VERTICAL MOTIONS OF SYNOPTIC SCALE

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Vertical motions observed in synoptic vortex (upward – in cyclone and downward – in anticyclone) have a primary influence on not only the formation and growth of clouds, but also spread of admixtures of both natural and anthropogenic origin, as well as levels of air pollution near the Earth's surface. The influence of such parameters as geostrophic and surface wind, surface roughness, thermodynamic stability of the ground layer of the atmosphere, and the horizontal size of vortex on the vertical velocity was estimated using equations of the similarity theory. The results are presented as tables, which allow calculation of the vertical velocity at any level of the boundary layer using the data of synoptic maps. A good agreement of results (in the boundary layer) on vertical velocity calculation within the framework of the similarity theory and using the equation approximating the vertical velocity within the entire troposphere suggests that the latter equation can be used in modeling of the atmospheric processes and phenomena.

In a number of studies,[1,2] it was shown that vertical motions of synoptic scale play a leading part in formation of not only stratus (*Ns–As–Cs*), but also stratocumulus (*Cu*, *Cb*) clouds.

In this case, to model the *Ns–As* clouds, it is sufficient to know that air executes an upward motion ($w > 0$). Certainly, the time of formation, the height of borders, and the profile of clouds depend significantly on the height distribution of vertical velocity (w), temperature (T), and the water vapor mass fraction (q) at the initial moment, as well as the turbulent exchange intensity, etc. Nevertheless, early or late, high or low, a cloud will necessarily be formed at $w > 0$.

Vertical motions of the synoptic scale also play the decisive part in the convective clouds (*Cu*, *Cb*) formation. However, in contrast to *Ns–As*, when modeling the convective clouds it is a prime necessity to take into account not only the sign, but also the height distribution of w. The vertical temperature lapse rate ($\gamma = -\partial T/\partial z$) varies with time in response to w change with increasing altitude.

Vertical motions in the atmospheric boundary layer (ABL) are most completely studied.[3,4] In one of the recent papers in this line,[5] the model of height distribution of w was constructed, which takes into account (within the framework of the similarity theory) sufficiently fine features of ABL structure.

Using the equations of continuity and steady motion, it was shown in Ref. 4 that for the vertical velocity averaged over some area σ

$$\bar{w} = \frac{1}{\sigma} \iint_{(\sigma)} w \, d\sigma$$

the following expression is valid

$$\bar{w}(z) = \frac{1}{2\omega_z \rho\sigma} \int_{(l)} (\tau_{0l} - \tau_l) \, dl. \quad (1)$$

Here l is the contour enclosing the area σ; τ_l and τ_{0l} are the l projections of the turbulent stress at a height z and on the Earth's surface ($z = 0$), respectively; $2\omega_z$ is Coriolis parameter; ρ is an air density.

At the top of the boundary layer (H) where τ_l is close to zero, Eq. (1) takes the form

$$\bar{w}(H) = \frac{1}{2\omega_z \rho_H \sigma} \int_{(l)} \tau_{0l} \, dl. \quad (2)$$

Expression for τ_l is obtained in Ref. 5

$$\tau_{0l}/(2\,\omega_z\,\rho_H) = c_g\, z_1\, G/2; \quad (3)$$

$$G = \sqrt{\frac{c_1}{c_g} D \,\mathrm{Ro}} \times \left[1 - B\frac{c_1}{c_g}(\cos\alpha_0 - \sin\alpha_0)\right], \quad (4)$$

where c_g is velocity of the geostrophic wind; c_1 is the absolute value of the wind velocity at the level z_1; z_0 is the surface roughness parameter; $\mathrm{Ro} = c_g/(\omega_z \cdot z_1)$ is an analog of the Rosbi parameter; α_0 is the deflection angle of the wind velocity vector near the Earth's surface from the tangent to isobar (direction of the geostrophic wind) determined by the expression

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$$\cos\alpha_0 = \frac{1 + B^2\,(c_1/c_g)^2 - \mathrm{Ro}\,N\,(c_1/c_g)^3}{2\,B\,(c_1/c_g)}\,; \quad (5)$$

B, D, and N are the dimensionless parameters depending on ratios z_0/z_1 and z_1/z; L is the scale of the Monin-Obukhov ground layer; values of these parameters are tabulated in Ref. 5.

On the assumption that the area σ, over which the average value $\bar{w}$ is determined, is the circle of radius r, and parameters entering into Eqs. (3) and (4) weakly vary along the l contour, Eq. (2) takes the following form:

$$\bar{w}(H) = c_g\,z_1 \cdot G/r. \quad (6)$$

At fixed thermal stability and roughness of the Earth's surface (i.e., L/z_1 and z_0/z_1), the angle α_0 and the parameter G grow as Ro and c_1/c_g increase. Thus, at $L/z_1 = 50$, $z_0/z_1 = 0.1$ and $c_1/c_g = 0.5$ we have the following α_0 and G values for different Ro:

10^{-4} Ro	2	3	4	5	6	8	10
α_0, degs	27	36	41	45	50	55	62
G	94	132	170	220	292	400	550

At $L/z_1 = 20$, $z_0/z_1 = 0.1$ and $\mathrm{Ro} = 2\cdot 10^4$, α_0 and G values for different c_1/c_g are the following:

c_1/c_g	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
α_0, degs	11	20	30	38	45	51	56	66	76
G	12	24	48	72	98	122	146	196	246

At fixed dynamic parameters ($\mathrm{Ro} = 3\cdot 10^4$ and $c_1/c_g = 0.4$) and surface roughness ($z_0/z_1 = 0.1$), the angle α_0 and G increase as L/z_1 decreases, i.e. as the ground layer stability increases:

L/z_1	50	20	10
α_0	31	54	88
G	90	120	126

It is easily seen that the influence of different parameters on α_0 and G is determined by their influence on the friction stress (τ_0) near the Earth's surface. Indeed, since

$$\sqrt{\tau_0/\rho_0} = u_* = \varkappa(c_1/c_g)\,c_g/\ln(\eta_1/\eta_0)$$

and

$$\frac{\eta_1}{\eta_0} = \frac{z_1}{z_0}\,(1 + \frac{z_1}{2L} + \frac{z_1^2}{6L^2} + \ldots),$$

then:

1) at fixed c_g, c_1/c_g, and z_1/L, increase in z_0/z_1 leads to growth of τ_0 and, as a consequence, to an increase of α_0 and G;

2) at fixed z_0/z_1, and z_1/L, decrease in c_g or c_1/c_g is followed by decrease in τ_0 along with decrease in α_0 and G;

3) at fixed c_g, c_1/c_g, and z_0/z_1, increase in L/z_1, i.e. approaching the neutral stratification, leads to growth of τ_0, α_0, and G. However, in this case, in addition to τ_0, the altitude of the ground layer (h) influences the deflection angle (α) too. It is obvious that the higher is the h level, the closer is the wind at this level to the geostrophic wind, i.e., the less is α.

Since within the ground layer the angle α varies only slightly with increasing altitude ($\alpha_h \approx \alpha_0$), and the altitude of the ground layer is accepted to be $|L|$, the growth of L under the effect of altitude increase leads to decrease in α_0. The above data testify that this second effect is more essential than the influence of τ_0: as L increases, the angle α_0 decreases.

Transformation from strongly stable ($L/z_1 < 10$) to strongly unstable ($L/z_1 > -10$) stratification is followed by growth of the turbulent exchange intensity, as well as strengthening of the interaction between the ground layer and the upper part of the boundary layer and, as a consequence, decrease in the deflection angle α_0 (Table I).

Let us point out that at L/z_1 values exceeding 30–50, when the thermal stratification is close to neutral, the dimensionless parameters entering Eqs. (3) and (4) can be written to sufficiently high accuracy in the form:

$$B = \ln\left(\beta\,\frac{L}{z_1}\,\frac{z_1}{z_0}\right) \Big/ \ln\left(\frac{z_1}{z_0}\right);$$

$$D = \varkappa^2\,\beta\,\frac{L}{z_1} \Big/ \ln\left(\frac{z_1}{z_0}\right); \quad (7)$$

$$N = \frac{1}{2}\,\frac{\varkappa^2}{\beta} \Big/ \ln^3(z_1/z_0),$$

$\beta = (e - 1)/e$ at $\gamma < \gamma_a$ and $\beta = 1 - e$ at $\gamma > \gamma_a$,

$e = 2.7128 \ldots$.

It follows from the data presented in Table I that all the four parameters: Ro, c_1/c_g, z_0/z_1, and L/z_1 essentially influence the deflection angle. In this case, α_0 varies widely from 5–10 to 80–90°.

The results of calculation of the parameter G derived in Ref. 4 are presented in Table II. Like the angle α_0, the parameter G varies widely depending on Ro, c_1/c_g, and z_0/z_1. An increase in each of these parameters leads to the growth of G. However, the parameter G depends on the L/z_1 ratio far more weakly than α_0 does.

Vertical velocity in the boundary layer most strongly depends on the geostrophic velocity (pressure gradient). At $\mathrm{Ro} = 4\cdot 10^4$, $z_0/z_1 = 0.1$, $L/z_1 = 50$ and $c_1/c_g = 0.5$, the parameter G equals 170 according to the data from Table II.

TABLE I. Deflection angle α_0 (degrees) *at strongly unstable* (L/z_1 = − 10, *numerator) and stable* (L/z_1 = + 10, *denominator) thermal stratification of the ground layer.*

10^{-4} Ro	c_1/c_g								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
					z_0/z_1 = 0.01				
3			−/8	−/21	−/29	−/35	−/41	15/49	24/57
4	−/11	−/20	11/32	20/40	26/48	30/54	34/60	42/71	48/81
5	10/20	17/29	26/42	32/51	38/60	43/68	48/76	56/−	63/−
6	16/23	23/33	32/48	40/61	46/71	52/81	57/−	67/−	78/−
8	17/23	25/36	36/54	45/68	53/82	60/−	66/−	78/−	90/−
10	14/22	25/38	39/60	50/77	58/−	67/−	74/−	89/−	−/−
					z_0/z_1 = 0.1				
2	−/3	−/25	−/43	24/57	34/69	42/79	49/90	62/−	73/−
3	14/32	27/47	43/69	55/88	65/−	75/17	84/−	−/−	−/−
4	28/41	41/60	59/−	75/−	89/−				
5	34/47	49/72	73/−						
6	38/53	57/84	86/−						
8	40/58	64/−							
10	43/65	71/−							

TABLE II.

Parameter G										
10^{-4} Ro	L/z_1	c_1/c_g								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
						z_0/z_1 = 0.01				
3	10	−	−	18	31	42	53	63	84	106
	− 10	−	−	−	−	−	−	−	80	105
4	10	8	17	33	49	66	82	99	133	165
	− 10	−	−	32	52	69	87	104	139	174
5	10	10	21	44	68	93	118	142	−	−
	− 10	12	25	49	73	98	123	148	198	249
6	10	11	25	57	91	126	161	−	−	−
	− 10	14	29	60	93	127	162	197	268	339
8	10	9	29	72	119	166	−	−	351	−
	− 10	11	29	70	114	160	207	255		445
10	10	6	32	91	152	−	−	−	−	−
	− 10	5	28	81	138	199	261	324	447	−
						z_0/z_1 = 0.1				
2	50	12	24	48	71	94	117	141	189	238
	10	9	23	47	70	94	116	138	−	−
3	50	2	17	52	91	132	174	218	308	401
	10	20	41	83	125	−	−	−	−	−
4	50	−	−	35	100	170	242	318	472	630
	10	28	62	−	−	−	−	−	−	−
5	50	−	−	−	106	219	337	457	601	942
	10	37	89	−	−	−	−	−	−	−
6	50	−	−	−	118	293	470	649	1002	−
	10	48	123	−	−	−	−	−	−	−
8	50	−	−	−	145	400	654	905	−	−
	10	63	−	−	−	−	−	−	−	−
10	50	−	−	−	196	550	895	−	−	−
	10	82	−	−	−	−	−	−	−	−

Setting $z_1 = 10$ m and $r = 500$ km, we obtain the following values of $\bar{w}(H)$ at different c_g:

c_g, m/s	5	10	15	20	25
$\bar{w}(H)$, cm/s	1.7	3.4	5.1	6.8	8.5

If the ratio $c_1/c_g = 0.8$ at the same Ro, z_0/z_1, L/z_1, z_1, and r values, then

c_g, m/s	5	10	15	20	25
$\bar{w}(H)$, cm/s	4.7	9.4	14.2	18.9	23.5

Let us note that at given Ro the velocity c_g at the given latitude cannot exceed the value equal to

$$(c_g)_{\max} = 7.29 \cdot 10^{-5} \sin\varphi \, z_1 \, \text{Ro}.$$

This maximum value grows as φ increases. On the other hand, the parameter Ro at observed c_g values can reach the greater values, the smaller is the latitude. Thus, at the latitude 10°, as c_g varies from 10 to 40 m/s, the parameter Ro increases from $7.9 \cdot 10^4$ to $3.2 \cdot 10^5$.

Even at $\text{Ro} = 10^5$, $L/z_1 = 50$; $z_0/z_1 = 0.1$; $c_1/c_g = 0.6$; $r = 500$ km, the vertical velocity $\bar{w}(H)$ increases from 8.9 cm/s at $c_g = 5$ m/s to 71.6 cm/s at $c_g = 40$ m/s.

In tropical cyclones, $w(H)$ reaches particularly large values. Since the velocity of the geostrophic wind in them amounts up to 30–80 m/s, whereas the latitude is, as a rule, less than 20°, the parameter Ro can take values exceeding $(3–5) \cdot 10^5$.

Thus, at the latitude 20° at $c_g \approx 25$ m/s, the parameter $\text{Ro} \approx 10^5$. Since, as follows from Table II, as Ro doubles, the parameter G increases 2–3 times, we obtain the following estimations for $\bar{w}(H)$ at the same $\varphi = 20°$:

at $z_0/z_1 \approx 0.01$, $c_1/c_g = 0.5$; $L/z_1 = -10$, $r = 200$ km

=) $c_g = 25$ m/s, $\text{Ro} = 10^5$, $G \approx 200$,
$\bar{w}(H) \approx 25$ cm/s;
b) $c_g = 75$ m/s, $\text{Ro} = 3 \cdot 10^5$, $G \approx 500$,
$\bar{w}(H) = 1.88$ m/s;

at $z_0/z_1 \approx 0.1$, $c_1/c_g = 0.5$; $L/z_1 = 50$ and $r = 200$ km

=) $c_g = 25$ m/s, $\text{Ro} = 10^5$, $G = 550$,
$\bar{w}(H) = 69$ cm/s;
b) $c_g = 75$ m/s, $\text{Ro} = 3 \cdot 10^5$, $G = 1375$,
$\bar{w}(H) = 5.16$ m/s.

At lesser latitude, values of $\bar{w}(H)$ may be even more considerable (at the latitude of 10°, for example, the above-presented values of $\bar{w}(H)$ are nearly doubled).

The obtained estimations of $\bar{w}(H)$ should be taken into account when modeling the tropical cyclones. It is known that, in an attempt to obtain the w values about $10^0–10^1$ m/s, researchers arbitrarily invoke some additional vertical velocity, which is absolutely inconsistent with the solution of the set of equations.

The vertical velocity $\bar{w}(H)$ at an arbitrary altitude z, being averaged over the same area $\sigma = \pi r^2$, is calculated in accordance with Ref. 1 using the equations

$$\bar{w}(z) = c_g \, z_1 \, (G - G_z)/r + \bar{w}_h; \tag{8}$$

$$G_z = \sqrt{\frac{c_1}{c_g} D \, \text{Ro}} \left\{ \left[1 - B \frac{c_1}{c_g} (\cos\alpha_0 - \sin\alpha_0) \right] \times \right.$$
$$\times \cos \frac{a}{z_1} (z - h) + \left[1 - B \frac{c_1}{c_g} (\cos\alpha_0 + \sin\alpha_0) \right] \times$$
$$\left. \times \sin \frac{a}{z_1} (z - h) \right\} \exp \left[- \frac{a}{z_1} (z - h) \right], \tag{9}$$

where $a = \sqrt{c_g/(c_1 D \text{Ro})}$; G is the value of parameter G_z at $z = h$; $h = |L|$.

The velocity w_h at the top of the ground layer in Eq. (8) is determined in the following way. Let us calculate G_z using Eq. (9), as well as the first term in Eq. (8) at $z = 2h$. Let it be equal to b_{2h}. Then, according to Eq. (8), $\bar{w}(2h) = b_{2h} + \bar{w}_h$. Since at a small altitudes the profile of w is close to linear, $\bar{w}_h = \bar{w}(2h)/2$. It follows from the two last relations that: $\bar{w}_h = b_{2h}$.

In the previous studies,[6,7] the following equation for $w(z)$

$$w(z) = 4 \, w_{\text{m}} \frac{z}{H_{\text{c}}} \left(1 - \frac{z}{H_{\text{c}}} \right), \tag{10}$$

has been widely used when modeling clouds and fogs. In Eq. (1), H_{c} is the level (most often the top of cyclone or anticyclone) where w becomes zero a second time (in addition to the Earth's surface); w_{m} is maximum (in height) value of w achieved at the level $z_{\text{m}} = H_{\text{c}}/2$.

Equation (10) was obtained by integration of the continuity equation on the assumption that the divergence of the horizontal wind velocity is a linear function of height.

Figure demonstrates the agreement between the results of $w(z)$ calculations using Eqs. (8)–(9) (solid curves) and those calculated from Eq. (10).

Example I was calculated by Eqs. (8)–(9) at the following values of parameters: $\text{Ro} = 4 \cdot 10^4$, $c_1/c_g = 0.26$, $z_0/z_1 = 0.1$, and $L/z_1 = -20$ (with $\alpha_0 = 34.2°$, $B = 2.74$, $D = 0.80$, and $N = 4.68 \cdot 10^{-4}$). Example II corresponds to $\text{Ro} = 4 \cdot 10^4$, $c_1/c_g = 0.39$, $z_0/z_1 = 0.1$, and $L/z_1 = 50$ (here $\alpha_0 = 35.4°$, $B = 2.93$, $D = 1.98$, $N = 1.87 \cdot 10^{-4}$).

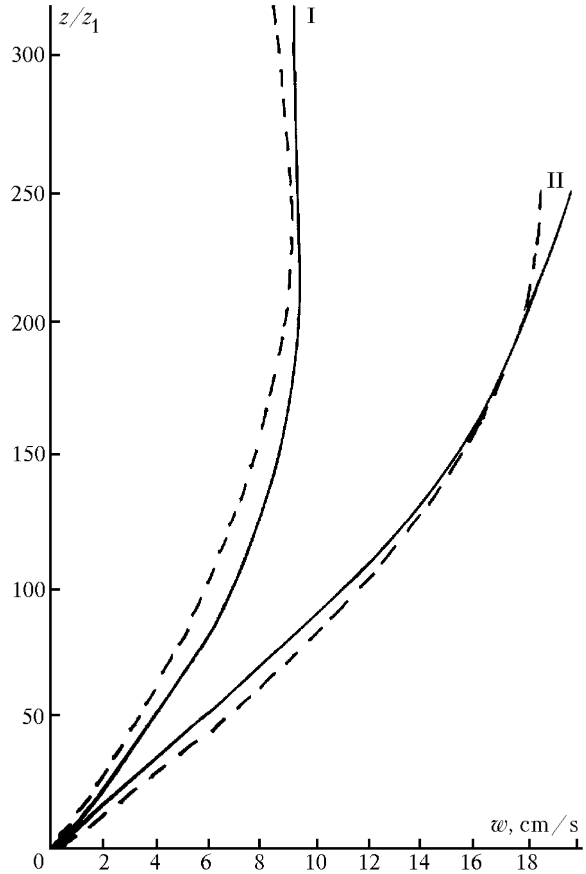

FIG. 1.

The calculation of w by Eq. (10) was performed on the assumption that the maximum value w_m coincides with the value of w at the top of the boundary layer ($w_m = w_n$), while the ratio $H_c/z_1 = 500$.

The vertical velocity calculated by Eqs. (8)–(9) keeps practically the constant value above the boundary layer; Eq. (10) provides for decrease of w in the middle and upper troposphere as the height grows.

In the boundary layer (lower troposphere), the results of the calculation using Eq. (10) agree quite satisfactory with the data coming from the theory accounting for sufficiently fine features of the boundary layer structure.

It follows from the above estimations of w_n that the vertical velocity varies widely under the effect of different parameters (c_g, z_0, L, c_1). Owing to this, the errors which may arise in w calculation using Eq. (10) because of neglect of some factors (nonstationary motion, for example) will be entirely absorbed by the errors appearing due to the above-mentioned parameters: they are frequently determined in actual practice with a considerable error (the pressure gradient and, especially, z_0 and L can be, by no means everywhere, retrieved from maps with needed accuracy).

It is not accidental, then, that the results of w calculations using different methods seem to be essentially different (even to the point of signs alternation[8]).

In view of these comments, we can conclude that Eq. (10) describes the height distribution of vertical velocity in vortex of synoptic scale with the required accuracy not only in the boundary layer, but throughout the troposphere as well.

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