THREE-DIMENSIONAL MODELS OF SOLAR-RADIATION TRANSFER IN THE ATMOSPHERE

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Three-dimensional models of the transfer of natural visible radiation in the atmosphere and their applications in the investigation of the transfer properties of the atmosphere and the solution of optical remote sensing problems are studied. The models are based on the theory of boundary-value problems for the integrodifferential radiation transfer equation in the system ground- atmosphere. A classification of boundary-value problems is given. In this classification the albedo nonuniformities, surface reflectance anisotropy and the horizontal nonuniformity of the scattering medium, and the polarization of the radiation are taken into account.

The applications of three-dimensional models of the transfer theory which are included in the review contain the following: methods for constructing the optical transfer operator of the atmosphere; calculations of the point-spread function and the optical space-frequency characteristics of a layer of a turbid medium, and profiles of the brightness and contrasts of natural objects observed through the atmosphere; calculations of the spatial resolution of satellite images taking into account the side illumination; investigations of the characteristics of radiation transfer in horizontally nonuniform clouds and the construction of algorithms for filtering a semitransparent cloud cover; methods for solving inverse problems of reconstructing the two-dimensional albedo, the reflection phase function and the reflection matrix of the ground; and, the problems of incorporating in the system for processing of remote sensing data a block which takes into account the effect of the atmosphere.

INTRODUCTION

The current status of the theory of solar-radiation transfer in the atmosphere is largely determined by progress in three-dimensional modeling. Three-dimensional models are used to describe the light fields in the presence of horizontal nonuniformities of the optical parameters of the medium and the reflective characteristics of the underlying surfaces; they are based on the solutions of boundary-value problems for the integrodifferential transfer equation. In Refs. 1–3 the three-dimensional problem of radiation transfer above a Lambertian surface with a nonuniform albedo as well as the related problems of calculating the optical transfer functions of the atmosphere were studied. In the reviews Refs. 4 and 5 a quite wide range of models was studied, but the authors cited mainly their own works and ignored foreign works. In addition, in the periodicals little attention is devoted to comparing investigations performed in this country and abroad; this often gives a misleading impression of the novelty of the results obtained. In this paper investigations in the field of solar-radiation transfer in a three-dimensional medium, excluding broken clouds, are generalized based on information published over the last ten years.

1. CLASSIFICATION AND ARCHITECTURE OF BOUNDARY-VALUE PROBLEMS

Boundary-value problems in the theory of transfer, on which three-dimensional models are based, are classified according to the degree of complexity. The degree of complexity is determined by a number of factors, including the anisotropy and horizontal nonuniformity of the luminance factors of natural surfaces, the horizontal nonuniformity of a layer of the atmosphere, and the polarization of radiation propagating in the medium. The term architecture refers to the construction of the solutions of complicated boundary-value problems, starting from the solutions of the simplest, basic problems. An illustration of this conceptual organization is given in Ref. 6.

Possible variants of the classification of scalar and vector problems are presented in Tables I and II. The classifications include one-dimensional boundary-value problems which are related in a natural manner with the three-dimensional problems. In Tables I and II the following notation is employed: D, $\bar{I}$, $\bar{I}'$, I, I', D', $\bar{J}$, J are the radiation brightnesses: D, $\bar{I}$, $\bar{I}'$, I, I', D', $\bar{J}$, J are the Stokes vectors; Ψ, Ψ', Ψ, Ψ' are the scalar,

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vector, and matrix optical space-frequency characteristics and Ψ_0, Ψ'_0, Ψ_0, Ψ'_0 are their values at $p = 0$; πS_λ is the solar constant; $\bar{L} = \mu \dfrac{d}{dz} + \bar{\alpha}(z)$, $L = (s, \nabla) + \bar{\alpha}(z)$, $\tilde{L} = (s, \nabla) + \alpha(z, r)$, $\hat{L} = \mu \dfrac{d}{dz} - i(p, s_\perp) + \bar{\alpha}(z)$ are the differential transfer operators;

$$\bar{S}: \bar{S}\Psi_0 = \frac{\bar{\sigma}(z)}{2} \int_{-1}^{1} \psi_0(z, \mu') f^0(\mu, \mu')\, d\mu',$$

$$S: S\bar{I} = \frac{\bar{\sigma}(z)}{4\pi} \int_{\Omega} \bar{I}(z, s') f(s, s')\, ds',$$

$$\tilde{S}: \tilde{S}J = \frac{\bar{\sigma}(z, r)}{4\pi} \int_{\Omega} J(z, r, s') f(s, s')\, ds',$$

$$\bar{P}: P\Psi_0 = \frac{\bar{\sigma}(z)}{2} \int_{-1}^{1} \Psi(z, \mu') F^0(\mu, \mu')\, d\mu',$$

$$P: P\bar{I} = \frac{\bar{\sigma}(z)}{4\pi} \int_{\Omega} \bar{I}(z, s') F(s, s')\, ds',$$

$$\tilde{P}: \tilde{P}J = \frac{\bar{\sigma}(z, r)}{4\pi} \int_{\Omega} J(z, r, s') F(s, s')\, ds'$$

are integral multiple scattering operators;

$$\bar{R}: \overline{RI} = \frac{\bar{q}}{\pi} \int_{\Omega_+} \bar{I}(h, \mu')\mu'\, ds',$$

$$\bar{R}': \bar{R}'\bar{I}' = \frac{1}{\pi} \int_{\Omega_+} \bar{\rho}(s, s') \bar{I}'(h, s')\mu'\, ds',$$

$$R: RI = \frac{q(r)}{\pi} \int_{\Omega_+} I(h, r, s')\mu'\, ds';$$

$$R: RI' = \frac{1}{\pi} \int_{\Omega_+} \rho(r, s, s') I'(h, r, s')\mu'\, ds';$$

$$\bar{\Gamma}: \bar{\Gamma}\bar{I} = \frac{\bar{q}}{\pi} \int_{\Omega_+} \boxed{1}\, \bar{I}(h, s')\mu'\, ds',$$

$$\bar{\Gamma}': \bar{\Gamma}'\bar{I}' = \frac{1}{\pi} \int_{\Omega_+} \bar{\Lambda}(s, s') \bar{I}'(h, s')\mu'\, ds',$$

$$\Gamma: \Gamma I = \frac{q(r)}{\pi} \int_{\Omega_+} \boxed{1}\, I(h, r, s')\mu'\, ds',$$

$$\Gamma': \Gamma' I' = \frac{1}{\pi} \int_{\Omega_+} \Lambda(r, s, s') I'(h, r, s')\mu'\, ds'$$

are the integral reflection operators; $l = \{1, 0, 0, 0\}$; $\bar{\alpha}(z)$, $\bar{\sigma}(z)$, $\alpha(z, r) = \bar{\alpha}(z) + \alpha'(z)\tilde{\alpha}(r)$, $\sigma(z, r) = \bar{\sigma}(z) + \bar{\sigma}'(z)\tilde{\sigma}(r)$ are the average and horizontally nonuniform attenuation and scattering coefficients; $f(s, s_0)$ is the scattering phase function; $F(s, s_0)$ is the angular matrix; $q(r)$, $\rho(r, s, s_0)$, $\Lambda(r, s, s_0)$ are the albedo, the luminance factor and the scattering matrix of the underlying surface; q, $\rho(s, s_0)$, $\Lambda(s, s_0)$ are the same quantities averaged over horizontal coordinates;

$$f^0(\mu, \zeta) = \frac{1}{2\pi} \int_0^{2\pi} f(s, s_0)\, d\varphi;$$

$$F^0(\mu, \zeta) = \frac{1}{2\pi} \int_0^{2\pi} F(s, s_0)\, d\varphi;$$

$$\boxed{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is the isotropic reflection matrix; E is a unit matrix; $s = \{\mu, s_1\}$ is a unit vector 0; $\mu = \cos\theta$; $s_\perp = \sqrt{1 - \mu^2} \times \{\cos\varphi, \sin\varphi\}$; θ and φ are the zenith and azimuthal angles; $s_0 = \{\zeta, s_{0\perp}\}$ is the direction of incidence of the solar rays; $\zeta = \cos\theta_0$; $s_{0\perp} = \left\{\sqrt{1 - \zeta^2}, 0\right\}$; θ_0 is the Sun's zenith angle; Ω is the unit sphere; Ω_+ and Ω_- are the lower and upper hemispheres; z is the vertical coordinate; $r = (x, y)$ is a vector in the horizontal plane; $\rho = (\rho_x, \rho_y)$ is a vector of the spatial frequencies; h is the height of the atmosphere: $z = 0$ and $z = h$ are the upper boundary of the atmosphere and the ground.

TABLE I. Classification of scalar boundary-value problems in the theory of transfer.

$$\begin{cases} \bar{L}D = SD; \quad D|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ D|_{z=h} = 0. \end{cases} \quad (1)$$

Flat layer, bounded by the ground.

$$\begin{cases} \hat{L}\Psi' = S\Psi'; \quad \Psi'|_{z=0} = 0; \\ \Psi'|_{z=h} = \delta(s - s). \end{cases} \quad (7)$$

Three-dimensional layer with an anisotropic source on the lower boundary.

$$\begin{cases} \bar{L}\Psi_0 = S\Psi_0; \quad \Psi_0\big|_{z=0} = 0 \\ \Psi_0\big|_{z=h} = 1. \end{cases} \tag{2}$$

Flat layer with an isolated source on the lower boundary.

$$\begin{cases} \bar{L}\Psi'_0 = S\Psi'_0; \quad \Psi'_0\big|_{z=0} = 0; \\ \Psi'_0\big|_{z=h} = \delta(s - s'). \end{cases} \tag{3}$$

Flat layer with an anisotropic source on the lower boundary.

$$\begin{cases} \bar{L}\ \bar{I} = \bar{S}\ \bar{I}; \quad \bar{I}\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ \bar{I}\big|_{z=h} = \bar{R}\ \bar{I}. \end{cases} \tag{4}$$

Flat layer bounded by an isotropically reflecting surface.

$$\begin{cases} \bar{L}\ \bar{I}' = S\bar{I}'; \quad \bar{I}'\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ \bar{I}'\big|_{z=h} = 0. \end{cases} \tag{5}$$

Flat layer bounded by an anisotropically reflecting surface.

$$\begin{cases} \hat{L}\Psi = S\Psi; \quad \Psi\big|_{z=0} = 0; \\ \Psi\big|_{z=h} = 1. \end{cases} \tag{6}$$

Three-dimensional with an isolated source on the lower boundary.

$$\begin{cases} LI = SI; \quad I\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ I\big|_{z=h} = RI. \end{cases} \tag{8}$$

Three-dimensional layer bounded by a nonuniformly reflecting Lambertian surface.

$$\begin{cases} LI' = SI'; \quad I'\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ I'\big|_{z=h} = RI. \end{cases} \tag{9}$$

Three-dimensional layer bounded by a nonuniformly and anisotropically reflecting surface.

$$\begin{cases} \tilde{L}D' = \tilde{S}D'; \quad D'\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ D'\big|_{z=h} = 0. \end{cases} \tag{10}$$

Horizontally nonuniform layer bounded by an absorbing surface.

$$\begin{cases} \tilde{L}\bar{J} = \tilde{S}\bar{J}; \quad \bar{J}\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ \bar{J}\big|_{z=h} = \bar{R}\ \bar{J}. \end{cases} \tag{11}$$

Horizontally nonuniform layer bounded by a uniformly reflecting surface.

$$\begin{cases} \tilde{L}J = \tilde{S}J; \quad \bar{J}\big|_{z=0} = \pi S_\lambda \delta(s - s_0); \\ J\big|_{z=h} = RJ. \end{cases} \tag{12}$$

Horizontally nonuniform layer bounded by a nonuniformly reflecting surface.

TABLE II. Classification of vector boundary-value problems in the theory of transfer.

$$\begin{cases} L\Psi' = P\Psi'; \quad \Psi'\big|_{z=0} = 0; \\ \Psi'\big|_{z=h} = E\delta(s - s'). \end{cases} \tag{19}$$

Three-dimensional layer with a matrix anisotropic source on the lower boundary.

$$\begin{cases} LI = PI; \quad I\big|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ I\big|_{z=h} = \Gamma I. \end{cases} \tag{20}$$

Three-dimensional layer bounded by a nonuniformly reflecting Lambertian surface.

$$\begin{cases} \bar{L}\ D = PD; \quad D\big|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ D\big|_{z=h} = 0. \end{cases} \tag{13}$$

Flat layer bounded by an absorbing surface.

$$\begin{cases} \bar{L}\ \Psi_0 = \bar{P}\ \Psi_0; \quad \Psi_0\big|_{z=0} = 0; \\ \Psi_0\big|_{z=h} = 1. \end{cases} \tag{14}$$

Flat layer with a unit vector source on the lower boundary.

$$\begin{cases} LI' = PI'; \quad I'|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ I'|_{z=h} = \Gamma' I'. \end{cases} \tag{21}$$

Three-dimensional layer bounded by a surface with nonuniform anisotropic matrix reflection.

$$\begin{cases} \tilde{L}D' = \tilde{P}D'; \quad D'|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ D'|_{z=h} = 0. \end{cases} \tag{22}$$

Horizontally nonuniform layer bounded by an absorbing surface.

$$\begin{cases} \tilde{L}\bar{J} = \tilde{P}\bar{J}; \quad J|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ J|_{z=h} = \bar{\Gamma} J. \end{cases} \tag{23}$$

Horizontally nonuniform layer bounded by a surface with uniform reflection matrix.

$$\begin{cases} \tilde{L}J = \tilde{P}J; \quad J|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ J|_{z=h} = \Gamma J. \end{cases} \tag{24}$$

Horizontally nonuniform layer bounded by a surface with nonuniform reflection matrix.

In the tables some intermediate combinations, as well as more complicated combinations which are not significant for applications, are omitted. Figure 1 shows the hierarchical tree, whose branches can be extended in accordance with the formulation of the boundary-value problem.

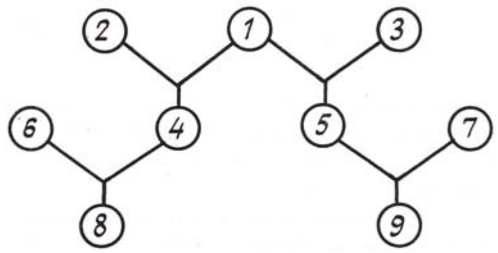

FIG. 1. Architecture of scalar boundary-value problems in the theory of transfer.

The architectural relations of the vector models can be represented analogously. We shall examine the most characteristic examples. The solution of the boundary-value problem Eq. (4) for $s \in \Omega_- \bar{I}$ can be expressed in terms of the solution of the simplest problems (1) and (2) in the form[7,8]

$$\bar{I} = D + \frac{\bar{q}E_0\Psi_0}{1 - \bar{q}c_0}, \tag{25}$$

where

$$\begin{cases} \bar{L}\,\Psi'_0 = P\Psi'_0; \quad \Psi'_0|_{z=0} = 0; \\ \Psi'_0|_{z=h} = E\delta(s - s'). \end{cases} \tag{15}$$

Flat layer with a matrix anizotropic source at the lower boundary.

$$\begin{cases} \bar{L}\ I = \bar{P}\ I; \quad I|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ I|_{z=h} = \bar{\Gamma}\ I. \end{cases} \tag{16}$$

Flat layer bounded by an isotropically reflecting surface.

$$\begin{cases} \bar{L}\ \bar{I}' = \bar{P}\ \bar{I}'; \quad \bar{I}'|_{z=0} = \pi S_\lambda l\delta(s - s_0); \\ \bar{I}'|_{z=h} = \bar{\Gamma}'\bar{I}'. \end{cases} \tag{17}$$

Flat layer bounded by a surface with uniform matrix reflection.

$$\begin{cases} L\Psi = P\Psi; \quad \Psi|_{z=0} = 0; \\ \Psi|_{z=h} = 1. \end{cases} \tag{18}$$

Three-dimensional layer with unit vector source on the lower boundary.

$$E_0 = 2\int_0^1 D^0(h,\ \mu,\ \zeta)\mu\ d\mu + \zeta S_\lambda \exp\left(-\tau_0/\zeta\right);$$

$$D^0(h,\ \mu,\ \zeta) = \frac{1}{2\pi}\int_0^{2\pi} D(h,\ s)\ d\varphi;$$

$$c_0 = 2\int_0^1 \Psi_0(h,\ \mu)\mu\ d\mu;$$

is the optical thickness of the atmosphere.

The solutions of the boundary-value problem with anisotropic reflection (5) were studied in Refs. 1, 7, and 9–13. It is useful to represent $\bar{I}$ as a sum of three terms

$$\bar{I} = D + \bar{I}_1 + \bar{I}_\Sigma. \tag{26}$$

The terms $\bar{I}_1$ and $\bar{I}_\Sigma$ satisfy the boundary-value problems

$$\left\{\bar{L}\ \bar{I}_1 = S\bar{I}_1;\ \bar{I}_1|_{z=0} = 0;\ \bar{I}_1|_{z=h} = \bar{R}'D + \bar{R}'I_{dir}\right\},$$

$$I_{dir} = \pi S_\lambda \delta(s - s_0)e^{-\tau/\zeta}, \quad \tau = \int_0^z \bar{\alpha}(z')\ dz',$$

$$\left\{\bar{L}\ \bar{I}_\Sigma = S\bar{I}_\Sigma;\ \bar{I}_\Sigma|_{z=0} = 0;\ \bar{I}_\Sigma|_{z=h} = \bar{R}'\bar{I}'_\Sigma + \bar{R}'\bar{I}_1\right\}.$$

In Ref. 11 it is shown that replacing the natural reflection $\bar{\rho}(s, s_0)$ by Lambertian reflection $\bar{q}$ when calculating I_Σ introduces an error of < 1%. Setting $\bar{R}'\bar{I}_\Sigma \approx \bar{R}\cdot\bar{I}_\Sigma$ and $\bar{R}'\bar{I}_1 \approx \bar{R}\cdot\bar{I}_1$ we obtain

$$\bar{I}_\Sigma \approx \frac{\Psi_0 \bar{R}\,\bar{I}_1}{1 - \bar{q}c_0}. \qquad (27)$$

The representation (26) and (27) makes it possible to simplify substantially the solution of the boundary-value problem (5) without reducing the computational accuracy.

The problem with nonuniform Lambertian reflection (8) was studied in Refs. 1, 6, 10, 11, and 14–25. The solution of this problem is represented in terms of Eqs. (4) and (5) as follows:

$$I = \bar{I} + \tilde{I} + \tilde{I}_\Sigma, \qquad (28)$$

where

$$\tilde{I} = \frac{E_0}{1 - \bar{q}c_0}\frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty}\frac{\Psi(z,\ p,\ s)}{1 - \bar{q}\bar{C}(p)}\hat{\tilde{q}}(p)e^{-i(p,r)}\,dp; \qquad (29)$$

$$C(p) = \frac{1}{\pi}\int\limits_{\Omega_+}\Psi(h,\ p,\ s)\mu\,ds;$$

$$\hat{\tilde{q}} = \int\limits_{-\infty}^{\infty}\tilde{q}(r)e^{-i(p,r)}dr; \quad \tilde{q}(r) = q(r) - \bar{q}.$$

The function $\Psi \equiv \Psi(z,\ p,\ s)$ plays a fundamental role in the theory of vision in turbid media. The radiation characteristics D, E_0, c_0, $C(p)$, Ψ_0, determine the effect of the optical transfer operator, which transforms the albedo of the underlying surface $q(r) = \bar{q} + \tilde{q}(r)$ into the brightness of the outgoing radiation. The quantity $\tilde{I}_\Sigma$ depends nonlinearly on $\tilde{q}(r)$. An exact expression for this quantity was found in Refs. 20 and 26. For real values of the optical thickness τ_0 of the layer of atmosphere and the average albedo $\bar{q}$ the contribution of $\tilde{I}_\Sigma$ to the total brightness I does not exceed 1%.

The questions of taking into account simultaneously the anisotropy of reflection and the nonuniformities of the albedo of the ground were investigated in Refs. 10, 11, and 13. For the case $\rho(r, s, s_0) = \bar{\rho}(s, s_0) + q(r)\rho'(s, s_0)$ the following approximation is proposed:

$$I = I + \tilde{I}' + \tilde{I}'_\Sigma, \qquad (30)$$

where $\tilde{I}' = \dfrac{\Psi'_0}{\Psi_0}\tilde{I}$. An expression for $\tilde{I}'_\Sigma$ can be presented in the Ref. 13. In the general case $\rho(r,\ s,\ s_0) = \sum\limits_{m=1}^{M} q_m(r)\rho_m(s, s_0)$ the solution can be constructed analogously. Like in the examples presented above, the boundary-value problem (9) reduces to the simpler problems (5), (7), (2), and (3).

The horizontal nonuniformity of the atmosphere is given by the coefficients[27,26]

$$\alpha(z,\ r) = \bar{\alpha}(z) + \bar{\alpha}'(z)\tilde{\alpha}(r),$$

$$\sigma(z,\ r) = \bar{\sigma}(z) + \bar{\sigma}'(z)\tilde{\sigma}(r)$$

The boundary-value problem (10) was used to describe radiation transfer in horizontally nonuniform clouds,[27–31] and in addition the factors $\tilde{\alpha}(r)$ and $\tilde{\sigma}(r)$ were represented by finite harmonic series. In the case of arbitrary $\tilde{\alpha}(r)$ and $\tilde{\sigma}(r)$ and $q(r) \neq 0$ the solutions of the problem (10) were studied in Refs. 24, 26, and 32.

The architecture of vector boundary-value problems is very similar to that of scalar boundary-values problems,[33–36] and for this reason we do not give here the corresponding examples.

The foregoing discussions makes clear the principle of classification of boundary-value problems based on the degree of complexity, which is determined by the specification of the optical properties of the atmosphere and the ground. Using the principle of solution construction for complicated problems with the help of solving the elementary problems, it is possible to construct a model in which all factors that form the spatial and angular structure of the brightness fields are taken into account simultaneously: anisotropy and nonuniformity, reflection of the ground, horizontal nonuniformity of the scattering medium, and polarization of the radiation. In solving practical problems, however, some factors are found to be unimportant. For this reason, in most works simplified models are studied.

2. METHODS FOR SOLVING BOUNDARY-VALUE PROBLEMS

We shall discuss the methods used to construct the solutions of three-dimensional boundary-value problems in a plane-parallel geometry.

2.1. Method of optical space-frequency characteristics (Ψ representation). The physical content of the method lies in the description of the transfer of an optical image from the standpoint of the theory of linear systems. Such a description reduces to calculating the impulse transfer function or the frequency characteristic of the system.[2] Methods for calculating the brightness fields of luminous objects in a scattering medium using the frequency-contrast characteristics of the medium were proposed in Refs. 37 and 38. The idea of a linear systems approach was developed further in Refs. 39–42. In Refs. 23 and 43–45 the frequency-contrast characteristics were obtained from solutions of boundary-value problems. The method of optical space-frequency characteristics was defined, on the basis of the boundary-value problems of transfer theory, in Ref. 17. It was further developed in Refs. 19–21 and in Refs. 26, 24, and 46 it was extended to the case of a horizontally nonuniform atmosphere and in Ref. 36 it was also extended to problems including polarization.

The system for transferring an optical image in a horizontally uniform atmosphere is interpreted as a nonlinear system described by a collection of transfer functions $\{\Psi_n\}$. The Ψ representation is convenient for analysis and calculations, since the solution is represented by a sum, averaged over the horizontal coordinates, of the variational and nonlinear (relative to $\tilde{q}(r)$) components. The nonlinear component is determined by rereflection of photons by optical nonuniformities of the reflecting surface (see the formulas (26) and (28)). The effect of the pedestal is small and is characterized by the denominator on the right side of the equation $W = \dfrac{\Psi}{1 - \bar{q}C(p)}$, where W is the optical space-frequency characteristic of a scattering layer with the albedo $\bar{q} = 0$. The most general interpretation of the method is presented in Refs. 5 and 26.

2.2. Method of multiple rereflections (*O*-representation). This method consists of counting the photons reflected from the surface a given number of times.[16,21,47,48] This method was developed for solving scalar and vector problems in a horizontally nonuniform atmosphere[16,35,48] and can be extended to the case of a horizontally nonuniform medium. In Ref. 21 it was shown, by direct verification that the Ψ and O representations give the same solution for the direct problem of radiation transfer above a surface with a nonuniform albedo (8).

Summing photons according to the number of times they are rereflected from the surface makes it possible to represent the solution of the boundary-value problem (8) in the form

$$I = D + \int_{-\infty}^{\infty} O(z,\ r - r',\ s)Z(r')\,dr', \tag{31}$$

where $Z(r) = R(E - RO_h)^{-1}(D + I_{gr})$ is the brightness distribution at the ground; E is the identity operator; $I^0_{gr} = \pi S_\lambda \exp(-\tau_0 / \xi)\delta(s - s_0)$ is the singular component of the incident radiation; O_h is a linear integral operator of the convolution type with the kernel $O(h,\ r,\ s)$; $O \equiv O(z,\ r,\ s)$ is the point-spread function, satisfying the boundary-value problem

$$\left\{LO = SO; \quad O|_{z=0} = 0; \quad O|_{z=h} = \delta(r)\right\}.$$

related with Ψ by a Fourier transform in the coordinates $r = \{x,\ y\}$. The method is described in detail in Refs. 48 and 35. The functions (and the associated operators) appearing in Eq. (31) are related with the notion of photon trajectories and have a probability interpretation (Fig. 2). Thus

$$A_0 = \int_{-\infty}^{\infty} O(z,\ r,\ s)\,dr, \qquad c_0 = \int_{-\infty}^{\infty} O_h(r)\,dr$$

are, respectively, the probabilities that a scattered photon from a point source on the surface will reach the observer or strike the surface. In Refs. 16 and 49 analytical representations of the functions $O(r)$ and $A(p)$ are presented.

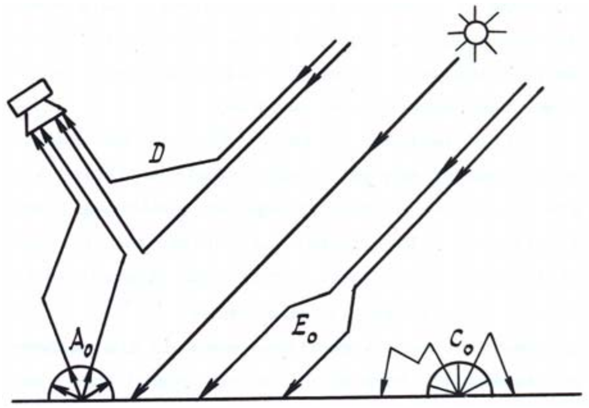

FIG. 2. Nonintersecting photon trajectories.

2.3. Numerical methods. Numerical methods consist essentially of calculating the specific functions in terms of which the solutions of the boundary-value problems are expressed. These include the point-spread function O, the optical space-frequency characteristic Ψ, the brightness of the haze D, the spherical albedo c_0, the illumination of the lower boundary of the layer πE, and other transfer functions.

In the course of investigation of three-dimensional models different algorithms based on the Monte Carlo method,[2,16,21,38,40,41, 47, 48,50,51] the method of iterations,[4,17,19,24,25,52] the method of spherical harmonics,[53-55] small-angle methods,[37,39,42,56] the source-function method,[2,49,54] and the method of discrete coordinates[57] have been developed. The most flexible method, which enables modeling with arbitrary optical parameters of the medium, is the Monte Carlo method. Another powerful computational tool is the method of integration of the transfer equation along characteristics with iterations and quadratures on a unit sphere; this method makes it possible to obtain simultaneously the values of the functions sought on all points of the difference grid in angular and spatial variables. Numerical investigations of most problems are now being performed precisely by these methods, whose accuracy can be controlled. These methods have the drawbacks that the volume of calculations is relatively large, especially in the case of large optical thicknesses; this makes it necessary to look for ways to accelerate the calculation and to employ approximations, which lower the computational accuracy.

Calculations in the small-angle approximation are of an approximate character and are valid for. strongly elongated scattering phase functions. The latest investigations of the small-angle approximation have made it possible to extend its range of application.[42,56,58] The accuracy of many algorithms based on the methods of spherical harmonics, source functions, and discrete coordinates is uncontrollable, and it is usually estimated with the help of comparative numerical tests. Comparing the calculations performed by different numerical methods is standard procedure in many works. The most representative comparisons were made in Refs. 52, 54, and 55. The advantage of this group of methods is that they are relatively fast.

The need for faster and more accurate algorithms is spurring further development. New methods and modifications which have already been t developed must be compared with existing calculations.

3. INVESTIGATION OF THE TRANSFER PROPERTIES OF THE ATMOSPHERE

According to Eqs. (25), (28), and (29) the atmospheric distortions consist of a superposition of atmospheric haze D, rereflection by the ground described by the average factor $(1-\bar{q}c_0)^{-1}$, followed by extinction in the atmosphere in accordance with $E_0\Psi_0$ and the action of Ψ. These factors taken together decrease the image contrast and reduce the real resolution of objects. The latter is a consequence of side illumination, caused by diffusion of photons in the atmosphere towards regions lying above weakly reflecting sections of the ground.

The three-dimensional transfer functions of a scattering horizontally uniform layer is the optical space-frequency characteristic

$$\Psi = \left[Ae^{i\Phi} + T\right]\exp\left[i\left(p,\ \tilde{r}\right)\right], \qquad \tilde{r} = \left(h - z\right)s_{\perp}/|\mu|$$

and the corresponding impulsive transfer characteristic

$$N = O\left(z,\ r - \tilde{r},\ s\right) + T\delta\left(r - \tilde{r}\right)$$

$$T = \exp\left(-\left(\tau_0 - \tau\right)/|\mu|\right)$$

where A and Φ are the amplitude and phase characteristics. In the first investigations A was calculated using an approximate formula in the small-angle approximation.[37] Then the function W was investigated for the model of a homogeneous atmosphere illuminated by the sun and bounded by a uniform underlying surface for different values of τ_0, $f(s,\ s')$, and q.[15,17,19] Later the functions W, Ψ, and O were calculated by different numerical methods[21,25,38,40,52,54,55,57] using models of a real atmosphere. Tables of values of the functions A and Φ for models of a continental aerosol are given in Refs. 52, 54, and 55.

Side illumination

$$H_{side} = \frac{E_0}{1 - \bar{q}c_0}\int\limits_{-\infty}^{\infty} O\left(z,\ r - \tilde{r} - r',\ s\right)\tilde{q}\left(r'\right)\, dr'$$

is a measure of the radiation reaching the detector from the direction s and reflected from sections outside the point of direct observation.[49,59,60] In Refs. 14, 25, 44, 45, and 49 side illumination from real natural formations on the ground is investigated using test objects. Side illumination is important above dark objects surrounded by a bright background. This physically quite obvious fact has been evaluated quantitatively. The effect of the background depends on the dimensions of the object and the difference of the object and background albedo. The smaller the object and the larger the indicated difference, the stronger the side illumination is when the background is brighter than the object.

A three-dimensional model of radiation transfer was apparently first used to analyze the interaction of radiation images of objects and the background in Ref. 14, where the brightness field of outgoing radiation for observation at the nadir at wavelengths of $\lambda = 0.5$, 0.65, 0.75, and 0.95 μm and the values of the meteorological visibility range $V = 3$, 10, and 40 km with different ratios of the albedo of the object (80 m in size) and albedo of the background, was modeled. It was shown that the brightness of the radiation above wheat and soil varies by 25 and 80%, respectively, if the surrounding earth cover uniform with the object is replaced by grass. In Ref. 45 an example of the simulation of a two-dimensional field (a section with $\bar{q} = 0.4$, surrounded by a field with $\bar{q} = 0.0$) at the top boundary of the atmosphere ($\lambda = 0.55$ μm, $\tau_0 = 0.64$) is presented. This example shows clearly the effect of the smearing of the boundaries of the section with $\bar{q} = 0.4$ on the image. In this case H_b is virtually independent of the height of the aerosol atmosphere, but it increases as τ_0 increases. In a cloud-free atmosphere the horizontal diffusion of photons has virtually no effect on the image of natural objects that are brighter than the surrounding location.[2,61] Significant distortions appear when viewing through fog or semitransparent clouds.[41] The effect of the phase characteristic Φ was investigated in Ref. 62. It was shown that in the case of a slightly turbid atmosphere the effect of the phase characteristic is negligibly small. Such estimates have not been made for the case of strong turbidity. Thus the three-dimensional model of radiation transfer has made it possible to extend the theory of transfer of brightness contrasts through the atmosphere.

In the observation of extended objects the concept of contrast makes sense near the boundaries of these objects. This is why calculations of the profiles of the brightness of reflected radiation above the interface of two natural media with different albedos are interesting.

The exact formula for the profile of the brightness above the boundary of two isotropically reflecting half-planes whose albedos differ by Δq has the form

$$I = \bar{I} + \frac{\Delta q}{2} \frac{E_0}{1 - \bar{q}c_0} \left[T\mathrm{sign}(x - \tilde{x}) + \right.$$

$$\left. + \frac{2}{\pi} \int_0^\infty \frac{A(p_x)\sin\left[p_x(x - \tilde{x}) - \Phi(p_x)\right]}{p_x} dp_x \right], \quad (32)$$

where $\tilde{x}$ is the component of the displacement vector $\tilde{r}$ along the x axis. In the calculations performed in Refs. 11, 15, 17, 18, 22, 25, 43–45, 63, and 64 very diverse numerical schemes were employed. The computed and experimental LANDSAT images of the land-sea boundary were compared in Refs. 43–45. The theoretical calculations were confirmed experimentally and the results agreed well with one another. In this case this agreement was also obtained because the correct optical parameters of the atmosphere were chosen. The most detailed investigations for different models of the atmosphere were performed in Refs. 43 and 64.

It is well known that the atmosphere contains mobile aerosol layers and semitransparent clouds. This is why it is of interest to investigate the possibilities of viewing through these scattering objects. In Ref. 59 the effect of the position of a scattering layer on side illumination in viewing systems was clarified. In Refs. 40 and 41 the effect of the position of the scattering layer between the object and the radiation detector on image quality was investigated. It was proved experimentally and theoretically that when the scattering layer moves from the source toward the detector the visibility of the object changes nonmonotonically. This phenomenon is described for both small and extended objects.[60] Based on the obtained results it can be concluded that the position of aerosol layers in the atmosphere affects the quality of aerospace images of the earth's surface.

The effect of the atmosphere on the spatial structure of a space image was modeled in Ref. 65. In Ref. 44 the frequency-contrast characteristic of the system detector-atmosphere, employed for estimating the real resolution of photographs, was calculated. It was shown that atmospheric turbidity (the aerosol optical thickness $\tau_a = 0.5$) reduced the 30-m resolution of thematic LANDSAT maps to 100 m. Therefore, when performing remote sensing and classification of fields with diameter $d > 200$ m it is pointless to improve the resolution of the sensor without introducing an atmospheric correction. In Refs. 66 and 67 it is shown that the composite effect of atmospheric scattering and the nonuniformity of the surface albedo strongly affects the accuracy of the classification of fields on the earth's surface. In some works the spatial distortions of the structure of the image were investigated with the help of the point-spread function O. In Ref. 68 the function O was calculated by the Monte Carlo method for a model of the real atmosphere ($\tau_a = 0.16$ and $\tau_a = 0.332$). It was found that the effect of O in simulating an image of an agricultural landscape was insignificant. The authors explain this by the fact that O has a sharp central peak and extended wings. But it is assumed that even an insignificant spread of the image can result in degradation of classification.

It is very difficult to implement numerical schemes in models with horizontal nonuniformity and polarization. The problem of radiation transfer in horizontally nonuniform clouds was first formulated without invoking the optical space-frequency characteristic for the particular cases of the r dependences of $\tilde{\alpha}(r)$ and $\tilde{\sigma}(r)$. The first four orders of the perturbation of the radiation brightness for reflection from a semi-infinite layer under conditions of isotropic scattering[28] where obtained, the asymptotic behavior of the light reflection and transmission functions was determined for a horizontally nonuniform layer,[30] and the reflectance of a horizontally nonuniform optically dense cloud with anisotropic scattering was calculated. It was found that the spatial distribution of the brightness field in the cloud can change the phase, i.e., the brightness maxima become minima and *vice versa*.[31] The concept of the optical space-frequency characteristic has made it possible to make progress in the investigation of this multidimensional problem. This characteristic of a horizontally nonuniform cloud was calculated in Ref. 69. The nonlinear distortions of a space image owing to horizontal nonuniformity of a cloud-free atmosphere were estimated[46] and a simple model of radiation transfer in a semitransparent cloud cover was developed.[5] We note that a universal algorithm, suitable for investigating horizontally nonuniform objects with arbitrary optical thickness, has not yet been developed.

Models with polarization still are of limited usefulness because they are unwieldy and because of a lack of experimental data. Nonetheless analysis of the models makes it possible to find simple relations between the degree of polarization of the radiation being measured and the optical parameters of the medium and the reflecting surface.[70,71] These relations can be used to interpret and indicate the horizontal variations of the aerosol from measurements of I near the nadir.

4. REMOTE SENSING PROBLEMS

It is natural to use in remote sensing simple relations, obtained by solving the direct problems of the theory of transfer, between the measured quantities and the quantity sought. Examples of such relations are the formulas relatingthe degree of polarization of the reflected radiation with the albedo of the surface or the degree of horizontal nonuniformity of the atmosphere. In many cases, however, inversion of the direct operator is a computational problem and it is necessary to solve inverse problems. An important application of three-dimensional models is the solution of inverse problems of remote sensing concerning the reconstruction of the two-dimensional albedo, the scattering phase function, and the reflection matrix of the earth's surface based on remote sensing data. These solutions represent a step in the processing of satellite data for the purpose of interpreting and determining the state of natural resources.

The most detailed solutions of problems have been obtained under the assumption that the reflection of the earth's surface is isotropic.[6,8,16,17, 21, 25, 61,72,73] In Ref. 61 an iterative procedure was proposed for reconstructing a stepped albedo, depending on one coordinate. This procedure was tested successfully in reconstructing the albedo near the land-sea boundary from LANDSAT photographs. In Refs. 8, 17, and 21 a solution was obtained for the two-dimensional case by the method of optical space-frequency characteristics. It was employed in modeling the block in which atmospheric corrections are made in the satellite images[72] and in processing the NOAA satellite images.[74] An exact solution was obtained in Refs. 16 and 21:

$$q(r) = \frac{Z(r)}{E_0 + \int\limits_{-\infty}^{\infty} \bar{O}(r - r')Z(r')\, dr'}, \tag{33}$$

where

$$Z(r) = \frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty} \frac{\hat{Y}(p)\exp\left[-ip\left[r + \tilde{r}\right]\right]}{T + Ae^{i\Phi}}\, dp;$$

$$Z(r) = \frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty} \frac{\hat{Y}(p)e^{\left(-ip\left(r+\tilde{r}\right)\right)}}{T + Ae^{i\Phi}}\, dp; \tag{34}$$

$$\bar{O}(r) = \frac{1}{\pi}\int\limits_{\Omega_+} O(h, r, s)\mu\, ds,$$

$$\hat{Y}(p) = \int\limits_{-\infty}^{\infty} Y(r)\exp\left[-i(p, r)\right]\, dr, \qquad Y(r) = I - D.$$

According to Eq. (33) the solution is achieved in two steps. First the brightness at the ground $Z(r)$ is calculated using the fast Fourier transform algorithm. Then the convolution integral in Eq. (33) is calculated and the function $q(r)$ is determined. The solution is stable with respect to measurement errors, and the well-known formulas are obtained for $q(r) \equiv \bar{q}$ = const. The effect of the input optics can be taken into account with the help of the formulas relating the angles of arrival with the points on the image. The one-dimensional problem of reconstructing $\bar{q}$ was solved in Refs. 7, 8, and 25, where the corresponding numerical algorithms were developed. An algorithm for reconstructing $\bar{q}$, taking polarization into account, is given in Ref. 21.

Algorithms for reconstructing the average albedo taking into account the anisotropy of the reflection by the surface were developed in Ref. 74. A correct formulation of the problem of determining the brightness coefficient of the underlying surface includes the question of reconstructing the reflection phase function. The corresponding inverse problem was studied in Ref. 13. The solution of this problem involves a stereo survey or angular measurements performed with a scanner.

The method of multiple rereflections makes it possible to reduce the problem to the solution of two equations

$$Y(z, r, s) = Z(r - \tilde{r})T + \int\limits_{\Omega}\int\limits_{-\infty}^{\infty} \tilde{O}(z, r - \tilde{r} - r', s, s')Z(r', s')dr'ds' \tag{35}$$

and

$$Z(r, s) = \int\limits_{\Omega}\int\limits_{-\infty}^{\infty} Q(r, r - r', s, s')Z(r', s')\, ds' + 2\int\limits_0^1 \rho^0(r, \mu, \mu')D^0(h, \mu', \zeta)\mu'\, d\mu' + \zeta\rho(r, s, s_0)\exp(\tau_0/\zeta), \tag{36}$$

where

$$\tilde{O}(z, r - \tilde{r}, s, s') = O(z, r, s, s') - T\delta(r - \tilde{r});$$

$$Q(r, r'; s, s') = \frac{1}{\pi}\int\limits_{\Omega_+} \rho(r, s, s'')O(h, r', s'', s')\mu''ds'';$$

$$\rho^0(r, \mu, \zeta) = \frac{1}{2\pi}\int\limits_0^{2\pi} \rho(r, s, s_0)\, d\varphi,$$

$$D^0(z, \mu, \zeta) = \frac{1}{2\pi}\int\limits_0^{2\pi} D(z, s)\, d\varphi;$$

$$Y(z, r, s) = I(z, r, s) - D(z, s).$$

In the case $\rho(r, s, s_0) = q(r)$ the formulas (35) and (36) transform into Eqs. (33) and (34). In the case when the brightness coefficients are arbitrary the multidimensional equations (35) and (36) must be solved simultaneously. A simpler solution is to employ the approximate relation (26) in the case of a uniformly reflecting bottom or the relation (30) in the case when the albedo of the ground is nonuniform.

Algorithms for solving Eqs. (35) and (36) have not yet been developed. Analogous arguments are valid for vector problems. The question of reconstructing the reflection matrix was studied ..in Ref. 36. However three-dimensional problems with polarization are still far from practical application. In addition, taking polarization into account improves the accuracy of the calculations of the brightness field of the upward directed radiation, on the average, but not more than 1%.[9] As a result, in practice, polarization problems are of less interest than scalar problems.

Real subjects of an aerospace survey have a relief in which one element of the location greatly predominates over other elements. As a result some elements are shaded and distortions are introduced into the. reflected scattered radiation. These effects can be approximately taken into account in calculations of the brightness fields. Accurate solutions of the transfer equation in a layer with an uneven lower boundary have not yet been obtained.

CONCLUSIONS

The complexity of three-dimensional models of solar-radiation transfer in the atmosphere, which can be used to calculate light fields taking into account the horizontal nonuniformities of the brightness coefficients of the ground in a scattering medium, has now reached the level required for applications. Three-dimensional models are used in remote sensing primarily for developing algorithms for reconstructing the reflective characteristics of the earth's surface from remote sensing data. These algorithms, are employed for interpreting natural objects in the process of digital video information processing.

The solution of the problem of reconstructing the two-dimensional albedo of an isotropically reflecting ground from the space image has now been solved exactly. Different modifications of the three-dimensional model of radiation transfer above a surface with a nonuniform albedo have been used for experimental processing of data obtained from Soviet and nonsoviet satellites. The general conclusion of these investigations is that taking into account the optical transfer function of the atmosphere makes it much likely that the observed objects will be correctly identified. Versions of digital models of the atmospheric correction block for the video information processing system have„now been developed.

Because scanning systems are used extensively the problem of developing effective algorithms for taking into account surface reflectance anisotropy on the basis of the general formulation of the inverse problem remains open. Radiation transfer above an uneven surface has not been investigated much.

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