MULTICHANNEL SLIT ANALYZER OF A WAVE FRONT OF OPTICAL RADIATION

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We present here a description of a wave front analyzer based on the Foucault method. Such an analyzer offers a number of advantages as compared to the Hartmann sensors when applying to adaptive optical systems. Its construction is simpler, because a monolithic matrix, which has four times less number of elements and the same spatial resolution, can be used instead of the set of quadrant photodetectors. The measurement range is significantly wider (by one order of magnitude).

The wave front analyzer is one of the principal element of the adaptive optical systems (AOS), very often it is more complicated than other components. The Hartmann sensor[1] normally used under conditions of strong distortions of the wave front (WF) and high power of the optical radiation is not an exception. In spite of the simplicity of its operation principle, its construction is too sophisticated, and it needs for accurate adjustment. Another disadvantage characteristic of this sensor usually applied in combination with quadrant photodetectors is its limited linear range of the measurable wave front tilts in the subapertures limited by the diffraction angle θ_d (as a rule, it is some minutes of arc) what is insufficient for the majority of typical situations, when the linear and spherical components with the tilts greater than θ_d dominate in the wave front analyzed. Finally, these disadvantages result in the decrease of the efficiency of AOS with such sensors.

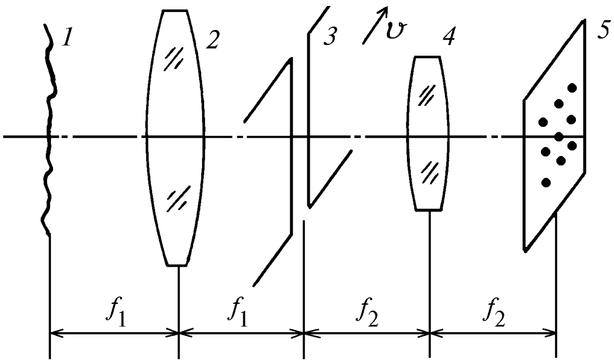

FIG. 1. Block diagram of the multichannel slit analyzer of a wave front of optical radiation.

However, researchers did not pay proper attention to the method of the analysis of WF proposed by Foucault in 1876 for investigation of the optical surfaces (so-called Foucault shadow method). There is a stable conception in the optical community on its practical inapplicability in AOS due to the difficulties in practical use. Nevertheless, the WF analyzers with some characteristics, including the ease of performance, better than the Hartmann sensors can be created on the base of this method after some modifications. The idea of Foucault method, initially proposed for the analysis of quality of concave spherical surfaces, is in the illumination of the surface under investigation by a parallel light beam and studying of the dependence of the image on a screen, distant from the focal plane on the position of another opaque screen (Foucault knife) moved in the focal plane. In other words, the idea of the method is in the combination of the operation of subsequent analyzer of the spatial spectrum of the image and parallel analyzer of the image capable of determining the WF distortions appearing due to defects in the surface under investigation. Possible variant of this method performed on the modern component base is presented in Fig. 1, where *1* is the wave front to be investigated, *2* and *4* are lenses forming the telescopic system, *3* is the spatial filter in the form of a narrow slit in an opaque screen, and *5* is the multielement detector of radiation. Let us study the dependence of output signals from the detectors on the position of the analyzer slit. The complex signal amplitudes at the input and output planes of such a system are related by the known relationship

$$s_{out} = F\ \{F\ \{s_{in}\}T\},$$

where $s_{in} = s_{in}(\mathbf{r})$ and $s_{out} = s_{out}(\mathbf{R})$ are the signal amplitudes at the input and output planes, respectively; $F\{...\}$ is the Fourier transform operator; and T is the modulation characteristic of the transparent (slit). After transformations we obtain

$$s_{out}(\mathbf{R}) = \gamma\ s_{in}(-\ \gamma\ \mathbf{R}) * F\ \{T\}, \tag{1}$$

where $\gamma = f_1/f_2$ is the magnification factor of the telescopic system, $*$ is the symbol of convolution

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operation. Then we have for the vertical slit displaced in the focal plane

$$F\{T\} = \mathrm{sinc}\,(x/\theta_d)\exp(-\,2\pi i x h\, f_1/\lambda)\ \delta(y), \qquad (2)$$

where x and y are the Cartesian coordinates normalized to f_2 in the output plane of the analyzer; $\theta_d = \lambda/af_1$ is the diffraction angle at the slit; λ is the radiation wavelength; a and h are the width and displacement of the slit relative to the optical axis, normalized to the focal length f_1; $\delta(...)$ is the delta-function; and sinc is the function defined by the equality

$$\mathrm{sinc}(t) = \sin(\pi t)/\pi t.$$

Let us set the input signal in the form $A(\mathbf{r})\exp(i\varphi(\mathbf{r}))$, where the characteristic scale on which the noticeable amplitude variations are observed, is significantly greater than the phase variation scale. Let us also select the slit width such that one can assume the WF phase variations to be linear within the limits of $\theta_d\, f_1$. Let us substitute Eq. (2) into Eq. (1) replacing the function sinc with the function of the window of unit width $\mathrm{rect}(x/2\theta_d)$ in order to obtain the analytical expression for the integral. Such a substitution does not essentially affect the final result, because the function $\mathrm{sinc}(x/2\theta_d)$ oscillates at $x \ge |\theta_d|$, and its absolute value quickly decreases. Let us remove the slowly changing function $A(\mathbf{r})$ out of the integral sign, and represent the function $\varphi(\mathbf{r})$ in the form of series, keeping only the linear term of the expansion. Then, making integration in Eq. (1), we obtain

$$s_{\mathrm{out}}(\mathbf{R}) = \gamma\, A(-\,\gamma\,\mathbf{R})\ \exp(i\varphi\ (-\,\gamma\,\mathbf{R}))\ \times$$
$$\times\ \mathrm{sinc}([q_x(-\,\gamma\,\mathbf{R}) - h]/a), \qquad (3)$$

where $q_x = \varphi'_x\,\lambda/2\pi$ is the tilt of WF with respect to the x-axis (to the direction of the analyzer slit movement). It follows from the expression obtained that the signal distribution over the output plane is the specular reflection of the input signal distribution with the scale of 1:γ. Then the output signal amplitude also depends on the analyzer slit position. When the angular position of the slit corresponds to the tilt of the reflected part of the wave front, the signal amplitude is maximum. From this follows the algorithm of the image processing at the output of the analyzer. Moving the slit in the entire range of the spatial spectrum of the signal, it is necessary to determine the angular position of the slit for each element of the photodetector matrix, for which the maximum of the signal is reached at the output of the element. Then, in order to avoid the losses of information, the step of the photodetector matrix should not exceed the value $\lambda/a\gamma$ determined using the theorem of readings (spatial frequency spectrum of the analyzer slit). One can significantly simplify the algorithm, if the slit moves at a constant velocity. In this case one will observe the pulses of the same shape but with different amplitude and temporal position at the photodetector outputs.

$$u_i(t) \sim A^2(-\,\gamma\,\mathbf{R}_i)\mathrm{sinc}^2([q_x(-\,\gamma\,\mathbf{R}_i) - v(t - t_0)]/a), \quad (4)$$

where $\mathbf{R}_i$ is the coordinate of the center of the ith element of the photodetector matrix, v is the angular velocity of the slit movement, and t_0 is the time during which the analyzer slit crosses the optical system axis. So the signal processing is reduced to determining of the moment when a pulse of known shape, fixed by its maximum value, arrives, what is the standard radiotechnical problem. If one has the bulk of measured delays $\{t_i\}$, he can estimate the tilts of the corresponding parts of WF, based on the expression

$$q_x(-\,\gamma\,\mathbf{R}_i) = v(t_i - t_0). \qquad (5)$$

To determine the another WF component, it is necessary to have another one analyzer slit oriented and moved perpendicularly to the first one. The same cycle of measurements should be repeated with it. The most simple way to make such analyzers in practice is to use cuts in the synchronously rotating disks. The disks should have big radii, in order the movement of each cut in the spectrum analysis zone to be assumed parallel.

Finishing the description of the slit analyzer of the wave front, let us consider its basic characteristics. The absolute measurement error in the proposed scheme is caused by the instability of the velocity of the slit movement and by the fluctuation error related to the measurements of the pulse arrival time. This error has the order of $a/\sqrt{q^2}$, where q^2 is the signal-to-noise ratio at the output of the radiation detector matrix element. As to the first component, the velocity of movement can be measured and taken into account when obtaining the estimates.[5] The fluctuation component can be done essentially less than in the Hartmann sensor. First, it is connected with the fact that measurements in each resolution element are provided by the same detector of radiation, and not by four detectors as in the widely applied schemes of the Hartmann sensor. Secondly, in the IR range, one can use high-sensitive cooled photodetector matrix, while the set of the cooled four-plate detectors of radiation is necessary for providing similar sensitivity of the Hartmann sensor, that is very difficult (so, as a rule, the quadrant pyroelectrical photodetectors are applied in such cases). The error in measuring the WF tilts is several per cent of the angular width of the analyzer slit for the values $q^2 \sim 1000$ usually realized in such schemes.

Let us now determine the linear range of the WF tilts measurements by this method. It is limited by the angular field of the optical system formed by the lenses *2* and *4* (see Fig. 1). It is known in the literature as the Kepler telescopic system and is widely used in astronomy and geodesy. The angular field, in which the diffraction quality of the image is realized, is not less than 30...60 minutes of arc for such a telescope, that noticeably exceeds the range of the possible WF tilts normally occurring in AOS. As to such an important characteristic of the analyzer as the operation rate, it is caused by the use of mechanical analyzers in the scheme. Their general disadvantage is that they have the limitation of the rotation speed resulting from the strength condition. The increase of the analyzer slits on one disk will result in the increase of its size, that is often unacceptable due to the construction considerations. So one use only the frequencies of ~ 3 kHz in the instruments with mechanical modulation, that however is quite enough for the analyzer operating in the AOS contour, because the transmission band of adaptive mirrors usually does not exceed 100...1000 Hz (Ref. 1), and, hence, the band of the entire adaptation contour can not be made wider.

Thus, the multichannel slit analyzer based on the Foucault method has a number of essential advantages against the Hartmann sensors used for analogous purposes. Firstly, such an analyzer has simpler construction, because the monolithic matrix with up to four times less number of elements and the same spatial resolution can be used instead of the set of quadrant photodetectors usually used in the Hartmann sensors. Secondly, at least one order of magnitude wider range of measurable WF tilts is provided. Thirdly, such an analyzer is significantly more sensitive in the IR range due to the possibility of using cooled photodetecting matrix. These advantages allow us to assume it to be much promising for investigating the characteristics of the optical radiation and for creation of more effective systems of the adaptive optics.

REFERENCES

1. V.G. Taranenko and O.I. Shanin, *Adaptive Optics* (Radio i Svyaz', Moscow, 1990), 111 pp.