TRANSFER EQUATION FOR THE OPTICAL DEPTH APPLIED TO ESTIMATING THE INTENSITY OF AN AEROSOL POLLUTION SOURCE

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This paper presents an investigation of the transfer equation for the optical depth obtained on the basis of the prognostic equation of atmospheric impurity transfer. It is shown that the equation obtained for the optical depth is identical to that for a linear source. Formulas are derived in the paper which allow one to estimate the intensity of an aerosol effluent by detecting the extremum in the measured values of the optical depth of the ground atmosphere layer.

Estimating and forecasting the air pollution intensity of an industrial area from the optical characteristic atmosphere is an applied problem of current interest in atmospheric optics. Pursuing that task, the authors of Ref. 1 introduced into consideration a transfer equation for certain conserved atmospheric optical parameters, in particular, the optical depth. Reference 2 presented an equation for the dynamics of the optical depth for a prescribed aerosol fraction in a form which allowed the precipitation of aerosol to the ground due to its sedimentation and diffusion to be estimated. The present paper considers the possibility of applying the solution of that equation to estimating the intensity of an isolated source of aerosol pollution from optical sensing data.

If the wind field and eddy diffusion coefficient are known, the propagation of the atmospheric pollutants can be estimated from the model prognostic equation,[3] which has the form

$$u\frac{\partial q_i}{\partial x} - w_i\frac{\partial q_i}{\partial z} = \frac{\partial}{\partial z}k_z\frac{\partial q_i}{\partial z} + \frac{\partial}{\partial y}k_y\frac{\partial q_i}{\partial y} - \alpha_i q_i. \qquad (1)$$

The x axis here is oriented in the direction of the average wind velocity u; z and ω_i are the vertical coordinate and the precipitation velocity for the ith aerosol fraction; q_i is the mass concentration of the ith aerosol pollutant; k_z and k_y are the vertical and horizontal eddy diffusion coefficients; α_i is a coefficient which determines the change in the concentration q_i due to pollutant conversions.

Consider a plume from a stationary source of intensity

$$uq_i = Q_i\delta(y)\,\delta(z - H), \qquad (2)$$

located at the point $x = 0$, $y = 0$, $z = H$. We single out the ith aerosol fraction, prescribing its radius r_i and its complex refractive index m_i. The aerosol mass extinction coefficient, which is given by

$$\alpha^0_{\lambda ai} = \frac{3}{4}\frac{1}{r_i}\frac{1}{\rho^T_{ai}}\left[K_a(r_i, m_i, \lambda) + K_s(r_i, m_i, \lambda)\right], \qquad (3)$$

will then be constant; here ρ^T_{ai} is the true density of the aerosol material, λ is the sensing wavelength, and K_a and K_s are the absorption and scattering coefficients.

The concentration transfer equation for the ith aerosol fraction q_i under such conditions can be transformed into an equation describing the optical depth transfer of the given aerosol fraction along the chosen path. To do this, we have to multiply it by $\alpha^0_{\lambda ai}$ and to integrate over the prescribed path. In particular, integrating Eq. (1) along a horizontal cross-plume path ($-\infty < y < +\infty$) we obtain the transfer equation for the optical depth $\tau^y_{\lambda ai}(x, z)$ in the form

$$u\frac{\partial\tau^y_{\lambda ai}}{\partial x} - w_i\frac{\partial\tau^y_{\lambda ai}}{\partial z} = \frac{\partial}{\partial z}k_z\frac{\partial\tau^y_{\lambda ai}}{\partial z} - \alpha_i\tau^y_{\lambda ai}, \qquad (4)$$

$$\tau^y_{\lambda ai} = \int_{-\infty}^{+\infty}\alpha^0_{\lambda ai}q_i\,dy. \qquad (5)$$

A particular feature of this equation is that its dimensionality is lowered by one, so that it coincides in its form with the equation for a linear source.[3] Therefore the solution of Eq. (1) for $q_i(x, y, z)$ expressed in terms of $\tau^y_{\lambda ai}(x, y)$ may be expressed by the relation

$$q_i(x, y, z) = \frac{\tau^y_{\lambda ai}(x, y)}{2\alpha^0_{\lambda ai}\sqrt{\pi k_0 x}}\exp\left(-\frac{y^2}{4k_0 x}\right). \qquad (6)$$

In the derivation of Eq. (6) it was assumed that $k_0 = k_y/u$ is constant, and u, ω, k_z, and α_i do not depend on the horizontal coordinate.

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Expressions for the surface concentration and its extremum were given by Berlyand,[3] who considered a light pollutant ($\omega_i = \alpha_i = 0$) and assumed power-law dependences for u and k_z: $u = u_1 z^n$, $k_z = k_1 z$.

The corresponding expressions for the surface optical depth then become

$$\tau^y_{\lambda ai}(x, 0) = \frac{Q_i \alpha_{\lambda ai}}{(1+n)k_1 x} \exp\left[-\frac{u_1 H^{1+n}}{(1+n)^2 k_1 x}\right]; \quad (7)$$

$$\left[\tau^y_{\lambda ai}\right]_{max} = \frac{Q_i \alpha^0_{\lambda ai}(1+n)}{e\, u_1 H^{1+n}}; \quad (8)$$

$$x_{max} = \frac{u_1 H^{1+n}}{(1+n)^2 k_1}. \quad (9)$$

Note that a simple expression for the source power follows from Eqs. (8) and (9), which is expressed in terms of the surface aerosol optical depth extremum and its coordinate

$$Q_i = \frac{e}{\alpha^0_{\lambda ai}}(1+n)k_1\left[\tau^y_{\lambda ai}\right]_{max} x_{max}. \quad (10)$$

For the heavy particles, whose sedimentation velocity is given by the Stokes relationship $\omega_i = 1.3 \cdot 10^{-2} \rho^T_{ai} r_i^2$ (ω_i, cm/s; ρ^T_{ai}, g/cm^3; r_i, μm), the corresponding expressions for the surface optical depth then take the form

$$\tau^y_{\lambda ai}(x, 0) = \frac{Q_i \alpha^0_{\lambda ai} H^{\omega_i(1+n)} u_1^{\omega_i}}{(1+n)^{1+2\omega_i}\Gamma(1+\omega_i)k_1^{1+\omega_i} x^{1+\omega_i}} \times$$

$$\times \exp\left[-\frac{u_1 H^{1+n}}{(1+n)^2 u_1 x}\right]; \quad (11)$$

$$\left[\tau^y_{\lambda ai}\right]_{max} = \frac{Q_i \alpha^0_{\lambda ai}(1+n)(1+\omega_i)^{1+\omega_i}}{e^{1+\omega_i} u_1 H^{1+n}\Gamma(1+\omega_i)}; \quad (12)$$

$$x_{max} = \frac{u_1 H^{1+n}}{(1+\omega_i)(1+n)^2 k_1},$$

$$\omega_i = w_i / k_1 (1+n). \quad (13)$$

Hence, the source power of a heavy aerosol pollutant is expressed in terms of the maximum optical depth as follows:

$$Q_i = \frac{1}{\alpha^0_{\lambda ai}} \frac{e^{1+\omega_i}(1+\omega_i)^{\omega_i}}{\Gamma(1+\omega_i)}(1+n)k_1\left[\tau^y_{\lambda ai}\right]_{max} x_{max}. \quad (14)$$

From a methodological point of view it is important to determine the possibility of measuring the maximum optical depth under actual- environmental conditions. The flatter is the maximum, i.e., the lower is the curvature, the more difficult it becomes to identify its actual position. Therefore we consider the curvature of the maximum surface optical depth for a light pollutant

$$\kappa_\tau = \frac{1}{R_\tau} = \frac{\partial^2}{\partial x^2}\left[\tau^y_{\lambda ai}\right]\Big|_{x_{max}} = -\left[\tau^y_{\lambda ai}\right]_{max}\frac{1}{x^2_{max}} \quad (15)$$

or

$$R_\tau = \frac{e\, u_1^3 H^{3(1+n)}}{(1+n)^2 Q_i \alpha^0_{\lambda ai} k_1^2} = -\frac{x^2_{max}}{\left[\tau^y_{\lambda ai}\right]_{max}}, \quad (16)$$

and also estimate the base length (i.e., the length of the Δx section of the plume) at which such a maximum may be identified with stable instrument accuracy $\Delta\tau_{instr}$. It follows from Fig. 1 that the optical depth increment along the base Δx is given by

$$\Delta\tau = R_\tau - \sqrt{R_\tau^2 - (\Delta x)^2} = \frac{(\Delta x)^2}{2R_\tau} + O(\Delta x^4). \quad (17)$$

Substituting R_τ from Eq. (16) into Eq. (17), we finally obtain

$$\Delta\tilde{\tau} = \frac{\Delta\tau}{\tau_{max}} = \frac{1}{2}\left(\frac{\Delta x}{x_{max}}\right)^2 \quad (18)$$

and, similarly, for a heavy pollutant

$$\Delta\tilde{\tau} = \frac{1}{2}\left(\frac{\Delta x}{x_{max}}\right)^2 (1+\omega_i). \quad (19)$$

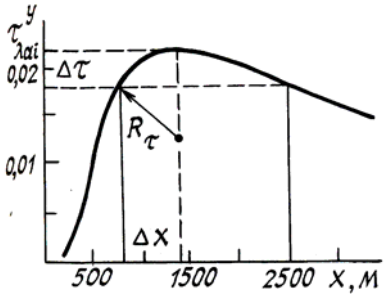

FIG. 1. Determination of the curvature at the maximum, of the surface optical depth. The reference values for calculating $\tau^y_{\lambda ai}(x, 0)$ *are:* $H = 50$ m, $u_1 = 4$ m^{1-n}/s, $n = 0.2$, $k_1 = 0.2$ m/s, $\rho = 2000$ kg/m^3, $r_i = 10$ μm, $Q_i = 10$ kg/s, and $\alpha^0_{\lambda ai} = 2$ m^2/kg.

We see that the position of the maximum can be identified if the following condition is satisfied

$$\Delta\tilde{\tau}_{instr} \le \Delta\tilde{\tau} \ . \qquad (20)$$

During lidar measurement of the horizontal optical depth, difficulties arise in choosing a normal cross section of the plume. It has been found from numerical simulations that the optical depth of the slant path *BB* (Fig. 2) is related to the optical depth of the path normal to the beam cross section *AA*, by the relation

$$\tau^{AA}_{\lambda ai} \simeq \tau^{BB}_{\lambda ai} \sin\Theta \ . \qquad (21)$$

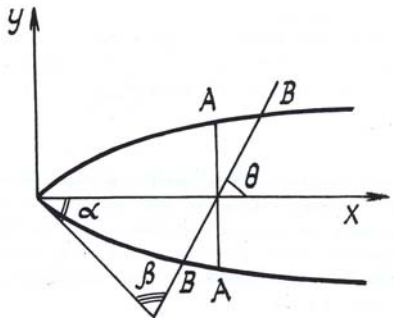

FIG. 2. The sounding paths: AA — the path perpendicular to the plume; BB — the slant path.

Relations (21) is satisfied by virtue of the following three factors: first, the main variation in the optical depth is concentrated close to the plume axis; second, the values of the optical depth at points of the slant path do not differ too much from those at the corresponding points of the normal cross-section path; third, deviations of these values in remote parts of the slanted beam path in the plume from those in the normal cross section are compensated, to a large extent, by such deviations in the near part of the plume.

Consider the simultaneous emission of two heavy fractions of a pollutant — a coarse one and a fine one — from the same source. We may estimate the relative contribution that the fine fraction (index "2") makes to the total optical depth at the extremum point of the coarse fraction (index "1") $\delta\tau_{12}$:

$$\delta\tau_{12} = \frac{\tau_2(x_{1m},\ 0)}{\tau_1(x_{1m},\ 0)} = \frac{Q_2\alpha^0_{\lambda ai}\Gamma(1+\omega_1)}{Q_1\alpha^0_{\lambda ai}\Gamma(1+\omega_2)}\,(1+\omega_1)^{\omega_2-\omega_1}. \qquad (22)$$

The contribution of the coarse fraction to the total optical depth at the point where the fine fraction reaches its maximum is given by

$$\delta\tau_{21} = \frac{\tau_1(x_{2m},\ 0)}{\tau_2(x_{2m},\ 0)} = \frac{Q_1\alpha^0_{\lambda ai}\Gamma(1+\omega_2)}{Q_2\alpha^0_{\lambda ai}\Gamma(1+\omega_1)}\,(1+\omega_2)^{\omega_1-\omega_2}. \qquad (23)$$

Under the conditions of equality of the optical emission intensities of the two fractions, $Q_2\alpha^0_{\lambda a2} = Q_1\alpha^0_{\lambda a1}$, we obtain the curves for $\Delta\tau_{ij}$ presented in Fig. 3 for the cases a) $i = 1$, $r_1 = 100$ μm (curve 1), and b) $i = 2$, $r_2 = 1$ μm (curve 2).As can be seen it; is practically impossible to separate the two extrema in the second case, where the particle size of the first fraction is 1 μm, and that of the second one remains below 40 μm, or if the particle size of the second fraction is 100 μm, while the first remains in excess of 80 μm. At large separations of the modes of the optical depths of the two fractions can be distinguished.

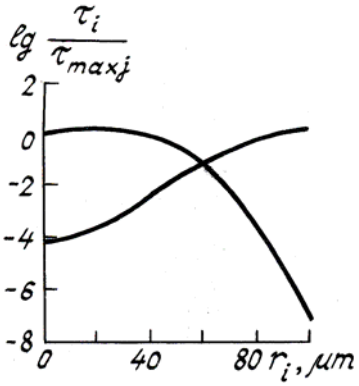

FIG. 3. The relative mutual influence of the optical depths of a two-fraction aerosol emission: 1) $\log(\tau_i/\tau_{max100})$*; 2)* $\log(\tau_1/\tau_{max1})$.

Relations (10) and (14) may be used to estimate the aerosol emission intensity from the surface optical depth measurements. It is enough then to identify the position and level of the extremum of the optical depth and to retrieve the coefficients which describe the state of the surface atmospheric layer (k_1, u, ω_1). If, on the contrary, the composition of the aerosol and it emission rate are known, such relations may b used to determine the atmospheric surface layer parameters.

The results presented here illustrate ne possibilities in modeling atmospheric pollution which are opened up by the equations describing the transfer of the conserved optical parameters.

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