FUNDAMENTAL BOUNDARY–VALUE PROBLEMS OF THE IMAGE TRANSFER AS APPLIED TO AN ACTIVE OPTO–ELECTRONIC SYSTEM OF OBSERVATION THROUGH A RANDOMLY ROUGH INTERFACE BETWEEN THE MEDIA

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A new method of constructing an optical transfer operator for an active opto–electronic system of observation through a randomly rough interface in the "atmosphere––ocean" system is proposed. Based on the methods of the Green's functions and perturbation theory the general boundary–value problem is decomposed into a number of the simplest problems whose solutions are sought in the small–angle approximation of the radiative transfer equation. The expressions obtained take into account a radiation correlation on the randomly rough surface. They are compared with the results obtained by various authors.

The simulation of the image transfer through the randomly rough interface (RRI) between two turbid media provides the basis for optimizing the active opto–electronic systems (OES) of observation of the underwater objects from the atmosphere and space.

Phenomenological approach to the description of the image transfer[1–4] based on the physically obvious assumptions makes it possible to obtain the analytical expressions for the basic characteristics of the light field; however, in this case it is rather difficult to indicate the limits of its applicability and its correctness.

By virtue of the insufficient mathematic rigour in the phenomenological description a necessity has arisen in a rigourous mathematic formulation of the corresponding boundary–value problem with nonlinear boundary conditions[5–8] and its decomposition into a set of the simplest problems whose solutions can be found in one or another approximation.

Initially, this formalism was developed for a plane turbid layer[6,8,10] upon exposure to natural radiation. The formulas for the image transfer through the RRI were obtained in Refs. 9 and 10 but they disregard the boundary correlation of radiation multiply passed through the RRI.

Let us consider the general scheme of observing through the combination of two turbid media with allowance made for the RRI. As is shown below, an increase of the number of layers introduces no principal changes into equations which can be easily generalized for this case. For definiteness, we refer to the first medium as the atmosphere while to the second – as the ocean. The coordinate system is chosen as shown in Fig. 1. The underlying surface at the depth $h + z$ is characterized by the diffuse reflection coefficient $\rho(\mathbf{r}')$. The unit vectors are denoted by the symbol $\wedge$, the elementary solid angle is designated by $d\hat{l}$, and the radius vectors are assumed to lie in a horizontal plane. The positions of the illumination source S with the radiant flux Φ_0 and the receiver of radiation R are specified by the radius vectors $\mathbf{r}_S$ and $\mathbf{r}_R$. The unit vectors $\hat{\mathbf{l}}_S$ and $\hat{\mathbf{l}}_R$ specify the directions of the axes of directional patterns of the source $\omega_S(\mathbf{r}, \hat{\mathbf{l}}_S)$ and the receiver $\omega_R(\mathbf{r}, \hat{\mathbf{l}}_R)$, respectively. The rest of designations are shown in Fig. 1.

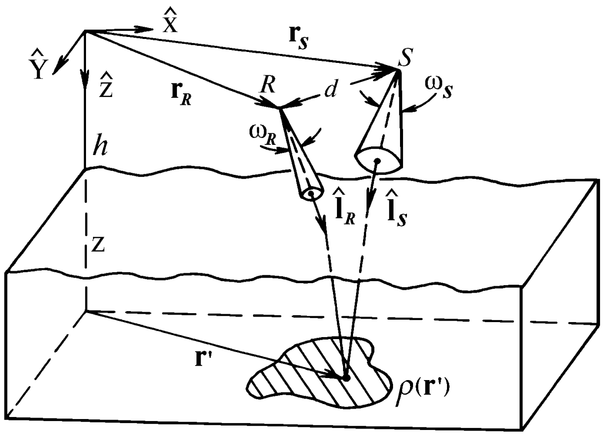

FIG. 1. Generalized scheme of the opto–electronic system of observation.

Let us use the following linear operators:

$$\boldsymbol{A}L = \frac{1}{\pi}\,\rho(\mathbf{r}')\int\limits_{\Omega_+} L(\mathbf{r}', \hat{\mathbf{l}}')(\hat{\mathbf{z}}, \hat{\mathbf{l}}')d\hat{\mathbf{l}}' = L(\mathbf{r}, \hat{\mathbf{l}})\,\Big|\,\hat{\mathbf{l}} \in \Omega_- , \quad (1)$$

$$\boldsymbol{R}_{\pm}L = \frac{1}{\pi}\int\limits_{\Omega_{\pm}} \rho(\mathbf{r}', \hat{\mathbf{l}}, \hat{\mathbf{l}}')\,L(\mathbf{r}'', \hat{\mathbf{l}}')(\hat{\mathbf{N}}, \hat{\mathbf{l}}')d\hat{\mathbf{l}}' = L(\mathbf{r}, \hat{\mathbf{l}})\,\Big|\,\hat{\mathbf{l}} \in \Omega_{\pm}, \quad (2)$$

$$\boldsymbol{T}_{\pm}L = \frac{1}{\pi}\int\limits_{\Omega_{\pm}} \tau(\mathbf{r}', \hat{\mathbf{l}}, \hat{\mathbf{l}}')\,L(\mathbf{r}', \hat{\mathbf{l}}')(\hat{\mathbf{N}}, \hat{\mathbf{l}}')d\hat{\mathbf{l}}' = L(\mathbf{r}, \hat{\mathbf{l}})\,\Big|\,\hat{\mathbf{l}} \in \Omega_{\pm} , \quad (3)$$

where $\Omega_{\pm}$ are the lower and upper hemispheres, respectively; $L(z, \mathbf{r}, \hat{\mathbf{l}})$ is the brightness of the light field at the point $(z, \mathbf{r})$ in the direction $\hat{\mathbf{l}}$; $\boldsymbol{A}$ is the operator of diffuse reflection by the underlying surface; $\boldsymbol{R}_{\pm}$ and $\boldsymbol{T}_{\pm}$ are the reflection and transmission operators of the RRI in the upper and lower hemispheres, respectively; $\hat{\mathbf{N}} = \hat{\mathbf{N}}(\mathbf{r})$ is the normal to the RRI. Below the argument z is omitted when it does not lead to misunderstanding.

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The local reflectance and the refractive index of the RRI have the form

$$\rho(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}') = \frac{\pi}{(\hat{\mathbf{l}}', \hat{\mathbf{N}}')} \delta(\hat{\mathbf{l}} - \hat{\mathbf{l}}' - 2\hat{\mathbf{N}} (\hat{\mathbf{N}}, \hat{\mathbf{l}}')) \rho_F(\hat{\mathbf{l}}', \hat{\mathbf{N}}) ; \quad (4)$$

$$\tau(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}') = \frac{\pi}{(\hat{\mathbf{l}}', \hat{\mathbf{N}})} \delta\Bigg\{\hat{\mathbf{l}} - \frac{n'}{n}\hat{\mathbf{l}}' - $$

$$- \left[\hat{\mathbf{N}} \sqrt{1 - \left(\frac{n'}{n}\right)^2 [1 - (\hat{\mathbf{N}}, \hat{\mathbf{l}}')^2]} - \frac{n'}{n} \hat{\mathbf{N}}(\hat{\mathbf{N}}, \hat{\mathbf{l}}')\right]\Bigg\} \tau_F(\hat{\mathbf{l}}', \hat{\mathbf{N}}), \quad (5)$$

where ρ_F and τ_F are the corresponding Fresnel coefficients; $\delta(\cdot)$ is the Dirac delta function; n' and n are the refractive indices of the atmosphere and ocean.

Let us use the following notation:

$$\begin{aligned} \Gamma_1 &= \{(z, \mathbf{r}, \hat{\mathbf{l}}) : z = 0, \hat{\mathbf{l}} \in \Omega_+\} , \\ \Gamma_{2+} &= \{(z, \mathbf{r}, \hat{\mathbf{l}}) : z = h, \hat{\mathbf{l}} \in \Omega_+\} , \\ \Gamma_{2-} &= \{(z, \mathbf{r}, \hat{\mathbf{l}}) : z = h, \hat{\mathbf{l}} \in \Omega_-\} , \\ \Gamma_3 &= \{(z, \mathbf{r}, \hat{\mathbf{l}}) : z = z + h, \hat{\mathbf{l}} \in \Omega_-\} . \end{aligned} \quad (6)$$

The radiative transfer equation (RTE) is valid for the layers

$$\boldsymbol{D}L = \boldsymbol{S}L , \quad (7)$$

where $\boldsymbol{D}L = (\hat{\mathbf{l}}, \nabla)L + \varepsilon(z)L$ is the differential transfer operator; $\boldsymbol{S}L = \frac{\sigma(z)}{4\pi} \oint x(z, \hat{\mathbf{l}}, \hat{\mathbf{l}}')L(z, \mathbf{r}, \hat{\mathbf{l}}'')\, d\hat{\mathbf{l}}'$ is the operator of scattering; ε and σ are the extinction and scattering coefficients of the medium.

The corresponding boundary conditions have the form

$$\left.\begin{aligned} L^a|_{\Gamma_1} &= \Phi_0\omega_S , \\ L^a|_{\Gamma_{2-}} &= \boldsymbol{R}_+L^a + \boldsymbol{T}_-L^0 , \end{aligned}\right\} \quad (8)$$

$$\left.\begin{aligned} L^0|_{\Gamma_{2+}} &= \boldsymbol{T}_+L^a + \boldsymbol{T}_-L^0 , \\ L^0|_{\Gamma_3} &= \boldsymbol{A}L^0 , \end{aligned}\right\} \quad (9)$$

where L^a and L^0 are the brightnesses of the light field in the atmosphere and ocean. Let us represent $L^a = I^a + D^a$ and $L^0 = I^0 + D^0$, where D^a and D^0 are the brightnesses of the hazes of the atmosphere and ocean and I^a and I^0 are the brightnesses caused by re–reflections from the RRI and underlying surface. The form of the RTE is the same for each component because of linearity. In this case we obtain the following system of the boundary–value problems:

$$\left.\begin{aligned} D^a|_{\Gamma_1} &= \Phi_0\omega_S , \\ D^a|_{\Gamma_{2-}} &= 0 ; \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} I^a|_{\Gamma_1} &= 0 , \\ I^a|_{\Gamma_{2-}} &= \boldsymbol{R}_+(I + D^a) + \boldsymbol{T}_-(I^0 + D^0) ; \end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned} D^0|_{\Gamma_{2+}} &= \boldsymbol{T}_+(I^a + D^a) , \\ D^0|_{\Gamma_3} &= 0 ; \end{aligned}\right\} \quad (12)$$

$$\left.\begin{aligned} I^0|_{\Gamma_{2+}} &= \boldsymbol{R}_-(I^0 + D^0) , \\ I^0|_{\Gamma_3} &= \boldsymbol{A}(I^0 + D^0) . \end{aligned}\right\} \quad (13)$$

Assuming that $I^0 = I^{\uparrow} + I_{\downarrow}$ let us transform boundary–value problem (13)

$$\left.\begin{aligned} I_{\downarrow}|_{\Gamma_{2+}} &= \boldsymbol{R}_-(D^0 + I^0) , \\ I_{\downarrow}|_{\Gamma_3} &= 0 ; \end{aligned}\right\} \quad (14)$$

$$\left.\begin{aligned} I^{\uparrow}|_{\Gamma_{2+}} &= 0 , \\ I^{\uparrow}|_{\Gamma_3} &= \boldsymbol{A}(D^0 + I^0) . \end{aligned}\right\} \quad (15)$$

The system of boundary–value problems (10)–(12), (14), and (15) is solved by the method of the Green's functions. Below the operation of spatial and angular superposition is denoted by the symbol "○". Let us assume that

$$D^a = \Phi_0\, l^a \circ \omega_S(\mathbf{r}, \hat{\mathbf{l}}) , \quad I^a = l^a_q \circ (\boldsymbol{R}_+D^a + \boldsymbol{T}_-L^0) ; \quad (16)$$

where $l^a = l^a(\mathbf{r}, \hat{\mathbf{l}} \to \mathbf{r}', \hat{\mathbf{l}}')$, $\mathbf{l}^a_\rho = \mathbf{l}^a_\rho(\mathbf{r}, \hat{\mathbf{l}} \to \mathbf{r}', \hat{\mathbf{l}}')$ are some functions. From Eqs. (10) and (11) on account of Eq. (16) we obtain the boundary–value problems for l^a and $\mathbf{l}^a_\rho$

$$\left.\begin{aligned} l^a|_{\Gamma_1} &= \delta(\mathbf{r} - \mathbf{r}')d(\hat{\mathbf{l}} - \hat{\mathbf{l}}') , \\ l^a|_{\Gamma_{2-}} &= 0 ; \end{aligned}\right\} \quad (17)$$

$$\left.\begin{aligned} l^a_\rho|_{\Gamma_1} &= 0 , \\ l^a_\rho|_{\Gamma_{2-}} &= \delta(\mathbf{r} - \mathbf{r}')d(\hat{\mathbf{l}} - \hat{\mathbf{l}}') + \boldsymbol{R}_+\mathbf{l}^a_\rho \end{aligned}\right\} . \quad (18)$$

Thus, Eq. (17) corresponds to the boundary–value problem for the point unidirectional source (PU source) in the atmosphere, while Eq. (18) – to the PU source in the atmospheric layer with the reflecting RRI.

Considering the perturbations caused by the RRI to be small, we expand l^a_ρ in a series of the perturbation theory in terms of the multiplicity of reflections from the RRI $l^a_\rho = \sum_{n=0}^{\infty} l^{(n)}_\rho$. In this case the relation $l^{(n)}_\rho = \boldsymbol{R}_+ l^{(n-1)}_\rho$ is valid for the boundary conditions. In addition, $l^0_\rho = \delta(\mathbf{r} - \mathbf{r}')\delta(\mathbf{l} - \mathbf{l}')$ for $n = 0$. As a result we obtain

$$l^a_\rho = \sum_{n=0}^{\infty} l^{(n)}_\rho = \sum_{n=0}^{\infty} (l^a \circ \boldsymbol{R}_+)^n\, l^a , \quad (19)$$

where boundary–value problem (17) corresponds to the Green's function l^a.

In analogy with Eq. (10) and under the assumption that $D^0 = l^0 \circ \boldsymbol{T}_+ I^a$ boundary–value problem (12) can be reduced to the superposition of the RTE with the Green's function $l^0 = l^0(\mathbf{r}, \hat{\mathbf{l}} \to \mathbf{r}', \hat{\mathbf{l}}')$ for the PU source in the ocean.

In analogy with the transformations made for Eq. (18) we derive from Eqs. (14) and (15)

$$I_{\downarrow} = l_1 \circ (\boldsymbol{R}\_D^0 + \boldsymbol{R}\_l_2(\boldsymbol{A}D^0 + \boldsymbol{A}I_{\downarrow}))\,,$$
$$I^{\uparrow} = l_2 \circ (\boldsymbol{A}D^0 + \boldsymbol{A}l_1 \circ (\boldsymbol{R}\_D^0 + \boldsymbol{R}\_I^{\uparrow}))\,, \tag{20}$$

where $l_1 = \sum\limits_{n=0}^{\infty} (l^0 \circ \boldsymbol{R}\_)^n l^0$, $l_2 = \sum\limits_{n=0}^{\infty} (l^0 \circ \boldsymbol{A})^n\, l^0$.

Correspondingly, solving the system of Eqs. (20) we obtain

$$\left.\begin{aligned} I_{\downarrow} &= \sum_{n=0}^{\infty} (l_1 \circ \boldsymbol{R}\_l_2 \circ \boldsymbol{A})^n l_1 \circ \boldsymbol{R}\_(1 + l_2 \circ \boldsymbol{A})D^0\,, \\ I^{\uparrow} &= \sum_{n=0}^{\infty} (l_2 \circ \boldsymbol{A}l_1 \circ \boldsymbol{R}\_)^n l_2 \circ \boldsymbol{A}(1 + l_1 \circ \boldsymbol{R}\_)D^0\,. \end{aligned}\right\} \tag{21}$$

Taking into account that $L^0 = I^{\uparrow} + I_{\downarrow} + D^0$ on the basis of Eqs. (16) and (21) we have

$$L^a = I^a + D^a = (1 + l^a_{\rho} \circ \boldsymbol{R}_+)D^a + l^a_{\rho} \circ \boldsymbol{T}\_\boldsymbol{O}I^a\,, \tag{22}$$

where $\boldsymbol{O}$ is the operator of the radiation transfer through the RRI and ocean layer

$$\boldsymbol{O} = \Bigg[1 + \sum_{n=0}^{\infty} (l_1 \circ \boldsymbol{R}\_l_2 \circ \boldsymbol{A})^n l_1 \circ \boldsymbol{R}\_(1 + l_2 \circ \boldsymbol{A}) +$$
$$+ \sum_{n=0}^{\infty} (l_2 \circ \boldsymbol{A}l_1 \circ \boldsymbol{R}\_)^n l_2 \circ \boldsymbol{A}(1 + l_1 \circ \boldsymbol{R}\_)\Bigg] l^0 \circ \boldsymbol{T}_+\,.$$

The solution of integral equation (22) can be represented by the Neumann series

$$L^a = \sum_{n=0}^{\infty} (l^a_{\rho} \circ \boldsymbol{T}\_\boldsymbol{O})^n (1 + l^a_{\rho} \circ \boldsymbol{R}_+)\, l^a \circ \omega_S. \tag{23}$$

On the basis of the constructed optical transfer operator of the ocean with allowance made for the RRI, series (23) makes it possible to analyze the contribution of the individual components to a random realization of the resulting brightness distribution over the input pupil of the OES using the Green's functions of the RTE obtained preliminary for the atmosphere and ocean.

Based on Eq. (23) we can obtain the relations for any multiplicity of the radiation re–reflection from the RRI and underlying surface. For simplicity we take into consideration only those terms of the expansion in perturbations which are shown in Fig. 2, since for wide class of applied problems the reflection from the RRI and backscattering in the atmosphere and ocean can be assumed to be negligible.[2–4,10] In this case we have

$$L^a = D^a + D^0 + B + S\,, \tag{24}$$

where

$$D^a = \Phi_0\, l^a \circ \omega_S\,,\quad D^0 = \Phi_0\, l^a \circ T\_l^0 \circ \boldsymbol{T}_+ l^a \circ \omega_S\,,$$
$$B = \Phi_0\, l^a \circ \boldsymbol{R}_+ l^a \circ \omega_S\,,\quad S = \Phi_0\, l^a \circ \boldsymbol{T}\_l^0 \circ \boldsymbol{A}l^0 \circ \boldsymbol{T}_+ l^a \circ \omega_S. \tag{25}$$

Here B denotes the glint reflections of radiation from the RRI and S is the valid signal.

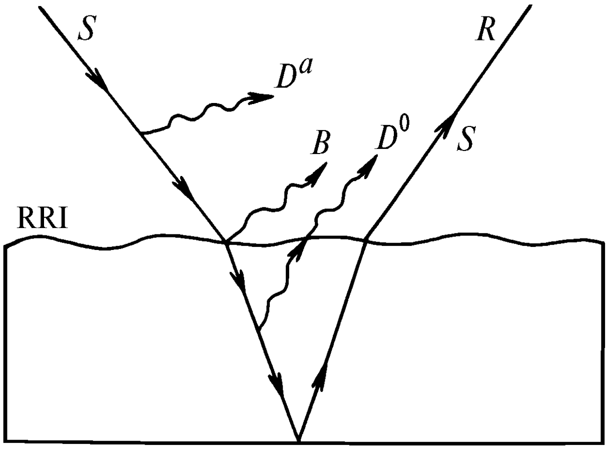

FIG. 2. Structure of the optical signal in the OES of observation. Here D^a is the haze of the atmosphere, D^0 is the haze of the ocean, B is the glint reflection from the RRI, and S is the valid signal.

Based on the optical reciprocity theorem describing the relation between the volume l_V and surface l^0 Green's functions[13] we obtain

$$\int\limits_{\Omega_+} l^0(\mathbf{r}', \hat{\mathbf{l}}' \to \mathbf{r}, \hat{\mathbf{l}})\mathrm{d}\hat{\mathbf{l}}' = \int\limits_{X_+} l_V(\mathbf{r}', \hat{\mathbf{l}}' \to \mathbf{r}, \hat{\mathbf{l}})(\hat{\mathbf{l}}', \hat{\mathbf{z}})\mathrm{d}\hat{\mathbf{l}}' =$$
$$= e^0(\mathbf{r}' \to \mathbf{r}, \hat{\mathbf{l}})\,, \tag{26}$$

where $e^0(\mathbf{r}' \to \mathbf{r}', \hat{\mathbf{l}})$ is the Green's function of a point diffuse source in the ocean (PD source). Hence

$$l^0\boldsymbol{A}l^0 = \frac{1}{\pi}\int \rho(\mathbf{r}')\, e^0(\mathbf{r}' \to \mathbf{r}_1, \hat{\mathbf{l}}_1)\, e^0(\mathbf{r}' \to \mathbf{r}_2, \hat{\mathbf{l}}_2)\mathrm{d}^2\mathbf{r}' \equiv \boldsymbol{Q}\, e_1 e_2. \tag{27}$$

Taking into account the receiving aperture and after averaging over all possible realizations we obtain the average valid signal in the form

$$\langle P_R \rangle = \Phi_0 \boldsymbol{Q}\omega_R \circ l^a \circ e_1 \circ \langle \boldsymbol{T}\_\boldsymbol{T}_+ \rangle e_2 l^a \circ \omega_S =$$
$$= \Phi_0 \boldsymbol{Q}\omega_R \circ l^a \circ \boldsymbol{O}_1 \circ l^a \circ \omega_S = \Phi_0 \int \rho(\mathbf{r}')\omega_R(\hat{\mathbf{n}}_R \to \hat{\mathbf{l}}_R) \times$$
$$\times\, l^a(\mathbf{r}_2, \hat{\mathbf{l}}'_2 \to \mathbf{r}_R, \hat{\mathbf{l}}_R)\, \boldsymbol{O}_1(\mathbf{r}'; \mathbf{r}_1, \hat{\mathbf{l}}'_1 \to \mathbf{r}_2, \hat{\mathbf{l}}'_2) \times$$
$$\times\, l^a(\mathbf{r}_S, \hat{\mathbf{l}}'_S \to \mathbf{r}_1, \hat{\mathbf{l}}_1)\, \omega_S(\hat{\mathbf{n}}_S \to \hat{\mathbf{l}}_S)\mathrm{d}\hat{\mathbf{l}}_1 \mathrm{d}\hat{\mathbf{l}}_2 \mathrm{d}\hat{\mathbf{l}}_S \mathrm{d}\hat{\mathbf{l}}_R \mathrm{d}^2 r_1 \mathrm{d}^2 r_2, \tag{28}$$

where $\boldsymbol{O}_1 = e_1 \circ \langle \boldsymbol{T}\_\boldsymbol{T}_+ \rangle e_2$ is the first approximation of the operator of image transfer through the RRI and ocean layer while the angular brackets denote the operation of statistical averaging.

Let us assume that the field of slopes of the RRI obeys the normal distribution.[3] Taking into consideration that $n' = 1$ in the approximation of small incidence angles of radiation $((\hat{\mathbf{N}}, \hat{\mathbf{l}}') \approx 1$ in paraxial optics approximation) we obtain

$$\langle \boldsymbol{T}\_\boldsymbol{T}_+ \rangle = \frac{t^2 n^2}{\left(2\sigma^2\sqrt{1-\Gamma^2}\right)^2 (n-1)^4} \int \mathrm{d}\hat{\mathbf{l}}_2 \mathrm{d}\hat{\mathbf{l}}_1 \theta(\mathrm{d}\hat{\mathbf{l}}'_1 \to \mathbf{l}_1, \mathbf{l}_2 \to \hat{\mathbf{l}}'_2), \tag{29}$$

where

$$\theta(\hat{\mathbf{l}}'_1 \to \mathbf{l}_1, \mathbf{l}_2 \to \hat{\mathbf{l}}'_2) =$$

$$= \exp\left[-\frac{(n\mathbf{l}'_1 - \mathbf{l}_1)^2 + (n\mathbf{l}'_2 - \mathbf{l}_2)^2 - 2\Gamma(n\mathbf{l}'_1 - \mathbf{l}_1)(n\mathbf{l}'_2 - \mathbf{l}_2)}{2\sigma^2(1-\Gamma^2)(n-1)^2}\right]$$

is the characteristic two–point operator of the RRI; $t = \tau_F(\hat{\mathbf{l}}', \hat{\mathbf{N}}) \approx \tau_F(1)$ is the transmittance of the RRI; σ^2 and Γ are the variance and correlation coefficient of the slopes of the RRI, respectively; $\mathbf{l}_1$, $\mathbf{l}'_1$, $\mathbf{l}_2$, and $\mathbf{l}'_2$ are the projections of the corresponding unit vectors onto the horizontal plane.

For the media with anisotropic scattering and small optical depths it is convenient to solve the RTE in the small–angle approximation (SAA). All forms of the SAA are equivalent,[14] therefore, to make the subsequent analysis more convenient, let us take the SAA in the form[3,4]

$$e(\mathbf{r}' \to \mathbf{r}, \hat{\mathbf{l}}) = \int \Phi(z, \mathbf{k}) \exp[i\mathbf{k}(\mathbf{r}' - \mathbf{r})/z] d^2k , \quad (30)$$

where $\mathbf{l} = \mathbf{r}/z$ and the function $\Phi(z, \mathbf{k})$ has the form

$$\Phi(z, \mathbf{k}) = z^{-2}\exp\left\{\int\limits_z^0 [-\varepsilon(\zeta) + \sigma(\zeta)\, x(\zeta k/z)]d\zeta\right\} . \quad (31)$$

On account of Eqs. (29), (30), and (31) the relation for $\boldsymbol{O}_1$ assumes the form

$$\boldsymbol{O}_1(\mathbf{r}'; \mathbf{r}_1, \hat{\mathbf{l}}'_1 \to \mathbf{r}_2, \mathbf{l}'_2) = \frac{t^2}{n^2}\int \Phi(z, \mathbf{k}_1)\Phi(z, \mathbf{k}_2) \times$$

$$\times \exp\left[-\frac{\sigma^2}{2}\left(\frac{n-1}{n}\right)^2(\mathbf{k}_1^2 + \mathbf{k}_2^2 + 2\Gamma\mathbf{k}_1\mathbf{k}_2) - \frac{i}{z}(\mathbf{r}_1\mathbf{k}_1 + \mathbf{r}_2\mathbf{k}_2) + \right.$$

$$\left. + \frac{i}{z}\mathbf{r}'(\mathbf{k}_1 + \mathbf{k}_2) + \frac{i}{n}(\mathbf{k}_1\mathbf{l}'_1 - \mathbf{k}_2\mathbf{l}'_2)\right]d^2k_1d^2k_2 . \quad (32)$$

For subsequent calculations the atmospheric transmission is assumed to be much better than the ocean transmission and the baseline between the receiver and emitter is much shorter than the distance to the object, i.e.,

$$l_a \circ \omega_S = \omega_S(\mathbf{l}'_1)\, \delta(\mathbf{r}'_1 - h\mathbf{l}'_1) , \; l_a \circ \omega_R = \omega_R(\mathbf{l}'_2)\, \delta(\mathbf{r}'_2 - h\mathbf{l}'_2). \quad (33)$$

Relations (33) correspond to the third observation scheme of Ref. 4 in which the image is formed by means of simultaneous scanning of the directional patterns of the source and receiver.

Under the assumption of statistical uniformity of the field of the slopes of the RRI $\Gamma(\mathbf{r}_1, \mathbf{r}_2) = \Gamma(\mathbf{r}_1 - \mathbf{r}_2) = \Gamma(\rho)$ on account of Eqs. (32) and (33) we derive from Eq. (28)

$$\langle P_R(\mathbf{r}_0)\rangle = \int \rho(\mathbf{r}')\, f(\mathbf{r}_0 - \mathbf{r}')d^2r' , \quad (34)$$

where

$$f(\mathbf{r}') = \frac{t^2\Phi_0}{n^2h^2}\int \Phi\left(z, \mathbf{K} + \frac{\mathbf{k}}{2}\right)\Phi\left(z, \mathbf{K} - \frac{\mathbf{k}}{2}\right)\omega_S\left(\mathbf{U} + \frac{H}{z}\mathbf{K}\right) \times$$

$$\times\, \omega_R\left(\mathbf{U} - \frac{H}{z}\mathbf{K}\right) \exp\left[-\frac{i}{z}2\mathbf{K}\mathbf{r}' - \frac{\sigma^2}{2}\left(\frac{n-1}{n}\right)^2 \times\right.$$

$$\left. \times \left[(1+\Gamma)\,\mathbf{K}^2 + (1-\Gamma)\frac{\mathbf{k}}{4}\right] - \frac{i}{2h}\rho\left(\frac{H}{z}\mathbf{k} + 2\mathbf{U}\right)\right] \times$$

$$\times\, d^2K\, d^2k\, d^2U\, d^2\rho \quad (35)$$

is the point spread function of the OES of observation through the turbid layer with allowance made for the RRI; $H = h + z/n$ is the reduced height on account of the refraction; $\mathbf{r}_0 = \mathbf{r}_0(\hat{\mathbf{l}}_S, \hat{\mathbf{l}}_R)$ is the coordinate of the sighting point in the object plane.

Let us introduce an optical transfer function (OTF) of the OES of observation

$$F(\mathbf{p}) = \int f(\mathbf{r}')\exp(i\mathbf{r}'\mathbf{p}/H)d^2\mathbf{r}' = \frac{z^2t^2\Phi_0}{n^2h^2} \times$$

$$\times \exp\left[-\frac{\sigma^2}{4}\left(\frac{n-1}{n}\right)^2\frac{H^2}{z^2}p^2\right]\int \Phi\left(z, \mathbf{x} + \frac{z}{2H}\mathbf{p}\right) \times$$

$$\times\, \Phi\left(z, \mathbf{x} - \frac{z}{2H}\mathbf{p}\right)\omega_S(\mathbf{y})\, \omega_R(\mathbf{y} - \mathbf{p}) \times$$

$$\times \exp\left[-\frac{\sigma^2}{4}\left(\frac{n-1}{n}\right)^2(1-\Gamma)\mathbf{x}\left(\mathbf{x} - \frac{z}{H}\mathbf{p}\right) - \frac{iH}{zh}\rho\left(\mathbf{x} - \frac{z}{H}\mathbf{y}\right)\right] \times$$

$$\times\, d^2\rho d^2x d^2y , \quad (36)$$

where $\mathbf{x} = \frac{1}{2}[z\mathbf{p}/H - \mathbf{k}]$, $\mathbf{y} = \mathbf{U} + \mathbf{p}/2$, and $\mathbf{p}$ is the angular spatial frequency.

Let us analyze the OTF of the system. When the plane wave is incident on the RRI $\omega_S(\cdot) = \delta(\cdot)$, Eq. (36) is equivalent to the well–known phenomenological expression[3,4] for observing the objects under conditions of solar illumination. Let us assume that the observation is carried out by means of the ideal electro–optical image converter (EOIC) with $\omega_R(\cdot) = 1$ for uncorrelated swell, i.e., for $\Gamma(\rho) = 0$. Then

$$F(\mathbf{p}) = \frac{z^4t^2\Phi_0}{n^2H^2}\exp\{-\sigma_M^2\rho^2\}\, \Phi(z, 0)\, \Phi\left(z, \mathbf{p}\frac{z}{H}\right) ,$$

$$\sigma_M^2 = \frac{s^2}{2}\left(\frac{n-1}{n}\right)^2\frac{H^2}{z^2} , \quad (37)$$

which corresponds to the well–known formula for the OTF for observation through the ocean surface. This formula was obtained for the first time by Yu.–A.R. Mullamaa.[1]

In applied calculations it is convenient to approximate the spectra of the real scattering phase functions of the radiation of the source and the directional patterns of the receiver by the Gaussian functions $\omega_S(\mathbf{y}) = \exp(-\omega_0^2\mathbf{y}^2/2)$ and $\omega_R(\mathbf{y}) = \exp(-\Omega_0^2\mathbf{y}^2/2)$ and represent the coefficient of the swell correlation in the form $\Gamma(\rho) = 1$ for $|\rho| \le \rho_0$ and $\Gamma(\rho) = 0$ for $|\rho| \ge \rho_0$, where ρ_0 is the effective length of the swell correlation. This representation is physically incorrect but it does not distort the final results in the case of appropriate choice of ρ_0 and is generally accepted in the literature.[2–4] In the cases $\Omega_0 \ll \omega_0$ or $\Omega_0 \gg \omega_0$ most often encountered in practice after corresponding transformations we finally obtain for the OTF of the OES

$$F(\mathbf{p}) = \frac{z^4 t^2 \Phi_0}{n^2 H^2} \exp\left[-\frac{\gamma^2}{2} p^2 - \sigma_M^2 p^2 \right] \Phi(z,\,0)\,\Phi(z,\,z\mathbf{p}/H) \times$$

$$\times \left\{ 1 + \int \left[1 - \exp\left(-2\sigma_M^2 \mathbf{x}(\mathbf{x}-\mathbf{p})\right) \right] \frac{\Phi(z,\,z\mathbf{x}/H)}{\Phi(z,\,0)} \times \right.$$

$$\left. \times \frac{\Phi(z - z(\mathbf{x}-\mathbf{p})/H)}{\Phi(z,\,z\mathbf{p}/H)}\, q(\mathrm{d}\,|\mathbf{x}-\mathbf{p}|\,\mathrm{d}^2x) \right\}, \qquad (38)$$

where $\dfrac{1}{\gamma^2} = \dfrac{1}{\omega_0^2} + \dfrac{1}{\Omega_0^2}$,

$$q(x) = \int\limits_0^{\rho_0/hd} \exp(-0.5\xi^2)\, J_0(\xi x)\xi \mathrm{d}\xi\,,\quad \mathrm{d} = \sqrt{\Omega_0^2 + \omega_0^2}\,,$$

and J_0 is the zero order Bessel function.

Analysis of Eq. (38) shows that the OTF of the active OES of observation can be represented in the most general form by the sum of two terms, the first of which is the product of the OTF of the ocean and the OTF of the RRI, while the second term is associated with the radiation correlation on the RRI and is nonlinearly dependent on the corresponding OTF.

On the basis of Eq. (25) we can obtain the relation for the backscatter interference (BSI) recorded by the receiver with the directional pattern $\omega_R(\cdot)$

$$P_{\mathrm{bsi}} = \Phi_0 \omega_R \circ l^{\,a} \circ \boldsymbol{T}_- l^{\,0} \circ \boldsymbol{T}_+ l^{\,a} \circ \omega_S\,, \qquad (39)$$

where $l^{\,0}$ can be found from the boundary–value problem similar to Eq. (17).

The error in the solution of the radiative transfer equations in the small–angle approximation becomes large when taking into account the radiation scattering at large angles[14] (larger than 60°). Therefore, let us represent $l^{\,0}$ as a series of the perturbation theory in a small parameter of backscattering while the integral operator of the RTE – as the sum of "sharp" and "blunt" terms

$$l^0 = \sum_{n=0}^{\infty} \varepsilon^n l_{\mathrm{s}}^{(n)}\,,\ \boldsymbol{S} = \boldsymbol{S}_{\mathrm{s}} + \varepsilon \boldsymbol{S}_{\mathrm{b}}\,,\ \varepsilon \to 0\,. \qquad (40)$$

In this case the RTE is reduced to the system of coupled equations

$$\boldsymbol{D} l_{\mathrm{s}}^{(n)} = \boldsymbol{S}_{\mathrm{s}} l_{\mathrm{s}}^{(n)} + \boldsymbol{S}_{\mathrm{b}} l_{\mathrm{s}}^{(n-1)}\,,\ \boldsymbol{D} l_{\mathrm{s}}^{(0)} = \boldsymbol{S}_{\mathrm{s}} l_{\mathrm{s}}^{(0)}\,, \qquad (41)$$

and the RTE in the small–angle approximation corresponds to the zeroth term of the series.

The solution of Eq. (41) can be represented as the superposition of the Green's function of the homogeneous radiation transfer equation with the source function $\boldsymbol{S}_{\mathrm{b}} l_{\mathrm{s}}^{(n-1)}$, i.e.,

$$l_{\mathrm{s}}^{(n)} = l^{\,0} \circ \boldsymbol{S}_{\mathrm{b}} l_{\mathrm{s}}^{(n-1)} = (l^{\,0} \circ \boldsymbol{S}_{\mathrm{b}})^n l^{\,0}\,. \qquad (42)$$

In real media backscattering can be considered negligible which makes it possible to take into consideration only the first term of expansion $l_{\mathrm{s}}^1 = l^{\,0} \circ \boldsymbol{S}_{\mathrm{b}} l^{\,0}$. For isotropic backscattering ($x_{\mathrm{b}}(\hat{\mathbf{l}},\,\hat{\mathbf{l}}') = x_\pi$) this leads to the equation

$$l_{\mathrm{s}}^{(1)} = \frac{\sigma x_\pi}{4\pi} \int\int e^0(\mathbf{r}'\to\mathbf{r}_2,\,\hat{\mathbf{l}}_2)\, e^0(\mathbf{r}'\to\mathbf{r}_2,\,\hat{\mathbf{l}}_2)\mathrm{d}^2 r'\,\mathrm{d}z = \boldsymbol{CB} e^0 e^0, \qquad (43)$$

where $\boldsymbol{C} f = \int f(\cdot)\mathrm{d}z$ and $\boldsymbol{B} f = \dfrac{\sigma x_{\mathrm{p}}}{4\pi} \int f(\cdot)\mathrm{d}^2 r$ are the new operators.

Hence, after averaging we obtain for Eq. (39)

$$\langle P_{\mathrm{bsi}} \rangle = \Phi_0 \boldsymbol{CB} \omega_R \circ l^{\,a} \circ \boldsymbol{O}_1 \circ l^{\,a} \circ \omega_S\,. \qquad (44)$$

Action of the operator $\boldsymbol{B}$ will be equivalent to that of the operator $\boldsymbol{A}$ in Eq. (1) if we set $\rho = \sigma x/4\pi$ which leads to the relation

$$\langle P_{\mathrm{bsi}} \rangle = \Phi_0 \boldsymbol{CA} \omega_R \circ l^{\,a} \circ \boldsymbol{O}_1 \circ\ l^{\,a} \circ \omega_S \Big|_{\rho = \sigma x_\pi/4\pi}\,. \qquad (45)$$

For short optical baseline and transparent atmosphere Eq. (45) can be simplified

$$\langle P_{\mathrm{bsi}} \rangle = \Phi_0 \int\limits_0^z \boldsymbol{A} \omega_R \circ l^{\,a} \circ \boldsymbol{O}_1 \circ l^{\,a} \circ \omega_S \mathrm{d}z = \Phi_0 \frac{\sigma x_\pi}{4\pi} \int\limits_0^z F(z,\,0)\mathrm{d}z\,, \qquad (46)$$

i.e., the statistically average backscatter interference can be expressed through an integral of the OTF.

On the basis of Eq. (25) we obtain for the statistically average signal of glint reflection from the RRI recorded by the receiver

$$\langle P_g \rangle = \langle \omega_R \circ \boldsymbol{B} \rangle = \omega_R \circ l^{\,a} \circ \langle \boldsymbol{R}_+ \rangle l^{\,a} \circ \omega_s\,, \qquad (47)$$

where

$$\langle \boldsymbol{R}_+ \rangle = \int\limits_{\Omega_+} w_1(\hat{\mathbf{N}})\rho_F(\hat{\mathbf{N}}_0,\,\hat{\mathbf{l}}')\mathrm{d}\hat{\mathbf{l}}'\,, \qquad (48)$$

$w_1(\hat{\mathbf{N}}_0)$ is the single–point function of the distribution of the slopes of the RRI, and $\hat{\mathbf{N}}_0$ is determined from the condition $\hat{\mathbf{l}} - \hat{\mathbf{l}}' - 2(\hat{\mathbf{N}}_0(\hat{\mathbf{N}}_0,\,\hat{\mathbf{l}}')) = 0$.

For the short optical baseline and transparent atmosphere and for observation with the help of an ideal electro–optical image converter Eq. (47) assumes the form

$$\langle P_g \rangle = h^{-2} \Phi_0 \omega_S(\hat{\mathbf{l}}_R) w_1(\hat{\mathbf{N}}_0) \rho_F(\hat{\mathbf{N}}_0 \hat{\mathbf{l}}_R)\,,$$

$$\hat{\mathbf{N}}_0 = (\hat{\mathbf{l}}_S - \hat{\mathbf{l}}_R)/|\hat{\mathbf{l}}_S - \hat{\mathbf{l}}_R|\,, \qquad (49)$$

which is equivalent to the well–known relation presented in Ref. 15.

To illustrate the possibilities of the method we carried out the calculations of the basic characteristics of the image transfer in the active pulsed opto–electronic system of observation. Figure 3 shows the curves of brightness distributions over the input pupil of the OES in observation of the disk with a radius of 1 m at a depth of $z = 0.5$ m from different altitudes h in the atmosphere on account of the backscatter interference, glint reflection, and swell variance σ^2. We considered the cases of observation in the positive and negative contrasts at different depths z_{st} of the strobe of illumination.

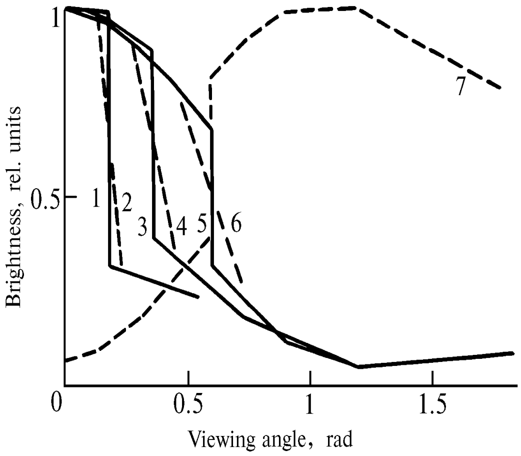

FIG. 3. Normalized brightness distribution over the input pupil of the OES. z_{st} = 0.01 m: *1)* h = 100 m, σ^2 = 0; *2)* h = 100 m, σ^2 = 0.2; *3)* h = 50 m, σ^2 = 0; *4)* h = 50 m, σ^2 = 0.2; *5)* h = 30 m, σ^2 = 0; *6)* h = 30 m, σ^2 = 0.2, and z_{st} = 0.51 m, and *7)* h = 30 m, σ^2 = 0.

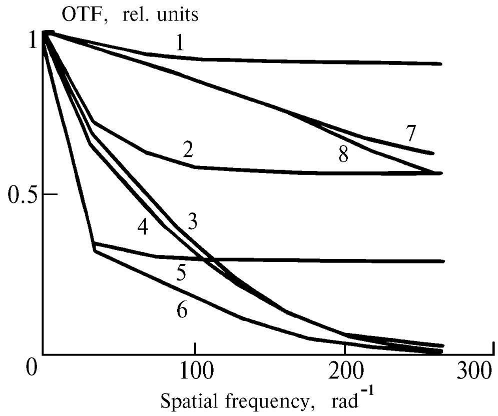

FIG. 4. Optical transfer function of the OES of observation. h = 30 m. *Narrow beam: 1)* z = 1 m, σ^2 = 0, ρ_0 = 0; *2)* z = 3 m, σ^2 = 0, ρ_0 = 0; *3)* z = 3 m, σ^2 = 0.2, ρ_0 = 0; *4)* z = 3 m, σ^2 = 0.2, ρ_0 = 0.5; *7)* z = 1 m, σ^2 = 0.2, ρ_0 = 0; *8)* z = 1 m, σ^2 = 0.2, ρ_0 = 0.5. *Plane wave: 5)* h = 3 m, σ^2 = 0 and *6)* z = 3 m, σ^2 = 0.2.

The OTF of OES of observation as functions of the depth, swell variance, and correlation length ρ_0 with illumination by a plane wave and narrow light beam with a divergence of 3° are shown in Fig. 4. The extinction and scattering coefficients were equal to 1 and 0.6 m^{-1}, respectively; the refractive index of water was 1.33. The calculations were performed for the Heneye–Greenstein scattering phase function with g = 0.97 and optical baseline of 0.5 m.

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