Analysis of factors affecting the accuracy of determination of the extinction coefficient by use of a mobile lidar

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Some ways are proposed for solving the problem caused by the radiation instability in determining the optical characteristics of a medium with a mobile lidar. The error in determination of the extinction coefficient is estimated for the case of signals backscattered from a set of common scatterers. The relation between the atmospheric (hydrospheric) situation (ε or $S_{\rm m}$), spatial resolution, and the measurement error of backscattered signals is described.

Nowadays, in connection with the deteriorating environmental conditions, there is a need in operative and systematic determination of aerosol and gas components of the atmosphere, as well as atmospheric pollutants. This problem cannot be solved on the global scale without the use of airborne and shipborne measurements, as well as measurements from other mobile platforms.

Lidar systems installed on mobile platforms (satellite, aircraft, ship, etc.) can play an important part in the global monitoring of the environment, because they possess the capabilities of both real-time measurements and remote monitoring.[1–3]

Almost all existing lidar methods for determining optical characteristics of scattering media (atmosphere, water)[1–3] are based on solving the lidar equation with the variable upper limit of integration:

$$S(R, r) = AP_0\sigma_\pi(r) \exp \left\{-2 \int\limits_R^r \varepsilon(r)\, {\rm d}r\right\}, \qquad (1)$$

where $S(R, r) = P(R, r)\,(r - R)^2$; $\sigma_\pi(r) = g_\pi(r)\,\varepsilon(r)$; $P(R, r)$ and P_0 are the power of the received and emitted signals, respectively; A is the instrumental constant; $\sigma_\pi(r)$, $\varepsilon(r)$, and $g_\pi(r)$ are the backscattering coefficient, extinction coefficient, and the backscattering phase function at the point r; R is the lidar position.

The solution of Eq. (1) with the variable upper integration limit r is equivalent to the solution of the first-order differential equation with two unknowns $g_\pi(r)$ and $\varepsilon(r)$:

$$\varepsilon'_\kappa(r) - \frac{S'_r(R, r)}{S_r(R, r)}\,\varepsilon(r) - \frac{g'_\pi(r)}{g_\pi(r)}\,\varepsilon(r) - 2\varepsilon^2(r) = 0. \qquad (2)$$

It can be solved given known boundary or the initial conditions. In practice, this requires either *a priori* information on the medium properties or additional measurements.

In case of a variable lower integration limit (dynamic method, in which the lidar moves and signals backscattered from the same scatterers are recorded) the differential equation equivalent to Eq. (1) has the form[4]:

$$S'_R(R, r) = 2\,\varepsilon(R)\,S_R(R, r)\,. \qquad (3)$$

Solution of Eq. (3) for $\varepsilon(R)$ does not require any assumptions or *a priori* information on the medium studied. However, measurement of $\varepsilon(R)$ in this case requires calculation of derivatives of the signals that are measured in the experiment with some error. Since such a problem is ill-posed, this can lead to large errors in $\varepsilon(R)$.

For measurements in the atmosphere and in water media, the mean values of ε measured actually at a sufficiently long interval ΔR. The equation for $\overline{\varepsilon}(\Delta R)$ in this case is written as follows [4]:

$$\overline{\varepsilon}(\Delta R) = -\frac{1}{2\Delta R} \ln \frac{S(R, r)}{S(R + \Delta R, r)}\,. \qquad (4)$$

Apparently, the values of the optical characteristics obtained by this method are independent of the position of a common scatterer r. Consequently, measuring signals backscattered from N common scatterers r_i, we can determine $\overline{\varepsilon}(\Delta R)$ as a simple mean of the values of $\varepsilon_i(\Delta R)$ obtained from independent processing of signals at each point r_i. This allows us to decrease the influence of measurement errors in statistically independent backscatter signals on the accuracy of $\overline{\varepsilon}(\Delta R)$ determination.

Let us compare the measurement errors in the extinction coefficient for the cases of a single scatterer and many scatterers. Using the method of finite increments,[5] it is easy to find the error in determination of $\overline{\varepsilon}$ using Eq. (4):

$$\delta\varepsilon(\Delta R) = \frac{1}{2\tau}\,[\delta S(R, r) + \delta S(R + \Delta R, r)], \qquad (5)$$

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where $\tau = \varepsilon \Delta R$; $\delta\varepsilon = \Delta\varepsilon/\varepsilon$; $\delta S = \delta/s$; σ is the rms deviation of the measured backscattered signals;

$$\Delta\varepsilon = \left|\frac{\partial\varepsilon}{\partial S(R, r)}\right| \Delta S(R, r) + $$
$$+ \left|\frac{\partial\varepsilon}{\partial S(R + \Delta R, r)}\right| \Delta S(R + \Delta R, r).$$

If the absolute errors in measuring the backscattered signals $S(R, r)$ and $S(R + \Delta R, r)$ that differ by the value of extinction due to the lidar displacement within the interval ΔR (spatial resolution) are assumed equal, then Eq. (5) takes the form

$$\delta\varepsilon = \frac{1}{\tau}\,\delta S.$$

In the case of many scatterers r_i ($i = 1, 2, \ldots, N$) to determine $\bar{\varepsilon}(\Delta R)$, the measurement error in the signal S_N averaged over N common scatterers is described by the equation

$$\delta S_N = \frac{\sqrt{D\left[\frac{1}{N}\sum\limits_{i=1}^{N} S_i\right]}}{\frac{1}{N}\sum\limits_{i=1}^{N} S_i} = \frac{\sqrt{\frac{1}{N^2}\sum\limits_{i=1}^{N} \sigma_i^2}}{\frac{1}{N}\sum\limits_{i=1}^{N} S_i}.$$

If we assume that σ_i, for the used scatterers at r_i are roughly equal (or σ_n does not exceed σ), then

$$\delta\bar{S}_N = \frac{1}{\sqrt{N}}\frac{\sigma}{\bar{S}_N},$$

and

$$\delta\varepsilon = \frac{2}{\sqrt{N}}\,\delta\bar{S}_N + \delta(\Delta R).$$

At the equal measurement errors δS and $\delta\bar{S}_N$ for the signal backscattered from a single scatterer and for the average signal for N common scatterers, the error in determination of $\varepsilon(\Delta R)$ in the case of many scatterers is roughly $\sqrt{N}$ times smaller. In practice, this reduces to the following: if the preset (needed) measurement error in the backscattered signal is achieved for the jth scatterer, then N other common scatterers used to decrease the error of determination of $\bar{\varepsilon}(\Delta R)$ must be located to the jth point.

Since the radiant energy of optical sources, including lasers, is characterized by some instability, it is obvious that this instability must manifest itself in the dynamic methods using straight-line motion, because these methods analyze scattered signals acquired at different time. The influence of this instability can be easily eliminated in the following way. A radiation source at the point R should emit pulses in two opposite directions. Then backscattered signals from these directions are received and transformed into electrical signals (Fig. 1).

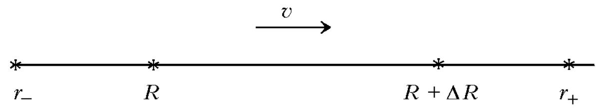

Fig. 1. Geometry of measurement of backscattered signals for eliminating instability of a radiation source.

After amplification of these signals proportionally to the square of the current time measured from the moment of emission of a light pulse, equation (1) describing the recorded signals takes the form

$$S'(R, r_+) = A_+ P'^{+}_{R}\,\sigma_\pi(r'_+)\exp\left\{-2\int\limits_{R}^{r_+}\varepsilon(r)\,\mathrm{d}r\right\};$$
$$S'(R, r_-) = A_- P'^{-}_{R}\,\sigma_\pi(r_-)\exp\left\{-2\int\limits_{r_-}^{R}\varepsilon(r)\,\mathrm{d}r\right\}, \qquad (6)$$

where A_+ and A_- are the constants of the receivers receiving the backscattered radiation from the opposite directions; P_R^+ and P_R^- are the power of radiation emitted at the point R along each of the directions; $\sigma(r_+)$ and $\sigma(r_-)$ are the backscattering coefficients at the points r_+ and r_- (in the opposite directions).

Similar equations can be written for a pulse emitted from the point $R + \Delta R$:

$$S'(R+\Delta R, r_+) = A_+ P'^{+}_{R}\,\sigma_\pi(r'_+)\exp\left\{-2\int\limits_{R+\Delta R}^{r_+}\varepsilon(r)\,\mathrm{d}r\right\};$$
$$S'(R+\Delta R, r_-) = A_- P'^{-}_{R}\,\sigma_\pi(r_-)\exp\left\{-2\int\limits_{r_-}^{R+\Delta R}\varepsilon(r)\,\mathrm{d}r\right\}. \qquad (7)$$

From Eqs. (6) and (7) it is easy to obtain the equation for $T(\Delta R) = \exp\left\{-\int\limits_{R}^{R+\Delta R}\varepsilon(r)\,\mathrm{d}r\right\}$:

$$T(\Delta R) = \sqrt[4]{S'^{+}/S'^{-}}, \qquad (8)$$

where

$$S'^{+} = S'(R, r_+)/S'(R + \Delta R, r_+),$$

$$S'^{-} = S'(R, r_-)/S'(R + \Delta R, r_-).$$

Equation (8) for the transmission for the case of the lidar moving within the interval ΔR does not include energy of the sounding pulses emitted along the counter directions. This, in its turn, indicates that the considered method is independent of the instability of radiation in these cases. In practice, the emission of a sounding pulse in the opposite directions in any version of the dynamic method is quite easy and does not complicate the method.

In a different version, in spite of the emission of sounding radiation in the opposite directions, it is proposed to deflect a part of radiation emitted along the direction of movement to the photodetector for recording. However, in this case one should know the reflection coefficient and conduct calibrating measurements to determine it.

The emission of sounding radiation along the direction of lidar motion and in the opposite one allows also the influence of distortion introduced by a moving object on the optical characteristics of the medium to be determined.

The measured signals $S(R, r)$ from a common scatterer at different positions of a lidar (source) differ from each other by the value of extinction within the lidar displacement interval. For the aquatic scattering media (ocean, rivers, lakes), the extinction is significant even for small intervals. However, for the atmosphere, especially transparent atmosphere, large distances are needed to obtain noticeable extinction. Therefore, there is a need to measure scattered signals with high accuracy that cannot be provided by current measuring equipment in many situations.

This peculiarity follows from the definition of an elementary scattering volume in optics of scattering media. By definition, the linear dimensions of an elementary scattering volume are about hundreds meters for the clear atmosphere and fractions of millimeter for a turbid colloid solution. Therefore, it is very important to determine the spatial resolution of the method proposed to meet the measurement accuracy achieved by the information and measurement systems under different meteorological conditions.

Below we present the equation relating the spatial resolution ΔR to the error of the measurement equipment and the character of the scattering medium (clear atmosphere, haze, fog, cloud, water) described by the meteorological visibility range $S_{\rm m}$.

We believe that the absolute measurement error in the backscattered signals $S(R)$ corrected for the squared distance cannot exceed their half-difference for two positions of the lidar ($\Delta \le \Delta S/2$), where $\Delta = S - S_{\rm tr}$; $S_{\rm tr}$ is the true value of a signal. This inequality is the condition of the maximum acceptable measurement error. Otherwise, one can obtain physically meaningless results (negative value for the extinction coefficient or the transmission larger than unity).

The difference of the signals ΔS can be presented in the following form:

$$\Delta S = S(R + \Delta R, r) - S(R, r) = AP_0\sigma_\pi(r) \exp\left\{-2\int\limits_{R+\Delta R}^{r} \varepsilon(r)\,{\rm d}r\right\} \times \left[1 + \exp\left\{-2\int\limits_{R}^{R+\Delta R} \varepsilon(r)\,{\rm d}r\right\}\right]. \quad (9)$$

The absolute error of signal measurement $\Delta = \delta S{\cdot}S$ (δS is the measurement error) is

$$\Delta = \delta S{\cdot}AP_0\sigma_\pi(r) \exp\left\{-2\int\limits_{R+\Delta R}^{r} \varepsilon(r)\,{\rm d}r\right\}. \quad (10)$$

Substituting Eqs. (9) and (10) in the inequality, we obtain

$$2\delta S < \left[1 + \exp\left\{-2\int\limits_{R}^{R+\Delta R} \varepsilon(r)\,{\rm d}r\right\}\right]. \quad (11)$$

From Eq. (11) it is easy to find

$$-\frac{1}{2}\ln(1 - 2\delta S) < \tau \quad (12)$$

or

$$[-S_{\rm m}\ln(1 - 2\delta S)]/7.8 < \Delta R, \quad (13)$$

where $S_{\rm m} = 3.9/\varepsilon$.

Equations (12) and (13) describe the relations among the spatial resolution, needed error of measurement of scattered signals, and the atmospheric situation (ε or $S_{\rm m}$). Thus, having known the capabilities of the measurement equipment we can find, from Eq. (12) or (13), the minimum spatial resolution for different meteorological situations.

Figures 2 and 3 show the calculated minimum resolution for the atmospheric conditions characterized by the meteorological visibility range S_m at different errors in the measured signals. As is seen, the spatial resolution strongly depends on the accuracy characterizing the measurement equipment. Thus, for a clear atmosphere ($S_{\rm m} = 39$ km) the increase in the accuracy of signal measurement from 1 to 0.5% decreases the spatial resolution from 100 to 50 m.

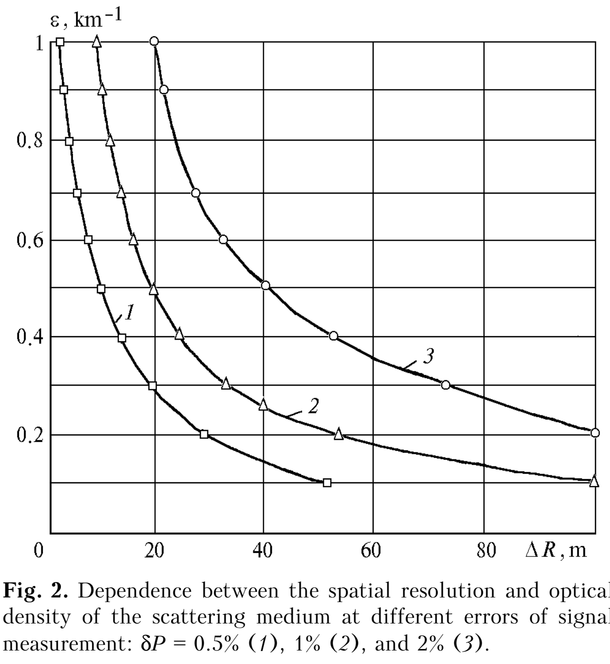

Fig. 2. Dependence between the spatial resolution and optical density of the scattering medium at different errors of signal measurement: $\delta P = 0.5\%$ (*1*), 1% (*2*), and 2% (*3*).

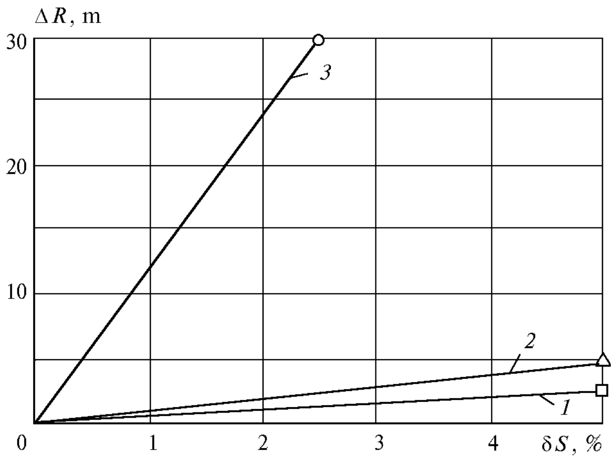

Fig. 3. Dependence between the spatial resolution and the error of signal measurement in optically dense scattering media: S_m = 0.2 (*1*), 0.4 (*2*), and 3.9 km (*3*).

For clouds and fogs the resolution of several meters is achieved at standard errors of the measurement equipment (δP = 2%), whereas tens meters are needed for the haze.

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