Step-by-step technique for taking into account thermal radiation scattering by aerosol formations in the atmosphere. Part 1. Basic computational scheme

A.V. Vasil'ev

Research & Development Institute of Physics at St. Petersburg State University

Received November 27, 2002

The computational scheme of the step-by-step technique for radiative transfer problems taking into account multiple scattering and multiple reflection from the surface for various natural radiation sources is described. Some peculiarities of implementing this technique for calculations of thermal radiation scattering by aerosol formations in the atmosphere are considered. Simplifications and approximations enabling one to develop efficient computational algorithms in this case are suggested.

Introduction

With the advent of high-accuracy instrumentation for measuring the intensity of thermal radiation from the atmosphere and the surface in the IR and microwave spectral regions, it has become important to develop adequate high-accuracy physical-mathematical modeling of findings from such measurements. In particular, such a modeling is especially urgent for interpretation of remote measurements, evaluation of their information content on the atmosphere and surface, and in other studies connected with solution of inverse problems of atmospheric optics. An important stage in improving the adequacy of the modeling algorithms is the account of multiple scattering by atmospheric aerosols, clouds, and precipitation.

The problem of numerical calculation of the field of scattered radiation in the horizontally homogeneous plane-parallel atmosphere can be now believed solved.[1–3] However, most of the numerical methods are considered only for traditional problems of transfer of scattered solar radiation, and therefore it is difficult to adapt them to the IR and microwave (MW) regions, in which every elementary atmospheric volume is a source of radiation. In my opinion, in these spectral regions we can use one of the oldest and simplest methods of the radiative transfer theory, namely, the step-by-step technique. It is not only simple in implementation, but also very convenient in research problems, because it allows one to find directly which of the elementary components determine the measured radiation and what are their contributions. The well-known disadvantage of this method is its poor convergence in the case of weak atmospheric absorption, but in the IR and MW regions, where molecular absorption is always strong, this disadvantage is insignificant. Note that once the step-by-step technique is adapted to particular applied problems, its efficiency can be increased by tens times due to application of simple computational approaches that will be mentioned in Part 2 of this paper.

Problem formulation

The instrumentally measured radiation intensity is determined by a convolution of the monochromatic intensity with the instrumental function. Since the latter is believed known, the problem reduces to calculation of intensity in the monochromatic case at a given frequency (or wavelength, wave number) ν. Geometrically, the monochromatic intensity is characterized by the height of measurement z above the surface, nadir angle ϑ, and sighting azimuth φ (the nadir angle of sighting varies from 0° for nadir sighting to 180° for zenith sighting). The sighting azimuth is measured in the horizontal plane from an arbitrarily selected direction (varies from 0 to 360°).

For the Earth's atmosphere, we use the plane, horizontally homogeneous model with the bottom z_0 and top z_∞ boundaries. First, let us consider the approximation of unpolarized radiation. All the mentioned parameters of the atmosphere and the surface (volume extinction and absorption coefficients and others) at the frequency ν are assumed known.

Radiation transfer through the atmosphere

The monochromatic intensity is calculated based on the radiative transfer equations and their solutions.[1–4] If radiation propagation is characterized only by extinction, then the sought intensity at radiation propagation from the initial height z_1 to the final one z_2 is determined by the Bouguer law

$$I(\nu, z_2, \vartheta, \varphi) = I(\nu, z_1, \vartheta, \varphi) P(\nu, z_1, z_2, \vartheta) . \qquad (1)$$

0235-6880/03/08 668-05 $02.00

Here

$$P(\nu, z_1, z_2, \vartheta) = \exp\left(-\frac{1}{\cos\vartheta}\int_{z_1}^{z_2}\alpha(\nu, z')\mathrm{d}z'\right)$$

is the transmission function, where $\alpha(\nu, z)$ is the volume extinction coefficient (of air). The concept of optical depth τ corresponding to the height z is introduced. By definition,[1,2] $\tau(\nu, z) = \int_{z}^{z_\infty}\alpha(\nu, z')\mathrm{d}z'$. The parameter $\tau_0(\nu) = \tau(\nu, z_0)$ is the optical depth of the entire atmosphere.

In the case of transfer of thermal atmospheric radiation neglecting scattering under conditions of local thermodynamic equilibrium (LTE) between the radiation and matter, the solution of the radiative transfer equation has the well known form

$$I(\nu, z_2, \vartheta, \varphi) = I(\nu, z_1, \vartheta, \varphi)P(\nu, z_1, z_2, \vartheta) +$$

$$+\frac{1}{\cos\vartheta}\int_{z_1}^{z_2}\kappa(\nu, z')B_e(\nu, T(z'))P(\nu, z', z_2, \vartheta)\mathrm{d}z', \quad (2)$$

where $\kappa(\nu, z)$ is the volume absorption coefficient; $B_e(\nu, T)$ is the Planck's function depending on the radiation frequency and the air temperature ($B_e(\nu, T) = 2h\nu^3/c^2[\exp(h\nu/kT) - 1]$. Here h is the Plank's constant; k is the Boltzmann constant; c is the speed of light). Note that Eq. (2) can be written in the traditional form, convenient for calculations, with integration of the derivative of the transmission function:

$$I(\nu, z_2, \vartheta, \varphi) = I(\nu, z_1, \vartheta, \varphi)P(\nu, z_1, z_2, \vartheta) +$$

$$+\int_{z_1}^{z_2}B_e[\nu, T(z')]P_\sigma(\nu, z', z_2, \vartheta)\frac{\partial P_\kappa(\nu, z', z_2, \vartheta)}{\partial z'}\mathrm{d}z', \quad (3)$$

where

$$P_\sigma(\nu, z_1, z_2, \vartheta) = \exp\left(-\frac{1}{\cos\vartheta}\int_{z_1}^{z_2}\sigma(\nu, z')\mathrm{d}z'\right)$$

and

$$P_\kappa(\nu, z_1, z_2, \vartheta) = \exp\left(-\frac{1}{\cos\vartheta}\int_{z_1}^{z_2}\kappa(\nu, z')\mathrm{d}z'\right)$$

are individual transmission functions for the scattering and absorption processes, respectively.

In the presence of scattering under LTE conditions and making allowance for transition to coordinates $\eta = \cos\vartheta$ and the optical depth τ, the radiative transfer equation takes the form[1,2,4]

$$\eta\frac{\mathrm{d}I(\nu, \tau, \eta, \varphi)}{\mathrm{d}\tau} = I(\nu, \tau, \eta, \varphi) -$$

$$-\frac{\omega_0(\nu,\tau)}{4\pi}\int_{-1}^{1}\mathrm{d}\eta'\int_{0}^{2\pi}\mathrm{d}\varphi' x(\nu, \tau, \chi)I(\nu, \tau, \eta', \varphi') -$$

$$-(1 - \omega_0(\nu,\tau))B_e(\nu, T(\tau)). \quad (4)$$

Here

$$\omega_0(\nu, \tau) = \sigma(\nu, \tau)/\alpha(\nu, \tau) = \sigma(\nu, \tau)/[\sigma(\nu, \tau) + \kappa(\nu, \tau)]$$

is the single scattering albedo, where $\sigma(\nu, z)$ is the volume scattering coefficient; $x(\nu, \tau, \chi)$ is the scattering phase function normalized as follows

$$\frac{1}{2}\int_{-1}^{1}x(\nu, \tau, \chi')\mathrm{d}\chi' = 1$$

and

$$\chi = \eta\eta' + \sqrt{(1-\eta^2)(1-(\eta')^2)}\cos(\varphi - \varphi').$$

Radiation sources

The sources in the considered class of the radiative transfer problem in the atmosphere can be the following: extraterrestrial sources — cosmic radiation incoming to the Earth, in particular, solar radiation; sources on the atmospheric bottom — thermal radiation from the surface and the radiation reflected from the surface; sources inside the atmosphere — thermal radiation from every elementary air volume. The radiation reflected from the surface depends on the radiation field in the atmosphere, while all other sources are independent of it.

Let us consider first the problem on calculating the radiation field neglecting reflection from the surface. In this case, the intensities of all sources are known. For simplicity and convenience, the function $B(\nu, \tau, \eta, \varphi)$ is introduced as a function of sources being at the optical depth τ and emitting along the direction (η, φ). According to Refs. 1 and 4, these sources are all the terms, except for the intensity, in the right-hand side of the radiative transfer equation written in the coordinates of the optical depth (4). The source function in this case is separated into the terms corresponding to sources of different types.

Consider first the source functions of the initial radiation, that is, neglecting the contribution of scattered radiation to them, and denote them as $B_0(\nu, \tau, \eta, \varphi)$. For the thermal radiation under LTE conditions, this, according to Eq. (4), immediately gives

$$B_0(\nu,\tau,\eta,\varphi) = [1 - \omega_0(\nu,\tau)]B_e[\nu,T(\tau)]. \quad (5)$$

After introduction of the source function, we can easily obtain the equations like the following one (see, for example, Ref. 1):

$$I(\nu, \tau, \eta, \varphi) = -\frac{1}{\eta}\int_{\tau(\eta)}^{\tau}B(\nu, \tau', \eta, \varphi)P(\nu, \tau', \tau, \eta)\mathrm{d}\tau', \quad (6)$$

where $\tau(\eta) = \tau_0$, if $\eta > 0$; $\tau(\eta) = 0$, if $\eta < 0$. For the source being at the optical depth τ' and characterized

by the radiation intensity $I_0(\nu, \eta, \varphi)$, Eq. (6) should transform to the Bouguer law (1), wherefrom it follows that

$$B_0(\nu, \tau, \eta, \varphi) = I_0(\nu, \eta, \varphi) \mid \eta \mid \delta(\tau - \tau'), \qquad (7)$$

where $\delta(\tau - \tau')$ is the delta function. Thus, in the absence of reflection, the function $B_0(\nu, \tau, \eta, \varphi)$ by Eqs. (5) and (7) is determined for all types of sources (both the sources of thermal radiation and, for example, sources at the atmospheric boundaries).

Because of the linear character of the radiative transfer equation (4) the sought radiation field, for an arbitrary distribution of sources, can be found as[1,4]

$$I(\nu, \tau, \eta, \varphi) =$$

$$= \int_0^{\tau_0} d\tau' \int_{-1}^{1} d\eta' \int_0^{2\pi} d\varphi' T(\nu, \tau, \eta, \varphi, \tau', \eta' \varphi') B_0(\nu, \tau', \eta', \varphi'), \qquad (8)$$

where the radiation intensity $I(\nu, \tau, \eta, \varphi)$ at any (arbitrary) coordinates (τ, η, φ) for the given source function $B_0(\nu, \tau', \eta', \varphi')$ at fixed (but again arbitrary) coordinates (τ', η', φ') is formally written as $I(\nu, \tau, \eta, \varphi) = T(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') B_0(\nu, \tau', \eta', \varphi')$. Equation (8) can be written in a compact operator form as $\mathbf{I} = \mathbf{T}\mathbf{B}_0$, where $\mathbf{I}$ and $\mathbf{B}_0$ are the radiation intensity and the source function, $\mathbf{T}$ is the linear radiative transfer operator (the action of a linear operator on a function, see Ref. 5, can be presented symbolically as a product; by definition $\mathbf{G} = \mathbf{AF}$ is equivalent to

$$g(x) = \int_a^b a(x, x') f(x') dx',$$

wherefrom we have for the operator product $\mathbf{C} = \mathbf{AB}$

$$c(x, x') = \int_a^b a(x, x'') b(x'', x') dx'',$$

the power of an operator is defined as $\mathbf{A}^n = \mathbf{A}\mathbf{A}^{n-1}$, $\mathbf{A}^1 = \mathbf{A}$).

In the absence of scattering, all the parameters for this case are denoted by the zero subscript, we obviously have from Eq. (6) that

$$\mathbf{I}_0 = \mathbf{T}_0 \mathbf{B}_0, \qquad (9)$$

where

$$T_0(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = -\frac{1}{\eta'} P(\nu, \tau', \tau, \eta') \delta(\eta' - \eta) \delta(\varphi' - \varphi),$$

if $\eta' > 0$ and $\tau' \geq \tau$ or $\eta' < 0$ and $\tau' \leq \tau$;

$$T_0(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = 0, \qquad (10)$$

if $\eta' > 0$ and $\tau' < \tau$ or $\eta' < 0$ and $\tau' > \tau$

is the transfer operator for the direct radiation.

The difficulty of taking into account the scattering consists in the fact that in this case Eq. (6) is no longer a solution, because the source function itself depends on the sought intensity [the term with the integral in Eq. (4)]. A standard approach[4] in taking into account the scattering is separation of the direct radiation and diffuse radiation, that is, the radiation after at least one scattering event. Actually, representation of the sought intensity as a sum

$$I(\nu, \tau, \eta, \varphi) = I_0(\nu, \tau, \eta, \varphi) + I_n(\nu, \tau, \eta, \varphi),$$

where $I_n(\nu, \tau, \eta, \varphi)$ is the intensity of scattered radiation (only scattered, without the direct radiation and the radiation reflected from the surface), gives the transfer equation for the scattered radiation, from which we have the integral equation for the source function[1,2,4]:

$$B_n(\nu, \tau, \eta, \varphi) = \frac{\omega_0(\nu, \tau)}{4\pi} \int_{-1}^{1} \frac{d\eta'}{\eta'} \int_0^{2\pi} d\varphi' x(\nu, \tau, \chi) \times$$

$$\times \int_{\tau(\eta')}^{\tau} d\tau' [B_n(\nu, \tau', \eta', \varphi') + B_0(\nu, \tau', \eta', \varphi')] P(\nu, \tau', \tau, \eta'). \qquad (11)$$

In applied calculations, the integral equation for the radiation intensity is often used.[3] Its solution immediately gives the sought value, but this equation includes the direct radiation intensity $\mathbf{I}_0$ in place of $\mathbf{B}_0$. Since the intensity should be calculated all over the coordinate grid, computations are more voluminous than in the case of integration of the source function by Eq. (6) already for a limited number of needed coordinates. Therefore, the scheme for calculation of just the source function for scattered radiation seems to be more economic.

Step-by-step technique

Equation (11) is the Fredholm integral equation of the second-kind; in the operator form it is written as follows

$$\mathbf{B}_n = \mathbf{T}_1 \mathbf{B}_n + \mathbf{T}_1 \mathbf{B}_0 \qquad (12)$$

where

$$T_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = \frac{\omega_0(\nu, \tau)}{4\pi\eta'} x(\nu, \tau, \chi) P(\nu, \tau', \tau, \eta'),$$

if $\eta' > 0$ and $\tau' \geq \tau$ or $\eta' < 0$ and $\tau' \leq \tau$;

$$T_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = 0, \qquad (13)$$

if $\eta' > 0$ and $\tau' < \tau$ or $\eta' < 0$ and $\tau' > \tau$

is the operator of transfer of the single scattered radiation. The formal solution of the Fredholm integral equation of the second-kind (12) is the Neumann series

$$\mathbf{B}_n = \mathbf{T}_1 \mathbf{B}_0 + \mathbf{T}_1^2 \mathbf{B}_0 + \mathbf{T}_1^3 \mathbf{B}_0 + \ldots . \qquad (14)$$

The series (14) is known as an expansion of the source function for scattered radiation in terms of the number of scattering events. In practice, expansion of Eq. (14) is realized as a recursion computational scheme — the step-by-step technique.

Taking into account reflection from the surface

Consider now the process of radiation reflection from the Earth's surface. In the general case, it can be specified as an intensity ratio between the radiation incident on the surface $I(\nu, \tau_0, \eta, \varphi)$ at $\eta < 0$ and the radiation reflected from the surface $I(\nu, \tau_0, \eta, \varphi)$ at $\eta > 0$. Denote the case $\eta < 0$ as $I^{\downarrow}(\nu, \tau_0, \eta, \varphi)$ and the case $\eta > 0$ as $I^{\uparrow}(\nu, \tau_0, \eta, \varphi)$. The relation between them can formally be written as

$$\eta I^{\uparrow}(\nu, \tau_0, \eta, \varphi) =$$

$$= -\int_0^{2\pi} d\varphi' \int_{-1}^{0} \eta' R(\nu, \eta, \eta', \varphi - \varphi') I^{\downarrow}(\nu, \tau_0, \eta', \varphi') d\eta'. \quad (15)$$

The function $R(\nu, \eta, \eta', \varphi - \varphi')$ characterizes the process of reflection, expressing the intensity of the radiation reflected from the surface along the direction (η, φ) through the intensity of the radiation incident along the direction (η', φ'). Thus, for an ideal specular reflection

$$R(\nu, \eta, \eta', \varphi - \varphi') = r(\nu, -\eta')\, \delta(\eta - (-\eta'))\delta(\varphi - \varphi'),$$

where $r(\nu, -\eta') = r(\nu, \eta)$ is the reflection coefficient calculated by the Fresnel equations, see, for example, Refs. 6 and 7. For isotropic reflection

$$R(\nu, \eta, \eta', \varphi - \varphi') = \eta \frac{A(\nu)}{\pi},$$

where $A(\nu)$ is the spectral albedo of the surface. In the operator form Eq. (15) can be written as

$$\mathbf{B}_{r,0} = \mathbf{R}_1 \mathbf{I}, \quad (16)$$

where $\mathbf{I}$ is the radiation intensity before reflection; $\mathbf{B}_{r,0}$ are additional source functions arising due to reflection; $\mathbf{R}_1$ is the single reflection operator:

$$R_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') =$$

$$= -\eta' R(\nu, \eta, \eta', \varphi - \varphi')\delta(\tau' - \tau_0)\delta(\tau - \tau_0),$$

$$\text{if } \eta > 0 \text{ and } \eta' < 0;$$

$$R_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = 0, \quad (17)$$

$$\text{if } \eta < 0 \text{ and } \eta' > 0.$$

Thus, taking into account the reflection reduces to re-calculation of the source functions by Eq. (16). As a result, additional sources arise on the surface. In the general case of present scattering, these sources again change the intensity of the radiation incident on the surface, and this process gives rise to the iteration series known as calculation of the sought intensity with the allowance for the number of the scattering events:

$$\mathbf{I} = \mathbf{T}_0(\mathbf{T} + (\mathbf{R}_1\mathbf{T}_0\mathbf{T}) + (\mathbf{R}_1\mathbf{T}_0\mathbf{T})^2 + (\mathbf{R}_1\mathbf{T}_0\mathbf{T})^3 + \ldots)\mathbf{B}_0, \quad (18)$$

where

$$\mathbf{T} = \mathbf{1} + \mathbf{T}_1 + \mathbf{T}_1^2 + \mathbf{T}_1^3 + \ldots \quad (19)$$

is the radiative transfer operator ignoring reflection; $\mathbf{B}_0$ is the distribution of the initial radiation sources in the atmosphere.

As to practical application, it is convenient to substitute Eq. (19) into Eq. (18) by introducing $\mathbf{T}_s = \mathbf{T}_1 + \mathbf{T}_1^2 + \mathbf{T}_1^3 + \ldots$ — the transfer operator for the scattered radiation ignoring reflection and direct radiation, and to take into account that from the explicit equations (17) and (10) for the operators $\mathbf{R}_1$ and $\mathbf{T}_0$ we have $\mathbf{R}_1\mathbf{T}_0\mathbf{R}_1\mathbf{T}_0 = \mathbf{0}$, which at $n \geq 2$ gives the following equation:

$$(\mathbf{R}_1\mathbf{T}_0 + \mathbf{R}_1\mathbf{T}_0\mathbf{T}_s)^n = \mathbf{R}_1\mathbf{T}_0\mathbf{T}_s(\mathbf{R}_1\mathbf{T}_0 + \mathbf{R}_1\mathbf{T}_0\mathbf{T}_s)^{n-1},$$

hence

$$\mathbf{I} = \mathbf{T}_0(\mathbf{1} + \mathbf{R}_1\mathbf{T}_0)\mathbf{B}_0 + \mathbf{T}_0(\mathbf{1} + \mathbf{R}_1\mathbf{T}_0)\mathbf{T}_s\mathbf{B}_0 +$$

$$+\mathbf{T}_0(\mathbf{R}_1\mathbf{T}_0\mathbf{T}_s + (\mathbf{R}_1\mathbf{T}_0\mathbf{T}_s)^2 + \ldots)\mathbf{R}_1\mathbf{T}_0\mathbf{B}_0 +$$

$$+\mathbf{T}_0(\mathbf{R}_1\mathbf{T}_0\mathbf{T}_s + (\mathbf{R}_1\mathbf{T}_0\mathbf{T}_s)^2 + \ldots)\mathbf{R}_1\mathbf{T}_0\mathbf{T}_s\mathbf{B}_0. \quad (20)$$

The four terms in the equation for intensity (20) have clear physical meanings: the first one is the contribution of direct radiation with the allowance made for the single reflection, the second is the contribution of scattered radiation with the allowance for single reflection, the third one is the contribution of multiple reflection from the reflected and scattered direct radiation, the fourth is the contribution of multiple reflection from the scattered radiation. Note that the latter two terms are usually small enough and can be neglected in some applied problems. Separation of the direct and scattered radiation in Eq. (20) allows us to lift some limitations imposed at problem formulation, because only the direct radiation can be calculated by Eqs. (3) and (9), for example, for the spherical model of the atmosphere.

Peculiarities of taking into account the scattering in the IR and MW regions

In the IR and MW spectral regions, the main source is thermal radiation of the atmosphere and the surface, the former being isotropic and the latter either also isotropic or depending on only the angle η, but independent of the azimuth φ for most of the model surfaces. If we consider only such sources, then the radiation intensity is independent of the azimuth because of the azimuth isotropy. Then the non-zero parts of the single scattering (13) and reflection (17) operators can be written in the form

$$T_1(\nu, \tau, \eta, \tau', \eta') = \frac{\omega_0(\nu, \tau)}{4\pi\eta'} p(\nu, \tau, \eta, \eta') P(\nu, \tau', \tau, \eta');$$

$$R_1(\nu, \tau, \eta, \tau', \eta') = -\eta' \rho(\nu, \eta, \eta')\delta(\tau' - \tau_0)\delta(\tau - \tau_0), \quad (21)$$

where

$$p(\nu,\tau,\eta,\eta') = \int_0^{2\pi} x(\nu,\tau,\eta\eta' + \sqrt{(1-\eta^2)(1-(\eta')^2)}\cos\varphi)\mathrm{d}\varphi;$$

$$\rho(\nu,\eta,\eta') = \int_0^{2\pi} R(\nu,\eta,\eta',\varphi)\mathrm{d}\varphi \qquad (22)$$

are the scattering phase function and reflection function averaged over azimuth. For the Rayleigh scattering phase function $x(\chi) = 3(1+\chi^2)/4$ we have:

$$p(\eta,\eta') = \frac{3}{4}\pi(3 + 3\eta^2(\eta')^2 - \eta^2 - (\eta')^2)\,.$$

Difficulties arise in this approach, if it is needed to take into account processes without azimuth isotropy, such as direct solar radiation and some complicated models of reflected surfaces, for example, rough water surface.[8] In the case of account of the direct solar radiation, the problem of its transfer can be solved separately based on independent summation of intensities from all the sources, and then the result can be added to solution of the problem for the thermal radiation. The methods for calculation of solar radiation field in the atmosphere are now well known, see, for example, Ref. 3. The azimuth anisotropy of reflection from the surface at illumination by the azimuth-isotropic radiation is low,[8] therefore it is quite feasible to calculate the scattered radiation ignoring it.

Thus, for the overwhelming majority of applied problems involving calculation of the scattered radiation in the IR and MW regions it seems possible to use the azimuth-average transfer model (21) and (22).

Another important feature of the IR and MW regions is the presence of a rather strong molecular absorption, which weakens the effect of scattering and reflection on the measured intensity. In taking this into account, assume $(\mathbf{R}_1\mathbf{T}_0\mathbf{T}_s)^n \approx (\mathbf{R}_1\mathbf{T}_0\mathbf{T}_1)^n$ in solution of (20). In this case, multiple reflection is actually taken into account only in the single scattering approximation. The Eq. (20) can be written for practical realization already as a recursion algorithm

$$\mathbf{I} = \sum_{n=0}\Bigg[\mathbf{T}_0(\mathbf{1} + \mathbf{R}_1\mathbf{T}_0)(\mathbf{T}_1^n\mathbf{B}_0) + \sum_{m=1}\mathbf{T}_0(\mathbf{R}_1\mathbf{T}_0\mathbf{T}_1)^m\mathbf{R}_1\mathbf{T}_0(\mathbf{T}_1^n\mathbf{B}_0)\Bigg], \qquad (23)$$

in which it is sufficient to keep in mind only the source function $\mathbf{B}_n = (\mathbf{T}_1^n\mathbf{B}_0)$ recalculated at any iteration. Note that Eq. (23) systematically underestimates the result as compared to rigorous equation (20), but this error can be neglected in most practical problems.

The algorithm implementing the step-by-step technique for the IR and MW regions with the allowance for these features will be given in Part 2 of this paper.

Conclusion

The presented general scheme of the step-by-step technique can be easily implemented on a computer (see also Part 2 of this paper). It, unlike many other methods, allows one to use various complex models of reflection from the surface and natural emission of the surface. The method described is especially useful in scientific and research calculations, because it allows one to assess the accuracy of various approximations and select the number of scattering and reflection events to be taken into account in particular computational problems.

References

1. V.V. Sobolev, *Light Scattering in Planetary Atmospheres* (Nauka, Moscow, 1972), 335 pp.
2. I.N. Minin, *Theory of Radiative Transfer in Planetary Atmospheres* (Nauka, Moscow, 1988), 264 pp.
3. J. Lenoble, ed., *Radiative Transfer in Scattering and Absorbing Atmospheres: Standard Computational Procedures* (A. Deepak Publ., Hampton, Va., 1985), 300 pp.
4. D.I. Nagirner, *Lectures on the Radiative Transfer Theory* (St. Petersburg State University Publishing House, St. Petersburg, 2001), 284 pp.
5. A.N. Kolmogorov and S.V. Fomin, *Elements of the Theory of Functions and Functional Analysis* (Nauka, Moscow, 1989), 624 pp.
6. D.V. Sivukhin, *General Course of Physics. Optics* (Nauka, Moscow, 1980), 751 pp.
7. C.F. Bohren and D.R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).
8. Yu.-A.R. Mullamaa, *Atlas of Optical Characteristics of Rough Sea Surface* (Academy of Sciences of the Estonian SSR, Tartu, 1964), 511 pp.