[1,2] the lidar contrast *k* of the object's image is

$$k = W_{vs}/W_2 = (W_1 - W_2)/W_2, \qquad (1)$$

where W_{vs}, W_1, and W_2 are, respectively, the power of the valid signal and those of signals from the object and from the background.

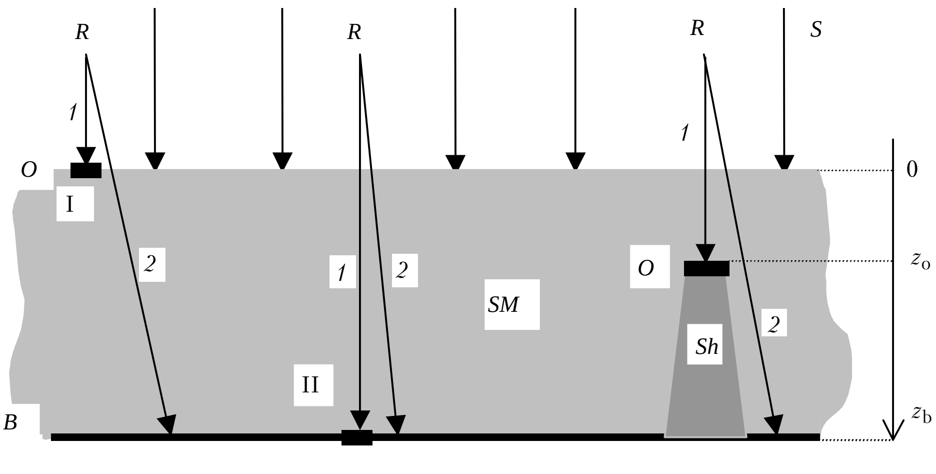

Fig. 1. Geometry of the problem: illumination source *S*, receiver *R*, object *O*, scattering medium *SM*, bottom (surface) *B*, object shadow zone *Sh*.

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As follows from Eq. (1), the valid signal here is the difference between the power of radiation coming to R at its orientation toward O (in the direction *1*) and toward the background (*2*). The condition of the object visibility is

$$|k| \geq k_{\rm th}, \quad (2)$$

where $k_{\rm th}$ is the threshold contrast. At visual observation, $k_{\rm th}$ is usually assumed equal to 0.02. We use only the condition (2) and ignore the signal-to-noise ratio, assuming that the source power is sufficient for exceeding the threshold value of the latter.[3]

We believe that the scattering phase function of the medium is strongly elongated in the forward direction (for example, observation through a cloud, fog or water layer) and use the quasi-single-scattering approximation,[2] i.e., take into account multiple scattering at propagation to the scattering point, single scattering at large angles, and multiple scattering on the way back to the source. A similar problem was solved in Ref. 4 at $A_{\rm b} = 0$ or for the layer of a scattering medium with a black bottom. Here we consider a more general case $A_{\rm b} \geq 0$. Then, in our approximation, the definition of the contrast (1) takes the form

$$k = \frac{[W_{\rm o}(\tau_{\rm o}) + W'_{\rm bs}(\tau_{\rm o}, \tau_{\rm b}) + W'_{\rm b}(\tau_{\rm o}, \tau_{\rm b})] - [W_{\rm bs}(\tau_{\rm b}) + W_{\rm b}(\tau_{\rm b})]}{W_{\rm bs}(\tau_{\rm b}) + W_{\rm b}(\tau_{\rm b})}, \quad (3)$$

where $W_{\rm o,\,bs,\,b}$ are powers of the signals coming to the receiver from O, SM, and B, respectively; the primed variables correspond to the signals with the allowance made for the shadow area; $\tau_{\rm o}$ and $\tau_{\rm b}$ are the optical depths to the object and that of the entire layer ($\tau = \varepsilon z$, ε is the extinction coefficient).

Let us write the variables entering into the right-hand side of Eq. (3) in the small-angle approximation[2,5] similarly to Ref. 4. This can be done in the analytical form if the object is small. The criterion of smallness for this case was introduced in Ref. 4. Thus, accurate to normalization factors identical for the numerator and denominator in Eq. (3), we have

$$W_{\rm o}(\tau_{\rm o}) = \frac{A_{\rm o}}{\pi} \exp\,[\tau_{\rm o}(\Lambda - 2)]; \quad (4)$$

$$W_{\rm bs}(\tau_{\rm b}) = \frac{a_{\rm w}}{\pi}\{1 - \exp\,[2\tau_{\rm b}(\Lambda - 1)]\}; \quad (5)$$

$$W'_{\rm bs}(\tau_{\rm o}, \tau_{\rm b}) = W_{\rm bs}(\tau_{\rm b}) - \frac{a_{\rm w}}{\pi} \exp\,(-\Lambda\tau_{\rm o})\{\exp\,[2\tau_{\rm o}(\Lambda - 1)] - \exp\,[2\tau_{\rm b}(\Lambda - 1)]\}; \quad (6)$$

$$W_{\rm b}(\tau_{\rm b}) = \frac{A_{\rm b}}{\pi} \exp\,[2\tau_{\rm b}(\Lambda - 1)]\,; \quad (7)$$

$$W'_{\rm b}(\tau_{\rm o}, \tau_{\rm b}) = \frac{A_{\rm b}}{\pi} \exp\,[2\tau_{\rm b}(\Lambda - 1)]\,[1 - \exp(-\Lambda\tau_{\rm o})]\,, \quad (8)$$

where $a_{\rm w} = \Lambda P_\pi/[8(1 - \Lambda)]$ is the brightness coefficient of the semi-infinite layer of SM at a standard illumination and observation conditions; P_π is the mean value of the scattering phase function in the backward direction; $\Lambda = \sigma/\varepsilon$ is the probability of photon survival in the medium; σ is the scattering coefficient. Upon substitution of Eqs. (4)–(8) into Eq. (3), we have

$$k = \frac{(A' - a')\exp[\tau_{\rm o}(\Lambda - 2)] + a'\exp(-\Lambda\tau_{\rm o})\exp[2\tau_{\rm b}(\Lambda - 1)]}{a'\{1 - \exp[2\tau_{\rm b}(\Lambda - 1)]\} + A_{\rm b}}, \quad (9)$$

where $A' = A_{\rm o} - A_{\rm b}$; $a' = a_{\rm w} - A_{\rm b}$. For a comparison, let us present here the corresponding equation for the contrast at $A_{\rm b} = 0$ derived in Ref. 4. Obviously, this equation follows from Eq. (9) too:

$$k = \frac{(A_{\rm o} - a_{\rm w})\exp[\tau_{\rm o}(\Lambda - 2)] + a_{\rm w}\exp(-\Lambda\tau_{\rm o})\exp[2\tau_{\rm b}(\Lambda - 1)]}{a_{\rm w}\{1 - \exp[2\tau_{\rm b}(\Lambda - 1)]\}}. \quad (10)$$

It is seen that Eqs. (9) and (10) are similar. Thus, many equations from Ref. 4 can be directly applied to the considered case of $A_{\rm b} > 0$ by replacing $A_{\rm o} \to A'$ and $a_{\rm w} \to a'$. The only significant difference between Eqs. (9) and (10) is the sign of the factor of the second term in the numerator. In Eq. (10) always $a_{\rm w} > 0$, whereas a' in Eq. (9) can be both positive and negative depending on the relation between $a_{\rm w}$ and $A_{\rm b}$. By analyzing the contrast as a function of $\tau_{\rm o}$ for extremum,[4] it was obtained that the dependence $k(\tau_{\rm o})$ can have maximum. Analyzing similarly Eq. (9), we can show that, under certain conditions depending on the object albedo and optical properties of the scattering medium, the contrast can have maximum (at $a' > 0$), being positive at this maximum, or minimum (at $a' < 0$), being negative at it, at some value of $\tau_{\rm o}^*$:

$$\tau_{\rm o}^* = \tau_{\rm b} + \frac{1}{2(1 - \Lambda)} \ln\left[\frac{2 - \Lambda}{\Lambda}\,(1 - A'/a')\right]. \quad (11)$$

This means that, at the $\tau_{\rm o}^*$ position inside the scattering layer, the object is visible better than at other depths, and it looks lighter or darker than the background. Based on the condition $0 < \tau_{\rm o}^* < \tau_{\rm b}$ (i.e., the extremum is inside the layer), we can find the range of the object albedo, within which the maximum of the absolute value of k exists. Actually, the necessary and sufficient condition of fulfillment of the above inequality (at the minimum of the contrast inside the scattering layer or at $k(\tau_{\rm o}^*) < 0$) is

$$A_2^* + A_{\rm b} < A_{\rm o} < A_1^* + A_{\rm b}, \quad (12)$$

where $A_1^* = 2a'(1 - \Lambda)/(2 - \Lambda)$, $A_2^* = a'\{1 - [\Lambda/(2 - \Lambda)]\exp[-2\tau_{\rm b}(1 - \Lambda)]\}$. If $A_{\rm o} < A_2^* + A_{\rm b}$ and $a' < 0$, then the contrast behaves as usual, that is, increases, remaining negative, with the depth, and the visibility of the object O becomes worse with the increasing τ_o. At $A_o > A_1^* + A_b$ and $a' < 0$, the contrast decreases monotonically with the increasing τ_o, and at the upper boundary ($\tau_o = 0$) $k > 0$. However, at $A_3^* + A_b > A_o > A_1^* + A_b$, where $A_3^* = a'\{1 - \exp[-2\tau_b(1 - \Lambda)]\}$, the function $k(\tau_o)$ becomes zero at $\tau_o = \tau_1$. In this case, as τ_o increases, the following takes place: the object becomes less visible, but near the upper boundary of the scattering medium it is lighter than the background, then near $\tau_o = \tau_1$ the object disappears or becomes invisible because of the low contrast as compared with the threshold value k_{th}. Deeper in the medium, the object becomes darker than the background and more visible (the absolute value of the contrast $|k|$ increases) down to the lower boundary of the scattering medium. The optical depth τ_1, at which the object disappears, can be evaluated analytically, as in Ref. 4. Besides, substituting Eq. (11) in Eq. (9), it is easy to find the maximum absolute value of the contrast and to study other conditions of object observation in the same way as in Ref. 4.

Figure 2 shows the dependence $k(\tau_o)$ at the different albedo A_o and the same optical thickness of the scattering layer $\tau_b = 2$. This figure illustrates the above-mentioned peculiarities in the behavior of the contrast of the object image. Figure 2*a* corresponds to the case of observation through a cloud (Cloud C-1 model[6]) at $A_b = 0$ (Ref. 4), and Fig. 2*b* corresponds to observation through a water layer against the background of the bottom with $A_b = 0.25$. The optical properties of water were taken in accordance with the correlation model from Ref. 7. For Fig. 2*a* $a' > 0$, and the peculiarities in the contrast behavior: object disappearance (curve *1*) near $\tau_o \approx 1.5$ and the maximum of $k(\tau_o)$ (curves *2* and *3*) inside *SM*, were analyzed in detail in Ref. 4.

Let us consider Fig. 2*b* more closely. At the low albedo A_o (curve *1*) the contrast increases, having negative values, and the object becomes worse visible with the depth in the scattering medium, as it follows from the inequality (12). When the condition (12) is fulfilled, the dependence $k(\tau_o)$ has a minimum, although non pronounced, inside the medium (curves *2* and *3*). As A_o varies within the limits set by the inequality (12), the position of the minimum shifts from the lower to the upper boundary of *SM*. Curve *3* illustrates the interesting behavior of the contrast. At the upper boundary ($\tau_o = 0$) the object is lighter than the background ($k > 0$). Deeper in the medium, the contrast falls down and the object visibility deteriorates. At the depth $\tau_o \approx 0.4$ the object is invisible because of low k. As τ_o further increases, the object becomes visible better ($|k|$ increases), but the object looks darker than the background, $k < 0$. Then, at the depth τ_o^* determined by Eq. (11), $|k|$ has a local maximum, and then the object visibility becomes worse. If we construct the dependence $k(\tau_o)$ for other values of the principal parameters A_o and (or) τ_b but using the *SM* optical properties corresponding to those in Fig. 2*b*, then $|k|$ may, for example, monotonically increase with the increasing τ_o. This means that the visibility of the object may increase monotonically with the depth down to the bottom. Note that these peculiarities follow from analysis of the simple analytical equation (9), and therefore we omit their consideration here.

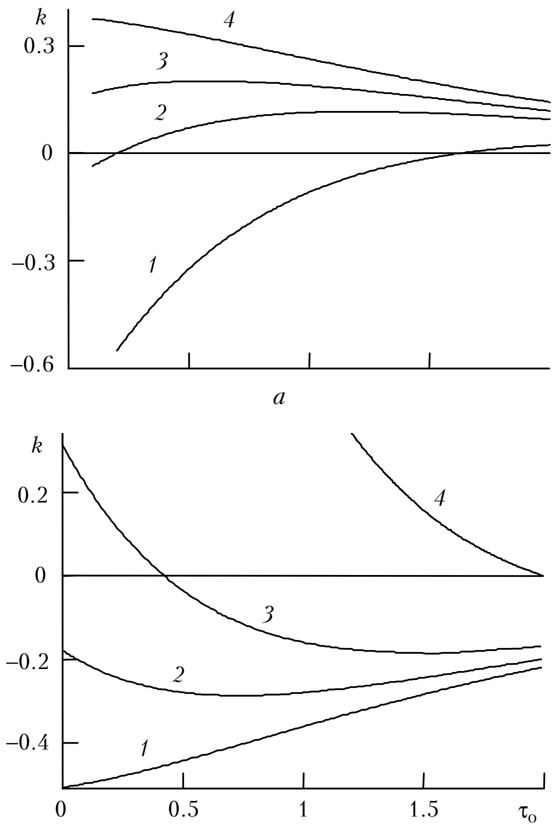

Fig. 2. Dependence of the contrast of an object image on τ_o at observation through a cloud layer with $\tau_b = 2$, $A_b = 0$ (*a*) and water layer with $\tau_b = 2$, $A_b = 0.25$ (*b*). The upper panel: $A_o = 0.05$ (curve *1*), 0.2 (*2*), 0.25 (*3*), and 0.3 (*4*); lower panel: $A_o = 0.02$ (*1*), 0.04 (*2*), 0.08 (*3*), and 0.25 (*4*).

Thus, coming back to the beginning of this paper, we should say yes to the question stated. Yes, there exist situations in which the object visibility increases with the increasing depth into a turbid medium.

In this paper, the improvement of the contrast with the increasing depth is described quantitatively based on the small-angle approximation at different optical properties of the medium and different values of the object's albedo. It is noted, in particular, that the point of the best visibility can be inside the scattering medium at some depth. This depth is estimated, and it is shown that the extreme value of the contrast can be both positive and negative depending on the relation between the brightness coefficient of the semi-infinite layer of the medium and the albedo of the object viewed through it.

References

1. L.S. Dolin and I.M. Levin, *Reference Book on Theory of Underwater Vision* (Gidrometeoizdat, Leningrad, 1991), 230 pp.
2. E.P. Zege, A.P. Ivanov, and I.L. Katsev, *Image Transfer through Scattering Media* (Nauka i Tekhnika, Minsk, 1985), 328 pp.
3. I.L. Katsev and E.P. Zege, Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana **25**, No. 7, 732–740 (1989).
4. V.V. Barun, Atmos. Oceanic Opt. **10**, No. 3, 165–170 (1997).
5. L.S. Dolin, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **7**, No. 2, 380–382 (1964).
6. D. Deirmendjian, *Electromagnetic Waves Scattering on Spherical Polydispersions* (American Elsevier, New York, 1969).
7. A.N. Dorogin, O.V. Kopelevich, I.M. Levin, and V.I. Feigels, in: *Abstracts of Papers at X Plenum on Ocean Optics* (State Optical Institute Publishing House, Leningrad, 1988), pp. 136–137.