Effect of coherence on parameters of a laser guide star

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Atmospheric inhomogeneities significantly affect optical waves and restrict the efficiency of present-day optoelectronic systems. It is well-known that one of the most radical methods to diminish this effect is application of adaptive systems. A key element in the optical scheme of an adaptive system is a reference source. This paper concerns some aspects of application of laser reference sources connected with the coherence of their radiation.

Atmospheric inhomogeneities significantly affect optical waves and restrict the efficiency of modern optoelectronic systems. It is well-known that one of the most radical methods to diminish this effect is application of adaptive systems. A reference source is a principal element of the optical scheme of any adaptive system.

We can find a mention on the reference source already in the first papers by V.P. Linnik[1] and J. Hardy.[2] It was assumed that some reference source might be presented by a natural source or by a specially made artificial source, as well as radiation backscattered from an object, and, finally, radiation backscattered (or re-emitted) by atmospheric inhomogeneities.[3–5] The most urgent task now is the development of reference sources employing an optical signal backscattered by atmospheric inhomogeneities. Astronomers figuratively call such sources as laser guide stars (LGS),[3] although in the Russian literature they were called laser beacons or artificial reference source.[4–7]

Specialists in laser sensing also deal with a signal reflected from atmospheric inhomogeneities.[9] Terms commonly used in laser sensing are “scattering volume” and “scattering cross section.” It should be noted that signals from LGS and signals of laser sensing of atmospheric inhomogeneities have the identical, in some sense, structure.[5–10] Thus, the signal describing a jitter of a scattering volume image at atmospheric sensing and the signal characterizing the global wave front tilt when measuring the LGS position actually have the same form.[8–10]

As is known, there are monostatic[3] and bistatic schemes in laser sensing, as well as monostatic and bistatic schemes in formation of laser guide stars.[8,10–13] Bistatic schemes assume no correlation between fluctuations of radiation propagated along forward and backward paths. The vector describing the jitter of the image of a scattering volume (or the vector of LGS angular position) in the objective’s focal plane has two noncorrelated components, namely,

$$\boldsymbol{\varphi}_{\rm bi} = \boldsymbol{\varphi}_{\rm l.b.} + \boldsymbol{\varphi}_F^{\rm s.s.}. \qquad (1)$$

Here $\boldsymbol{\varphi}_F^{\rm s.s.}$ is the vector characterizing the angular position of the image of a “secondary source” (laser guide star) observed in the aperture of the receiving objective, $\boldsymbol{\varphi}_{\rm l.b.}$ is the vector characterizing the angular position of the centroid of the laser beam forming the guide star (or scattering volume).

Using the results of Refs. 14–16, we can write the following equation for the variance in the angular jitter of the centroid of the laser beam emitted vertically upward from the ground:

$$<(\varphi_{\rm l.b.})^2> = 4\pi^2 x \int_0^1 d\xi (1-\xi)^2 \times$$
$$\times \int_0^\infty d\kappa \kappa^3 \Phi_n(\kappa, x\xi) \exp(-\kappa^2 a^2 q^2/2), \qquad (2)$$

where

$$q(\xi) = [\xi^2\Omega^{-2} + (1-\xi x/f)^2]^{1/2},\ \ \Omega = ka^2/x, \qquad (3)$$

where a is the initial size of the Gaussian laser beam, f is the curvature radius of the Gaussian beam phase front. Calculating by Eq. (2), we use the turbulence spectrum of the following form[4]

$$\Phi_n(\kappa,\ x\xi) = 0.033 C_n^2(x\xi)\kappa^{-11/3}\{1-\exp[-\kappa^2/\kappa_0^2]\}, \qquad (4)$$

accounting for the deviation from the Kolmogorov spectrum in the region of large scales of inhomogeneities of the atmospheric refractive index, κ_0^{-1} is the outer scale of turbulence.

Calculate the internal integral in Eq. (2) using the spectrum (4) and obtain

$$<(\varphi_{\rm l.b.})^2> = 4\pi^2 x 0.033 \frac{\Gamma(1/6)}{2^{5/6}} a^{-1/3} \times$$
$$\times \int_0^1 d\xi (1-\xi)^2 [q^{-1/3} - (q^2 + \frac{2}{a^2\kappa_0^{-2}})^{-1/6}] C_n^2(x\xi). \qquad (5)$$

For the collimated beam, that is, for $f = \infty$, we have

$$q(\xi) = (1 + \xi^2\Omega^{-2})^{1/2},$$

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if the beam is focused at a point, that is, $x/f = 1$, then

$$q(\xi) = [(1-\xi)^2 + \xi^2\Omega^{-2}]^{1/2};$$

if the beam is rather wide, that is, $\Omega = ka^2/x \gg 1$, then we have

$$q(\xi) = (1 - \xi).$$

As a result, for the focused beam we have

$$< (\varphi_{\mathrm{l.b.}})^2 >= 4\pi^2 x 0.033 \frac{\Gamma(1/6)}{2^{5/6}} a^{-1/3} \times$$

$$\times \int_0^1 d\xi (1-\xi)^2 \{(1-\xi)^{-1/3} - [(1-\xi)^2 + \frac{2}{a^2\kappa_0^2}]^{-1/6}\} C_n^2(x\xi). \tag{6}$$

If the outer scale of turbulence is thought to be large enough, i.e., $\kappa_0^{-1} \gg a$ then

$$< (\varphi_{\mathrm{l.b}})^2 >= 4\pi^2 x 0.033 \frac{\Gamma(1/6)}{2^{5/6}} a^{-1/3} \int_0^1 d\xi (1-\xi)^{5/3} C_n^2(x\xi). \tag{7}$$

However, for the collimated beam under the same conditions we have

$$< (\varphi_{\mathrm{l.b.}})^2 >= 4\pi^2 x 0.033 \frac{\Gamma(1/6)}{2^{5/6}} a^{-1/3} \int_0^1 d\xi (1-\xi)^2 C_n^2(x\xi). \tag{8}$$

The next step is the calculation of the variance in the jitter of a secondary source image, i.e., the scattering volume illuminated from the ground (or LGS) in the focal plane of the objective. Since light scattering by atmospheric inhomogeneities (molecular scattering, aerosol scattering, and stimulated emission at free atoms) is the process of scatter by independent scatterers, the resulting scattered wave field is fully incoherent.

The illuminated zone size within the scattering layer is calculated based on conclusions of the theory of light propagation in a turbulent medium. The distribution of the mean intensity of the Gaussian beam having passed through the layer of a turbulent medium was calculated in Refs. 14–16:

$$< I(\mathbf{R}, \xi) >= \frac{a^2}{a_{\mathrm{eff}}^2(\xi)} \exp(-R^2/a_{\mathrm{eff}}^2),$$

where

$$a_{\mathrm{eff}}^2(\xi) = a^2\{(1-\xi/f)^2 + \Omega^{-2} + \Omega^{-2}[1/2D_S(2a)]^{6/5}\}$$

is the effective size of the beam in the scattering medium, $D_S(2a)$ is the structural function for a spherical wave phase.

Then let us use the conclusions of the theory of coherence.[15–17] The van Cittert–Zernike theorem deals with propagation of the mutual coherence function of a field

$$\gamma(x; \mathbf{r}_1, \mathbf{r}_2) = \frac{< U^*(\boldsymbol{\rho}_1, x) U(\boldsymbol{\rho}_2, x) >}{\sqrt{I(\boldsymbol{\rho}_1, x) I(\boldsymbol{\rho}_2, x)}}$$

and quantitatively expresses the effect of incoherent radiation diffraction at light propagation from a LGS to the ground. The modulus of the complex degree of the coherence for some initially incoherent source after the light passage through a homogeneous layer is described by the following equation:

$$\gamma(x; \mathbf{r}_1, \mathbf{r}_2) = \frac{\left| \iint d^2 s\, I(\mathbf{s}) \exp[-ik\mathbf{s}(\mathbf{r}_1 - \mathbf{r}_2)/x] \right|}{\int d^2 s\, I(\mathbf{s})}. \tag{9}$$

In other words, the van Cittert–Zernike theorem states that modulus of the complex degree of spatial coherence of the fully incoherent source with small angular size is equal to modulus of the normalized Fourier transform $I(\mathbf{s})$ of the field intensity distribution on the source. Thus, for a circular incoherent homogeneously illuminated source of the size d in the initial plane, the modulus of the complex degree of coherence at the distance x is

$$|\gamma(x; \mathbf{r}_1, \mathbf{r}_2)| = 2J_1(k\alpha r/2)/(k\alpha r/2), \tag{10}$$

where $\alpha = d/x$ is the angular size of the source as seen from the distance x; $r = |\mathbf{r}_1 - \mathbf{r}_2|$. As a result, the spatial coherence radius of the radiation for this case is $\rho_c \approx 1.22\lambda x/d$. Naturally, these estimates are obtained for the case of radiation propagation in a homogeneous medium.

Consider the jitter of the secondary source image, i.e., a laser guide star or a scattering volume. The image position in the objective's focal plane is characterized by the vector of its centroid position[14]:

$$\boldsymbol{\rho}_F = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d^2\rho\, \boldsymbol{\rho} I_F(\boldsymbol{\rho}) \Bigg/ \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d^2\rho\, I_F(\boldsymbol{\rho}) \right], \tag{11}$$

where $I_F(\boldsymbol{\rho})$ is the intensity distribution in the objective's focal plane ($x = F$). Find this distribution in the case that a wave with the amplitude $U(\boldsymbol{\rho}) = U_0 \exp(\chi + iS)$ after passage through the turbulent layer is incident on the objective; here χ are fluctuations of the log amplitude, S are fluctuations of the optical wave phase. If the medium behind the objective is thought homogeneous, then the field here can be determined from the homogeneous wave equation

$$\Delta\varphi + k^2\varphi = 0, \tag{12}$$

supplemented with the boundary condition at $x = 0$

$$\varphi_0(\boldsymbol{\rho}) = U_0 \exp\{\chi(\boldsymbol{\rho}) + iS(\boldsymbol{\rho}) - ik\rho^2/2F\}, \tag{13}$$

that accounts for the phase shift introduced by the objective (in the approximation of a thin lens) and the condition of emission at infinity. As it is known, problem (12) has the following solution[16]:

$$\varphi(x, \boldsymbol{\rho}) = \iint_\Sigma \varphi_0(\boldsymbol{\rho}_0) G_0(x, \boldsymbol{\rho} - \boldsymbol{\rho}_0) d^2\rho_0, \tag{14}$$

where

$$G_0(x,\boldsymbol{\rho}-\boldsymbol{\rho}_0) = -\frac{1}{2\pi}\frac{\partial}{\partial x}\left\{\frac{\exp[ik\sqrt{x^2+(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2}]}{\sqrt{x^2+(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2}}\right\}, \quad (15)$$

and integration in an infinite domain in Eq. (14) is replaced by integration over the objective surface Σ, where the field is nonzero. In the approximation of Fresnel diffraction for $\varphi(x, \boldsymbol{\rho})$ at the point $(F, \boldsymbol{\rho})$ we obtain the equation

$$\varphi(F,\boldsymbol{\rho}) = \frac{k\exp(ikF)}{2\pi iF}\iint\limits_{\Sigma}\varphi_0(\boldsymbol{\rho}_0)\exp\{ik(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2/2F\}d^2\rho_0. \quad (16)$$

Hence, for $I(...) = \varphi\varphi^*$ we have

$$I_F(\boldsymbol{\rho}) = \frac{k^2}{4\pi^2F^2}\iint\limits_{\Sigma}\iint\limits_{\Sigma}\varphi(\boldsymbol{\rho}_{01})\varphi^*(\boldsymbol{\rho}_{02})\times$$
$$\times\exp\{ik(\boldsymbol{\rho}-\boldsymbol{\rho}_{01})^2/2F - ik(\boldsymbol{\rho}-\boldsymbol{\rho}_{02})^2/2F\}d^2\rho_{01}d^2\rho_{02},$$

where $\varphi(\boldsymbol{\rho}_0)$ is the field of the wave incident on the objective. Substitute the boundary condition (13) into Eq. (17):

$$I_F(\boldsymbol{\rho}) = \frac{k^2}{4\pi^2F^2}\times$$
$$\times\iint\limits_{\Sigma}d^2\rho_{01}\iint\limits_{\Sigma}d^2\rho_{02}U_0(\boldsymbol{\rho}_{01})U_0^*(\boldsymbol{\rho}_{02})\exp\{-ik\boldsymbol{\rho}(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02})/F\}. \quad (18)$$

Now determine a random position of the intensity distribution centroid in the focal plane by substituting Eq. (18) into Eq. (11). The integral in the denominator can be easily calculated using a well-known relationship of the theory of generalized functions

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\exp\{-ik\boldsymbol{\rho}(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02})/F\}d^2\rho =$$
$$= 4\pi^2\delta\left[\frac{k}{F}(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02})\right] = \frac{4\pi^2F^2}{k^2}\delta(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02}), \quad (19)$$

which gives the total light flux through the objective

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}d^2\rho I_F(\boldsymbol{\rho}) = \iint\limits_{\Sigma}d^2\rho I_F(\boldsymbol{\rho}) = U_0^2\iint\limits_{\Sigma}d^2\rho\exp[2\chi(\boldsymbol{\rho})]. \quad (20)$$

Quite similarly, but using the derivative of the δ-function, we can calculate the integral in the numerator (11). As a result, for the random shift of the centroid position $\boldsymbol{\rho}_F$ we obtain[14,15]

$$\boldsymbol{\rho}_F = \frac{F}{ik}\frac{\iint\limits_{\Sigma}d^2\rho U(\boldsymbol{\rho})\nabla U^*(\boldsymbol{\rho})}{\iint\limits_{\Sigma}d^2\rho I(\boldsymbol{\rho})}. \quad (21)$$

The shift $\boldsymbol{\rho}_F$ is a real parameter, therefore, calculating the half-sum of Eq. (21) and the complex conjugate, we can write $\boldsymbol{\rho}_F$ in the more convenient form

$$\boldsymbol{\rho}_F = \frac{F}{k}\frac{\iint\limits_{\Sigma}d^2\rho\,\mathrm{Im}[U(\boldsymbol{\rho})\nabla U^*(\boldsymbol{\rho})]}{\iint\limits_{\Sigma}d^2\rho I(\boldsymbol{\rho})}.$$

Having substituted $U(\boldsymbol{\rho}) = U_0\exp[\chi(\boldsymbol{\rho}) + iS(\boldsymbol{\rho})]$, we have

$$\boldsymbol{\rho}_F = -\frac{F}{k}\frac{\iint\limits_{\Sigma}d^2\rho\exp[2\chi(\boldsymbol{\rho})]\nabla S(\boldsymbol{\rho})}{\iint\limits_{\Sigma}d^2\rho\exp[2\chi(\boldsymbol{\rho})]}. \quad (22)$$

It is clearly seen from the last equation that phase fluctuations play the main role in the phenomenon of image jitter in the focal plane; if $S = 0$, then $\boldsymbol{\rho}_F = 0$, while amplitude fluctuations play the role of a second-order addition. Therefore, in the first approximation they can be neglected, if we take $\chi = 0$. Then

$$\boldsymbol{\rho}_F = -\frac{F}{k\Sigma}\iint\limits_{\Sigma}d^2\rho\nabla S(\boldsymbol{\rho}). \quad (23)$$

The mean square of the linear centroid shift is

$$\sigma_F^2 = \langle(\boldsymbol{\rho}_F)^2\rangle = \frac{F^2}{k^2\Sigma^2}\times$$
$$\times\iint\limits_{\Sigma}d^2\rho_1\iint\limits_{\Sigma}d^2\rho_2\nabla_{\rho_{01}}\nabla_{\rho_{02}}B_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02}), \quad (24)$$

where $B_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02}) = \langle S(\boldsymbol{\rho}_{01})S(\boldsymbol{\rho}_{02})\rangle$ is the correlation function of phase fluctuations, which is related to the phase structure function as

$$D_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02}) = 2[1 - B_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02})];$$
$$B_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02}) = [1 - 1/2D_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02})]. \quad (25)$$

The problem of coherence disturbance turns out to be closely related to phase fluctuations. For the complex degree of coherence

$$\gamma(\mathbf{R},\boldsymbol{\rho}) = \frac{\Gamma_2(\mathbf{R},\boldsymbol{\rho})}{[\Gamma_2(\mathbf{R}+\boldsymbol{\rho}/2,0)\Gamma_2(\mathbf{R}-\boldsymbol{\rho}/2,0)]^{1/2}}, \quad (26)$$

where

$$\Gamma_2(\mathbf{R},\boldsymbol{\rho}) = \langle U(\mathbf{R},\boldsymbol{\rho})U^*(\mathbf{R},\boldsymbol{\rho})\rangle, \quad \mathbf{R} = (\rho_1+\rho_2)/2,$$
$$\boldsymbol{\rho} = (\rho_1-\rho_2),$$

the following equation was derived[14]

$$2ik\frac{\partial\Gamma_2(\mathbf{R},\boldsymbol{\rho})}{\partial x} + 2\nabla_R\nabla_\rho\Gamma_2(\mathbf{R},\boldsymbol{\rho}) + \frac{i\pi k^3}{2}H(x,\boldsymbol{\rho})\Gamma_2(\mathbf{R},\boldsymbol{\rho}) = 0 \quad (27)$$

with the boundary conditions for the deterministic initial field

$$\Gamma_2(\mathbf{R},\boldsymbol{\rho}) = U_0(\mathbf{R}+\boldsymbol{\rho}/2)U_0^*(\mathbf{R}-\boldsymbol{\rho}/2).$$

The function $H(x, \boldsymbol{\rho})$ in Eq. (27) characterizes statistical properties of permittivity fluctuations

$$H(x,\boldsymbol{\rho}) = 8\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\Phi_n(x,\boldsymbol{\kappa})[1-\cos\boldsymbol{\kappa\rho}]\mathrm{d}^2\kappa. \quad (28)$$

If the initial field U_0 fluctuates, then we should substitute the function

$$\Gamma_2^0(\mathbf{R},\boldsymbol{\rho}) = \langle\langle U_0(\mathbf{R}+\boldsymbol{\rho}/2)U_0^*(\mathbf{R}-\boldsymbol{\rho}/2)\rangle\rangle.$$

as a boundary condition. Double angle brackets here denote averaging over the ensemble of source realizations. As an example, we present here a boundary condition in the case of a partly coherent light beam, whose field has the form

$$U_0(\boldsymbol{\rho}) = A(\boldsymbol{\rho})\exp[i\varphi(\boldsymbol{\rho})],$$

where $\varphi(\boldsymbol{\rho})$ is a random phase with zero mean that is distributed, for instance, by the Gauss law.

Let the initial beam be Gaussian

$$A(\boldsymbol{\rho}) = U_0\exp\{-\rho^2/2a^2 - ik\rho^2/2f\}.$$

In this case

$$\Gamma_2^0(\mathbf{R},\boldsymbol{\rho}) = |U_0|^2\times$$
$$\times\exp\{-R^2/a^2 - \rho^2/4a^2 - ik\boldsymbol{\rho}\mathbf{R}/f - E(\boldsymbol{\rho})/2\}, \quad (29)$$

where

$$E(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2) = \langle\langle[\varphi(\boldsymbol{\rho}_1)-\varphi(\boldsymbol{\rho}_2)]^2\rangle\rangle.$$

To facilitate the calculations, denote $E(\boldsymbol{\rho}) = \rho^2/2a_k^2$. Then

$$\Gamma_2^0(\mathbf{R},\boldsymbol{\rho}) = |U_0|^2\times$$
$$\times\exp\{-R^2/a^2 - \rho^2/4a^2 - ik\boldsymbol{\rho}\mathbf{R}/f - \rho^2/2a_k^2\}. \quad (30)$$

Here a_k is the initial spatial coherence radius of the radiation source.

In a random medium[14–16]

$$\Gamma_2(x,\mathbf{R},\boldsymbol{\rho}) = \frac{k^2}{4\pi^2x^2}\iint\mathrm{d}^2R'\iint\mathrm{d}^2\rho'\Gamma_2^0(\mathbf{R}-\mathbf{R}',\boldsymbol{\rho}-\boldsymbol{\rho}')\times$$
$$\times\exp\{ik\mathbf{R}'\boldsymbol{\rho}/x - \frac{\pi k^2}{4}\int\limits_0^x H[x', \boldsymbol{\rho}-\boldsymbol{\rho}'(1-x'/x)]\mathrm{d}x'\}. \quad (31)$$

Consider the limiting case in Eq. (30), namely, the transition to a fully incoherent initial (thermal) source. At $a_k \to 0$, we obtain

$$\Gamma_2^0(\mathbf{R},\boldsymbol{\rho}) = 4\pi|U_0|^2a_k^2\exp\{-R^2/a^2\}\delta(\boldsymbol{\rho}).$$

Note that this equation is a particular case of the general equation for the coherence function of a thermal source. In the general case, if $a_k \to 0$, then $\Gamma_2^0(\mathbf{R},\boldsymbol{\rho})$ can be approximated by the following expression:

$$\Gamma_2^0(\mathbf{R},\boldsymbol{\rho}) = b^2I(\mathbf{R})\delta(\boldsymbol{\rho}). \quad (32)$$

In this case, it turns out that $b = \lambda/\sqrt{2\pi}$, i.e., the coherence radius of a thermal source is comparable with the wavelength. Therefore, for a fully coherent source

$$\Gamma_2^0(\mathbf{R},\boldsymbol{\rho}) = \frac{\lambda^2}{2\pi}I(\mathbf{R})\delta(\boldsymbol{\rho}).$$

At the distance x in the turbulent medium

$$\Gamma_2(x, \mathbf{R},\boldsymbol{\rho}) = \frac{U_0^2k^2a^2a_k^2}{x^2}\times$$
$$\times\exp\{ik\mathbf{R}\boldsymbol{\rho}/x - \frac{k^2a^2\rho^2}{4x^2} - \frac{\pi k^2}{4}\int\limits_0^x H[x', \boldsymbol{\rho}x'/x]\mathrm{d}x'\}. \quad (33)$$

For modulus of the complex degree of coherence

$$|\gamma(x,\mathbf{R},\boldsymbol{\rho})| = \exp\{-\frac{k^2a^2\rho^2}{4x^2} - \frac{\pi k^2}{4}\int\limits_0^x H[x',\boldsymbol{\rho}x'/x]\mathrm{d}x'\}. \quad (34)$$

It is seen herefrom that there are two opposite tendencies in variation of the spatial coherence radius of the initially incoherent radiation. On the one hand, it increases proportionally to $d_0 = 2x/ka$ (due to the decrease of the visible angular size $\gamma_s = a/x$ of the source) and, on the other hand, decreases due to the loss of coherence in the turbulent medium.

We can estimate the modulus of the complex degree of coherence through the phase structure function as

$$|\gamma(x, \mathbf{R}, \boldsymbol{\rho})| = \exp\{-\frac{1}{2}D_S(x,\boldsymbol{\rho})\}. \quad (35)$$

Comparison of Eqs. (34) and (35) shows that

$$D_S(x, \boldsymbol{\rho}) = \frac{k^2a^2\rho^2}{2x^2} + \frac{\pi k^2}{2}\int\limits_0^x H(x',\boldsymbol{\rho}x'/x)\mathrm{d}x'. \quad (36)$$

As a result of transformations in Eq. (36) with the spectrum (4), it follows that under the condition $\kappa_0^{-1} \gg \rho$,

$$D_S(x, \boldsymbol{\rho}) = \frac{k^2a^2\rho^2}{2x^2} + 2.91k^2\rho^{5/3}\int\limits_0^x \mathrm{d}x'C_n^2(x')(x'/x)^{5/3}. \quad (37)$$

Now let us return to the equation for the variance in the jitter of the secondary source image – LGS[18–21] or scattering volume – based on Eqs. (24) and (25)

$$\langle(\varphi_F^{s,s})^2\rangle = \sigma_F^2/F^2 = \langle(\boldsymbol{\rho}_F)^2\rangle/F^2 =$$
$$= -\frac{1}{2k^2\Sigma^2}\iint\limits_\Sigma\mathrm{d}^2\rho_1\iint\limits_\Sigma\mathrm{d}^2\rho_2\nabla_{\rho_{01}}\nabla_{\rho_{02}}D_S(\boldsymbol{\rho}_{01}-\boldsymbol{\rho}_{02}). \quad (38)$$

After substitution of the equation for the structure function in the form (37) into Eq. (38), we have two terms. The first term in the integrand in Eq. (38) is

$$\nabla_{\rho_{01}}\nabla_{\rho_{02}}\rho^2 = \left(\frac{\partial}{\partial y_{01}}\frac{\partial}{\partial y_{02}} + \frac{\partial}{\partial z_{01}}\frac{\partial}{\partial z_{02}}\right)\rho^2 = - 4,$$

while the second one is

$$\nabla_{\rho_{01}}\nabla_{\rho_{02}}\rho^{5/3} = \left(\frac{\partial}{\partial y_{01}}\frac{\partial}{\partial y_{02}} + \frac{\partial}{\partial z_{01}}\frac{\partial}{\partial z_{02}}\right)\times$$

$$\times[(y_{01} - y_{02})^2 + (z_{01} - z_{02})^2]^{5/6} = -\frac{25}{9}\rho^{-1/3}.$$

As a result, in the case that the size of the secondary source is equal to a, we obtain the following equation for the variance of its image jitter:

$$<(\rho_F)^2>/F^2 = \frac{a^2}{x^2} + 4.85D^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3}. \tag{39}$$

For a wide ($\Omega = ka^2/x >> 1$) collimated beam, the variance of the centroid jitter can be written as

$$<(\varphi_{\text{l.b.}})^2> = <(\rho_{\text{l.b.}})^2>/x^2 =$$

$$= 4\pi^2 0.033\frac{\Gamma(1/6)}{2^{5/6}}a^{-1/3}\int_0^x \mathrm{d}x'(1 - x'/x)^2 C_n^2(x'), \tag{40}$$

$$4\pi^2 0.033\frac{\Gamma(1/6)}{2^{5/6}} = 4.04.$$

Sum up the calculated results and obtain that for the wide beam the variance of the image angular jitter (1) is

$$<\varphi^2> = \frac{a^2}{x^2} + 4.85D^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3} +$$

$$+4.04a^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(1 - x'/x)^2. \tag{41}$$

In this case the secondary source size is equal to a, and for the focused beam the secondary source size is a/Ω, as a result,[18–21]

$$<\varphi^2> = \frac{a^2}{\Omega^2 x^2} + 4.85D^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3} +$$

$$+4.04a^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(1 - x'/x)^{5/3}. \tag{42}$$

Here D is the diameter of the receiving objective, a is the size of the Gaussian laser beam forming the scattering volume (LGS). Naturally, for a wide beam the first term in Eq. (41) can be neglected. Besides, it should be noted that the first terms in Eqs. (41) and (42) should be small enough by definition.

We return once more to Eq. (36) for the phase structural function

$$D_S(x, \mathbf{R}, \rho) = \frac{k^2 a^2 \rho^2}{2x^2} + 2.91k^2\rho^{5/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3}.$$

If we take here the square approximation in the second term, then for the coherence radius of the initial incoherent source in the turbulent medium we obtain[16] the following equation:

$$\rho_{\text{coh}} = \frac{d_0(x)}{(1 + d_0^2(x)/\rho_t^2)}, \tag{43}$$

where $d_0(x) = 2x/(ka)$; ρ_t is the coherence radius for the spherical wave in the turbulent medium.[15, 16]

The use of laser guide stars for correction of images of extraterrestrial objects faces[3,5–8,10–13] some principal problems. Thus, when applying laser guide stars, it is impossible to provide for efficient correction for the total wave front tilt (TWFT). Correction of TWFT fluctuations for a natural star (NS) using only the LGS signal is known[3,8,10–13] to be inefficient. The traditional monostatic scheme using only the aperture of the very telescope (main) remains inefficient even with the use of the procedure of LGS signal optimization.[11]

In this connection, some papers declare the need of the simultaneous use of both LGS and natural stars for correction of the total wave front tilt. And since the angle of spatial correlation for TWFT fluctuations far exceeds the isoplanatism angle for higher aberrations of phase fluctuations of the optical wave having passed through the turbulent atmospheric layer, a sufficiently remote star can be used for TWFT correction. Another disadvantage of the LGS application to image correction in the ground-based telescopes is the cone effect or focal nonisoplanatism. Some authors propose to apply more than one guide stars to overcome this difficulty.[22]

It is shown in this paper that high coherence of laser radiation in the guide star could be achieved only if the visible star image is small enough. Therefore, almost all laser guide stars were formed using focused laser beams.

Recently, the use of wide collimated beams for formation of laser guide stars was reported.[23] It was assumed that the resulting guide star has a plane wave front, thus providing for elimination of the focal nonisoplanatism. However, it was ignored that the secondary source – guide star – has a significantly low spatial coherence (43) because of the lack of coherence in the process of light scattering at atmospheric inhomogeneities (molecular scattering, aerosol scattering, and re-emission of radiation at free atoms in the upper atmosphere), and the coherence radius of the scattered radiation ρ_{coh} turns out to satisfy the following relationship:

$$\frac{1}{\rho_{\text{coh}}^2} = \frac{k^2 a^2}{4x^2} + \frac{1}{\rho_t^2}.$$

Therefore, the coherence of the received radiation is always determined by two factors – the size of the "visible" domain of LGS from the measured telescope focus and the *spherical* wave coherence, because of the practically incoherent initial emission from LGS.

The coherence of the secondary radiation is always lower in a wide collimated beam than in a focused beam. For example, for a telescope with the

8-m aperture, if the star is formed by the collimated beam at the altitude of 20 km, the coherence radius of radiation from this star is about some fractions of millimeter (for the wavelength of 0.5 μm). It is clear that phase measuring at the aperture of several meters with such a small coherence radius is rather difficult.

However, if we take into account that the angular resolution of the atmosphere–telescope system (without adaptive correction) is expressed through the ratio λ/r_0, where r_0 is the coherence radius of radiation for the plane wave having passed through the entire atmosphere, within the telescope field of view it is possible to *separately* observe parts with the angular size equal to λ/r_0 of the laser-illuminated surface of the incoherently luminous LGS. As a result, the first term characterizing the LGS radiation coherence (calculated for vacuum) turns out to be equal to

$$\rho_{\rm coh} = \lambda/\pi\theta, \qquad (44)$$

where θ is the angular resolution of the telescope in the atmosphere, that is, $\theta = \lambda/r_0$. So, we have

$$\rho_{\rm coh} = r_0/\pi.$$

In the case of a wide focused beam (when the LGS spot cannot be "resolved" by the telescope) we can obtain the radiation from the secondary source with the coherence radius of about the size of the aperture focusing the laser radiation (naturally, this estimate is for a homogeneous medium). Under conditions of the turbulent atmosphere, the coherence radius of the secondary source can be calculated by Eq. (43). Nonetheless, in some cases, it should be believed that we deal with incoherent guide stars. Incoherent LGS can also be used efficiently, for example, for real-time measurements of the atmospheric optical transfer function on the path from the objective to the scattering volume (or LGS). This function, in its turn, can be used in the inverse convolution algorithm for post-detector image correction.

Certainly, this approach allows us to obtain more efficient correction as compared to "blind" inverse convolution, which assumes calculation of the atmospheric transfer function based on some atmospheric model. Besides, the guide star can be formed in almost any needed direction, for example, when forming the image of an extraterrestrial object in the telescope. One of restrictions on efficient application of such a star is the problem of focal nonisoplanatism connected with the fact that the LGS is always located at some finite distance in the atmosphere, while the object is always far beyond the atmosphere. Therefore, the object and the LGS are always seen in different planes, because they have wave fronts with different curvature. This, in its turn, causes different fluctuations for the waves coming to the objective from the object and from the reference source.

As is known, any adaptive system has a finite frequency band, which causes a lag between received and control signals. Therefore, there exist some limitations on the quality of correction of a moving object due to the time lag. At the same time, forming the LGS in a given direction partly compensates for the lag arising in any adaptive system and connected both with the evolution of random inhomogeneities in the channel and the fact that the object under study rapidly changes its position. Then the LGS will be formed in the position "predicting" the future position of the object.

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References

1. V.P. Linnik, Opt. Spektrosk. **25**, No. 4, 401–402 (1957).
2. J.W. Hardy, Proc. IEEE **66**, No. 6, 651–697 (1978).
3. R. Fugate, Opt. Photonics News **4**, No. 6, 14–19 (1993).
4. V.P. Lukin, *Atmospheric Adaptive Optics* (SPIE Optical Engineering Press, Vol. **PM23**, 1996), 275 pp.
5. V.P. Lukin and V.F. Matyukhin, Soviet J. Quant. Electron. **13**. No. 12, 1604–1610 (1983).
6. V.P. Lukin, Proc. ICO-16, "Active and Adaptive Optics," 521–524 (1993).
7. V.P. Lukin and B.V. Fortes, OSA Techn. Digest. **23**, 192–194 (1995).
8. M.S. Belen'kii, Proc. SPIE **2956**, 206–217 (1996).
9. V.M. Orlov, I.V. Samokhvalov, G.G. Matvienko, M.L. Belov, and A.N. Kozhemyakov, *Elements of Light Scattering Theory and Optical Detection and Ranging* (Nauka, Novosibirsk, 1982), 225 pp.
10. V.P. Lukin, Appl. Opt. **37**, No. 21, 4634–4644 (1998).
11. V.P. Lukin, Adaptive Optics-96, Technical Digest Series **13**, 150–152 (1996).
12. V.P. Lukin, B.V. Fortes, Atmos. Oceanic Opt. **10**, No. 1, 34–41 (1997).
13. V.P. Lukin, Atmos. Oceanic Opt. **11**, No. 11, 1076–1080 (1998).
14. V.I. Klyatskin, *Statistical Description of Dynamic Systems with Fluctuating Parameters* (Nauka, Moscow, 1975), 239 pp.
15. V.I. Tatarskii, *Wave Propagation in a Turbulent Medium* (McGraw Hill, New York, 1968).
16. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, *Laser Radiation in the Turbulent Atmosphere* (Nauka, Moscow, 1976), 277 pp.
17. S. Enguehard and B. Hatfield, *Incoherence and multiple laser guide stars. NOAO Workshop on the Reduction of Gemini AO Data*, Tucson, USA (2001).
18. V.P. Lukin, in: *Abstracts of Papers on V All-Union Symposium on Laser Radiation Propagation in the Atmosphere*, Part II, Tomsk (1979), pp. 33–36.
19. V.P. Lukin, Soviet J. Quant. Electron. **10**, No. 6, 727–731 (1980).
20. V.P. Lukin and M.I. Charnotskii, Soviet J. Quant. Electron. **12**, No. 5, 602–605 (1982).
21. V.L. Mironov, V.V. Nosov, and B.N. Chen, Izv. Vyssh. Uchebn. Zaved., Radiofizika **25**, No. 12, 1467–1471 (1982).
22. E. Viard, M. Le Louran, and N. Hubin, Appl. Opt. **41**, No. 1, 11–20 (2002).
23. D. Buscher, G. Love, and R. Myers, Opt. Lett. **27**, No. 3, 149–151 (2002).