WAVEFRONT VARIANCE MEASUREMENTS FROM SPECKLE PATTERN STATISTICS

A.V. Ivonin and L.A. Pushnoi

Institute of Atmospheric Optics,
Siberian Branch of the USSR Academy of Sciences, Tomsk
Received Nay 10, 1989

A new method for the measurement of the wavefront variance is proposed. The optimal measurement range for the method is found to be $\lambda/50 < \sigma_\pi < \lambda/8$, for which the accuracy of the method is about 6–30%.

Testing the performance of optimal systems by means of speckle techniques is sometimes more effective than the traditional methods of optical quality control.[4,5]

In 1978 Poltaratskii[1] reported the discovery of a speckle-pattern symmetry effect. The essence of this phenomenon is the point symmetry of the intensity distribution around the optical axis under certain conditions of the optical experiment:

$$I(\vec{r}) = I(-\vec{r}), \tag{1}$$

where $\vec{r}$ is the position vector of the observation point in the recording plane. In Refs. 6 and 7 it was demonstrated that Eq. (1) holds for coherent beam scattering by ensembles of large particles (i.e., for $\left|\vec{k}\right| d \gg 1$, where d is the particle size; $\left|\vec{k}\right| = 2\pi/\lambda$ and λ is the wave length) in those experiments in which $I(\vec{r}) \equiv \left\langle u(\vec{r}) u^*(\vec{r}) \right\rangle$ is determined by the Fourier transform of the transparency function $u_0(\vec{\xi})$ or the reflection of the scattering volume. In other words, the field in the recording plane is found as follows: $u(\vec{r}) \sim \int u_0(\vec{\xi}) \exp(-iB\vec{\zeta}\vec{r}) d\vec{\zeta}$, where is a proportionality coefficient which depends on the experiment setup. Note that a speckle pattern in which the intensity $I(\vec{r})$ is defined as above is called the Fourier speckle.[7] The properties of Gaussian statistics and the assumption of mutual independence of the scattering particles and phase irregularities make it possible to obtain simple expression for the correlation and variances of the intensity recorded in the Fourier speckle pattern[2,7]

$$KI \equiv \langle I(r)\, I(-r) \rangle - \langle I(r) \rangle \langle I(-r) \rangle = \left| \left\langle \sum_n \Phi_n^2 \right\rangle_\eta \right|^2 \left| \frac{1}{\xi_0^2} \int \langle A^2(\vec{\xi}) \rangle\, \xi d\vec{\zeta} \right|^2;$$

$$DI_r \equiv \langle I(r)\, I(r) \rangle - \langle I(r) \rangle \langle I(r) \rangle = \left\langle \sum_n |\Phi_n|^2 \right\rangle_\eta^2 \left[\frac{1}{\xi_0^2} \int \langle |A(\vec{\xi})|^2 \rangle\, \xi d\vec{\zeta} \right], \tag{2}$$

where the symbol $\langle \ldots \rangle_{\eta,\zeta}$ denotes averaging over the random parameters (i.e., η in the particle ensemble; ξ over the phase irregularities); Φ_n is the scattering amplitude of a single scatterer; $A(\vec{\zeta})$ is the complex field amplitude produced by the phase irregularities: $A(\vec{\zeta}) = \exp\{\ln \chi(\vec{\zeta}) + iS(\vec{\zeta})\}$, here $\ln\chi$ is the amplitude level, and S is the phase. Using the well-known relationship

$$\langle \exp\{ f(a, b) \} \rangle = \exp\left\{ \sum_{n,m}^{\to} \frac{1}{n!\,m!} \chi_{1,k}^{a,b} \right\},$$

where $\chi_{1.k}^{a,b}$ are the cumulants of the function $f(a, b)$, and assuming that the cumulants higher than second order are small:[7] $\chi_{n,m}^{a,b} \ll \chi_{1.k}^{a,b}$, $1 + k < 2$, it is possible to obtain a simple expression for the normalized intensity correlation function K. This expression relates K to the spatial variance of the wavefront V: $V \equiv \left\langle \{S_p(\vec{\zeta}) - S_\Gamma(\vec{\zeta})\}^2 \right\rangle \xi$:

$$K = \frac{KI}{\sqrt{DI_r\, DI_{-r}}} = \exp\{-4V^2\}. \tag{3}$$

Here $S_p(\vec{\zeta})$ is the value of the phase in the scalar approximation for radiation at the time t and the point $\vec{\zeta}$; the symbol $\langle \ldots \rangle_\zeta$ denotes averaging over the entire region in which the transparency function is defined, i.e., for $\left|\vec{\zeta}\right| \le \left|\vec{\zeta}_0\right|$.

Let us consider the conditions for satisfying Eq. (3):

i) Gaussian statistics must be valid for the Fourier speckle pattern for $\left\langle u(\vec{r}) \right\rangle = 0$;

ii) individual scatterers and phase irregularities must be mutually independent throughout the entire volume;

iii) cumulants higher than second order must be small for the complex field amplitude $A(\vec{\zeta})$.

Conditions (i) and (ii) can be easily realized in an experiment, e.g., by spatially separating the scatters from the phase irregularities (ii), or by choosing a large number of scatterers N (i).

According to the results of Ref. 8 the superposition of $N > 6$ scattered fields is estimated to be sufficient for Gaussian statistics to apply. Satisfaction of condition (iii) is possible in the case of a slightly turbulent medium; however, condition (iii) becomes questionable if while testing the optical systems regular wavefront distortions occur along with the local ones. To test condition (iii), we borrowed the results of analyses of the optics of large telescopes,[4] which yielded the following error pattern: random error 52 %, regular errors: coma 28%, astigmatism 17%, and zonal error 3 %. We analyzed the maps of the optics of these telescopes and found the amplitude distributions of the deviations of the optical surfaces. Twenty maps with 14×14 to 32×32 points were investigated. The density function $p(h)$ of the amplitude distribution $h(\bar{\zeta})$ was sought in the form[10] $p(h) = A\ \exp\{\ |h|\ (x_0)^{\alpha}\}$. For the initial maps the parameter α fell in the range $\alpha = (2.05 \pm 0.14)$, while for the local error maps $\alpha = (2.11 \pm 0.14)$, for the coma $\alpha = (2.1 \pm 0.1)$, and for the astigmatism $\alpha = (2.04 + 0.11)$. It follows from these data that the distribution of the amplitude errors or the wavefront phase remains Gaussian with a high probability of ~ 95%. The point-to-point spatial resolution of the maps varied from 1/14 to 1/32 of the respective aperture diameter (condition (ii)). As for condition (iii), it should be noted that the symmetry effect is observed only for weak phase fluctuations, when the higher-order moments and cumulants remain small.

To test the capabilities of the wavefront phase variance measurement technique (3), we made use of the fact that the value of V, defined with respect to the nearest sphere, is related, to within the accuracy of a scale factor, to the standard deviation σ of the defects of the optics forming the wavefront. The need for a method to measure σ^2 is now more acute because of a new All-Union standard demanding that optical devices be certified according to their σ^2 value. So far, the σ^2 value was been obtained only by measuring the local characteristic of the optics map, whereas using the speckle pattern allows one to get the total (integral) characteristic σ^2 for the optical system. This quality parameter or figure of merit σ^2 relates directly to the technological process, i.e., it defines the scale of the optical errors, whereas other integral parameters of the quality of the optical system are certificational but have no technological meaning.

The experimental setup for measuring σ^2 is shown in Fig. 1 (Ref. 3). The laser beam, expanded and collimated by the telescope *T1* passes through the object being tested (TO) and the revolving, randomly inhomogeneous amplitude screen, which has orifices of the same diameter d. These orifices are randomly distributed with a packing density $C = 0.04$. The objective *T2* of the second telescope produces the Fourier speckle image in the plane *IP* behind which the ocular is set. Varying the distance L, and the magnification of the ocular, we may scale the speckle pattern. In our experiment we chose $L = 2$ m, $d = 0.001$ m and the distance between the photodiodes $L = 0.06$ m. We found that the optimal distance of the photodetector from the optical axis $l/2$ is equal to ~ (0.4–0.6) r_0, where r_0 is the radius of the speckle pattern, which permits us to neglect the magnitude of the mean field: $\langle u(\bar{r})\rangle \simeq 0$. The accuracy of the symmetrical mount of the photodetectors with respect to the optical axis must be kept with 1/10 of the average size of the individual speckle (in our experiment this meant an accuracy of up to 0.0006 m).

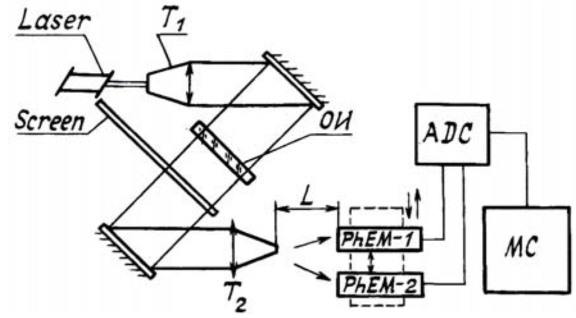

FIG. 1. The experimental setup for measuring the wavefront variance of optical components.

If phase perturbations introduced into the wavefront by the pattern-forming optics are small, we can observe a distinct symmetry effect. Upon introducing the tested optical piece with known σ^2_{TO} into the tract, the symmetry is broken. .The variance for radiation which has passed through the optical tract is then obtained from Eq. (3). Apparently, the total variance is equal to the sum of the variances of the wavefront-forming optics – $\sigma^2_{OS} + \sigma^2_{TO}$:

$$V = \left(\frac{2\pi}{\lambda}\right)^2 [\sigma^2_{OS} + \sigma^2_{TO}]. \tag{4}$$

Accounting for the dimension of the wave vector $\bar{k}$, we arrive at a scale for the defects of the TO. To obtain reliable data on the value of σ^2_{TO}, we tested two different optical setups:

1) the objective MTO-1000 (a mirror telescope objective with a 1:10 aperture and focal distance $F = 1000$ mm),

2) the Zeiss-50 telescope with a 1:8 aperture and $F = 400$ mm.

In the first case the value of K in Eq. (3) was founded to be $K = 0.3$. This corresponds to $\sigma^2_{OS} = \lambda/9$. The second system gave $K = 0.8$, i.e., $\sigma^2_{OS} = \lambda/25$ (Figs. 2b and c). These values of σ^2_{OS} already testify to the sensitivity of this method. We may conclude that the first optical system is unsuitable for measuring the optical components of low σ_{TO}.

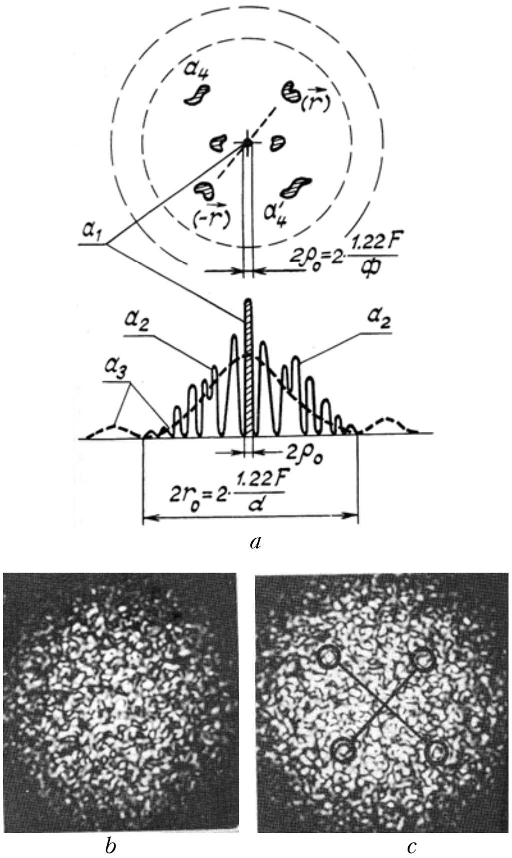

FIG. 2. The symmetry effect in the Fourier-speckle recording plane: a) stylized intensity distribution in the plane; b, c) speckle-pattern experimental images: b) K = 0.3 and c) K = 0.8. Circles connected through the center of the speckle pattern show the visually observed symmetry, α_1 *is the coherent part of the intensity distribution of width* 2 ρ_0 *for entrance aperture diameter Φ and focal distance F;* α_2 *is the intensity of the individual speckles;* α_3 *(dashed line) is the intensity distribution for a single scatterer of size d;* α_4 *and* α'_4 *are stylized images of symmetrical spots in the speckle pattern, likewise for (–r, r).*

To test the data so obtained, we checked a set of flat optical plates, 34 mm in diameter, with various values of σ^2_{TO}. To put the needed relief onto the surface of each plate, they were etched by a polishing tool. The required σ_{TO} was provided by varying the polishing time. All of the plates were tested on a Twyman-Green interferometer with light passing twice through the plate to obtain its interferogram. The images were measured automatically on a SKIF device with an accuracy of (1/60–1/45) λ (Ref. 5). The accuracy of this processing was checked repeatedly by comparing it with the results from the Hartman technique and data from manual measurements. The accuracy of σ^2_{TO} achieved for our pieces was better than 2%, thus making possible to compare it with the results from the interference measurements. The results of such comparisons for three separate pieces are presented in Table I.

TABLE I

Comparison of the σ_S *measurements with the interferometry data.*

detail number	measurement methods		
	Speckle-pattern measurement	Interferometer SKIF setup	Relative difference of two methods
1	λ/8	λ/5.0	> 37 %
2	λ/12.1	λ/13.1	9 %
3	λ/16.6	λ/15.6	6 %

Let us further analyze the optimal dynamic range of our method.[3] The measurement range for *KI* and *DI* when using an X6–4 instrument is close to 100, from which we get the values K_{min} = 0.01 and K_{max} = 0.99, from which we have $\lambda/132 < \sigma_{TO} < \lambda/5$. For either surface of a flat thin plate, the standard deviation is given by $\sigma_s = \sigma_{TO}/n$, where n = 1.5147 at λ = 0.6328 μm. The measurable range for σ is then $\lambda/200 < \sigma_s < \lambda/8$. The normalized relative accuracy may then be estimated by known techniques,[10] so that for our case$\Delta\sigma_s/\sigma_s$ = 000097 $(\lambda/\sigma_s)^2$.At a 95% probability level the statistical error of σ_s about 20%. The relative errors of the measured values {Δ*DI*/*DI*} and {Δ*K*/KI} amounted to 5 %.

Estimating the capabilities of the technique in general, we see from our experiment that a reliable result is obtained for the wavefront when $1/33 < \sigma_{TO} < \lambda/5$ and for either surface when $\lambda/50 < \sigma_s < \lambda/8$. The normalized accuracy of the σ_s measurements for these extreme ranges should then not exceed 30%. The value of σ_s for flat optical plate No. 1 (see Table I) does not fit this range, which accounts for the heavy disagreement with the interferometry data in this case. We estimate that the accuracy of this method will increase for lower σ_{TO} or σ_s.

Let us note certain particularities of the experiments:

1) To reduce the statistical error (up to 20% for a lag of ~ 0.1 s) the measurement time interval and the dynamic range for *DI* and *KI* should be increased.

2) When measuring low σ_s, optics with very small σ_{OS} ($\sigma_{OS} \ll \sigma_{TO}$) should be chosen.

3) The photodetectors must be aligned with the Fourier-speckle image plane with the highest possible accuracy. In our experiment we attained an accuracy of about 5 · 10^{-5} m.

Among the merits of the new technique mention should be made of its vibrational stability, the simplicity with which the measurements can be automated, and the sensitivity to low values of σ. This technique might be used to scrap faulty optics, e.g., microobjectives, mirrors, etc., by their σ_{TO}; for automatic fine adjustment of optical tracts (in that case one would not need to know the focal optical error map). The optical system might then be adjusted by changing the distance between the optical elements to minimize the value of σ_{TO}. One of the possible applications of this method might be to measure the high-frequency errors of large mirrors. This kind of defect increases the width of the point spread function and does not yield to simple control techniques. The new method might be useful for studies of atmospheric turbulence with rain or snow playing the role of the amplitude screen.[2]

The authors find it a pleasant duty to express their thanks to O.A. Reutova and F.P. Parshin, who took part in the realization of the experiment, and to A.G. Borovoi, with whom they discussed various stages of the work.

REFERENCES

1. B.F. Poltaratskii, Zn. Eksp. Teor. Fiz. Pis'ma **27**, No. 7, 406 (1978).
2. A.V. Ivonin and N.I. Vagin, Opt. Spectrosk. **58**, No. 1, 235 (1985).
3. Author's Certificate No. 1223033, SSSR, MCI SO 1 B 11/24, *Method for Control of Optical Systems Quality*, A.G. Borovoi, A.V. Ivonin, E.A. Vitrichenko, V.M. Kabanov, L.A. Pushnoi, and V.Ya. S"edin, Publ. in Bull. of Invent., No. 13 (1986).4. E.A. Vitrichenko, V.E. Zuev, V.P. Lukin, and L.A. Pushnoi, *Analysis of Errors in the Optics of Large Telescopes*, Dokl. Akad. Nauk SSSR 300, No. 2, 199 (1988).
5. L.A. Pushnoi, *Abstracts of Reports of the twelvth All-Union Conference on High-Speed Photography, Photonics, and Metrology of Fast Processes,* All-Union Scientific-Research Institute for Optical-Physical Measurements, **87** (1985).
6. A.G. Borovoi and A.V. Ivonin, Opt. Spectrosk., **53**, No. 6, 1049 (1982).
7. A.V. Ivonin and V.Ya. S"edin, *Abstracts of Reports of the eighth All-Union Conference on Laser and Acoustic Sensing of the Atmosphere,* Tomsk, Part 2, **54** (1984).
8. N.D. Ustinov, I.N. Matveev, and V.V. Protopopov, *Methods for Processing of Optical Fields in Laser Sounding* (Nauka, Moscow, 1983).
9. *Lenses: Classification of Wavefront Errors and Errors in the Manufacturing of Optical Elements,* Branch Standard, OST 3-5476-83 (USSR) Group 091. First introduced, July, 1983.
10. P.V. Novitskii and I.A. Zograf, Estimation of Measurement Errors (Energoatomizdat, 'Leningrad, 1985).