# Accuracy of estimates by the method of variational spectrum accumulation of the wind velocity in turbulent atmosphere from lidar data

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The accuracy of retrieval of the mean wind from lidar data by the method of variational spectrum accumulation (VSA) under conditions of wind-induced turbulence is studied. The analysis is performed by numerical simulation using the developed Virtual Instruments information-expert system. The method for simulation of inhomogeneous with height turbulent wind fields is described. It is shown that under conditions of wind-induced turbulence the accuracy of the VSA method decreases as compared to the case without velocity fluctuations, and, consequently, the height, up to which the mean wind can be retrieved from the spaceborne Doppler lidar data with the accuracy acceptable for further usage in weather-forecast models, decreases. At the same time, the accuracy of the VSA method turns out to be much higher as compared to that of the traditional method of wind velocity determination with prior estimation of the radial velocity.

## Introduction

The method of variational spectrum accumulation (VSA) for retrieving the vertical profiles of wind speed and direction directly from data of scanning coherent Doppler lidar without prior estimation of the radial velocity was proposed in Refs. 1 and 2. If the wind speed and direction are calculated as usual, from the array of estimated radial speed values obtained from return signals produced by separate sensing pulses at scanning, then, as was shown by numerical simulation,[1,2] the vertical profiles of the wind speed and direction can be retrieved with the acceptable accuracy only up to the altitudes of 2–3 km above the Earth's surface. With height, the aerosol concentration decreases (in a cloudless atmosphere), the signal-to-noise ratio decreases too, and this makes this method almost completely inapplicable to wind retrieval.

The VSA method assumes prior averaging of Doppler spectra over some spatial volume that results in the noise suppression and the signal at the Doppler frequency is summed up. Numerical simulation[2] shows that for a scanning spaceborne lidar operated at 2-μm wavelength and having pulse energy of 0.5 J, the pulse repetition frequency of 10 Hz, and the telescope diameter of 70 cm, under conditions with no turbulence in a cloudless atmosphere, the VSA method retrieves the wind speed accurate to 2 m/s and the wind direction to 20° in the atmospheric layer up to 20 km thick. This accuracy meets the demands of weather forecast services to wind sensing data for their further use in weather forecast models.

This paper presents analysis of the VSA accuracy in the presence of turbulent fluctuations of wind velocity.

## Algorithm for simulation of wind-induced turbulence

The effect of turbulence on the accuracy of the VSA method as applied to estimation of wind speed and direction was studied through numerical simulation using the Virtual Instrument information-expert computer system[3] developed to analyze the possibility of retrieving the wind velocity field from Doppler lidar measurements. The Virtual Instrument system consists of several blocks and enables simulating echo signal of a ground-based, airborne, or spaceborne scanning Doppler lidar.

The first block contains the initial information about the lidar parameters, sensing geometry, and the atmosphere. Virtual Instrument uses the real data of the German Weather Service (GWS) on the global distribution of atmospheric parameters in the period of January 19–30, 1998, what allows the capabilities of lidar wind sensing in the atmosphere under nearly realistic conditions to be assessed.

The second block simulates the process of lidar sensing. Its output data are realizations of the photocurrent arising in the lidar recording electronics.

In the third block, the data simulated are processed in order to obtain the vertical profiles of the radial wind velocity component with the preset height resolution. Then the calculated vertical profiles of the wind speed and direction are accumulated in the output block and can be compared with the initial GWS wind data. From this comparison one can judge on the efficiency of lidar measurements of wind fields.

In the atmospheric block, the GWS data on the global distribution of atmospheric parameters are set on a grid with the spatial resolution of 1.125° (longitude) × 1.121° (latitude). Simulation of wind

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turbulence involves the data on the zonal $V_z(z_i)$ and meridional $V_m(z_i)$ components of the mean wind velocity (m/s), temperature $T(z_i)$ (K), as well as the coefficients of turbulent viscosity $K_m(z_i)$ and turbulent heat exchange $K_h(z_i)$ (m$^2$/s), where $z_i$ is the height from 0 to 30 km, for which the values of the weather parameters are set, $i$ = 1, 2, ..., 20.

The procedure for simulation of random realizations of the radial speed includes two stages:

1) Calculation of the vertical profiles of the variance of wind speed fluctuations $\sigma_u^2(z)$ and the integral scale of turbulence $L_u(z)$ from the GWS data.

2) Simulation of random realizations of the 2D wind field in the plane of propagation of the sensing pulse using Karman model of the spatial spectrum of turbulent inhomogeneities.

In the atmospheric surface layer, the parameters $\sigma_u$ and $L_u$ are calculated by the turbulent energy balance equation. According to TKE (Turbulence Kinetic Energy) parameterization of the boundary layer,[4,5] for the dissipation rate of turbulent energy at the height $z_i$ in this case we have

$$\varepsilon(z_i) = K_m(z_i)\left[\left(\frac{dV_z}{dz}(z_i)\right)^2 + \left(\frac{dV_m}{dz}(z_i)\right)^2\right] - \frac{g}{\theta(z_i)} K_h(z_i) \frac{d\theta}{dz}(z_i), \quad (1)$$

where $\theta(z_i)$ is the potential temperature calculated from $T(z_i)$ through the adiabatic gradient,[6] $g$ is the acceleration due to gravity, in m$^2$/s. According to the theory of turbulence (see, for example, Refs. 7–9), the turbulent energy can be calculated as

$$E(z_i) = C_E\sqrt{\varepsilon(z_i)\, K_m(z_i)}, \quad (2)$$

where $C_E$ = 5.73 is an empiric constant. Hence, the variance (standard deviation) of fluctuations of the radial wind velocity component $\sigma_u(z_i)$ was calculated as

$$\sigma_u(z_i) = C_u\sqrt{E(z_i)}, \quad (3)$$

where $C_u$ = 1.04. According to Karman model of the turbulent spectrum, the integral scale of correlation is determined by the dissipation rate of the turbulent energy and the variance of velocity fluctuations:

$$L_u(z_i) = C_L\, \sigma_u^3(z_i)/\varepsilon(z_i), \quad (4)$$

where $C_L$ = 0.67. Equation (4) was used to find $L_u(z_i)$ from the calculated values of $\varepsilon(z_i)$ and $\sigma_u(z_i)$.

As known, normally the turbulent energy $E$ (and $\sigma_u$) in the atmospheric surface layer only slightly varies with height, while the outer scale increases proportionally to the height. Therefore, for the surface layer the following approximate equations were used to estimate the sought parameters:

$$\sigma_s = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \sigma_u^2(z_i))} \quad (5)$$

and

$$L_s = C_s z, \quad (6)$$

where $C_s = \frac{1}{n}\sum_{i=1}^{n} \frac{L_u(z_i)}{z_i}$; $\sigma_s^2$ is the variance, and $L_s$ is the correlation scale for the surface layer $z_i \in [0, h_s]$; $h_s$ is the height of the surface layer. To estimate $\sigma_s$ and $L_s$, we used the GWS data for three lower altitudes.

As the altitude in the boundary layer increases, the effect of air flow friction against the underlying surface becomes weaker, and for $z >> h_b$, where $h_b$ is the height of the atmospheric boundary layer, the friction effect can be neglected. In the free atmosphere at the altitudes $z >> h_b$, the turbulent energy is generated by other sources, and, as shown by the experimental data,[8] the turbulence intensity $\sigma_u(z)/U(z)$, where $U(z) = \sqrt{V_z^2(z) + V_m^2(z)}$, for wind velocity fluctuations with the spatial scales not exceeding 10 km is about 0.05. Parametric fitting of the measured spatial spectra of the turbulent wind field to the Karman model (9) gives the estimate of 500 m for the integral scale of velocity correlation in the free atmosphere.[8] Based on this data, we used for estimation of the vertical profiles of $\sigma_u^2$ and $L_u$ the following simple model:

$$\sigma_u^2(z) = \sigma_s^2 e^{-2(z/h_b)} + \sigma_f^2\,(1 - e^{-2(z/h_b)}), \quad (7)$$

$$L_u(z) = C_s z/(1 + C_s z/L_f), \quad (8)$$

where $\sigma_f^2 = (\gamma U(z))^2$ is the variance; $L_f$ is the integral scale of the velocity correlation in the free atmosphere, $\gamma$ = 0.05; $L_f$ = 500 m. The height of the boundary layer $h_b$ depends on the thermal stratification of the atmosphere and is assessed from the GWS data.

Random realizations of the radial speed were simulated using the spectral method.[10,11] It was assumed that the turbulent wind field is isotropic and its spatial spectrum is described by the Karman model.[9] According to this model, the 1D spatial spectra of the longitudinal $S_u(z, \kappa_z)$ and lateral $S_u(z, \kappa_x)$ wind velocity components and the 2D spatial spectrum of $S_u(z, \kappa_z, \kappa_x)$ can be written as

$$S_u(z, \kappa_z) = \frac{2\sigma_u^2(z)\, L_u(z)}{\{1 + [8.43L_u(z)\kappa_z]^2\}^{5/6}}, \quad (9)$$

$$S_u(z, \kappa_x) = \frac{\sigma_u^2(z)\, L_u(z)}{\{1 + [8.43L_u(z)\kappa_x]^2\}^{5/6}} \times \left[1 + \frac{5}{3}\frac{(8.43L_u(z)\,\kappa_x)^2}{1 + [8.43L_u(z)\kappa_x]^2}\right], \quad (10)$$

$$S_u(z, \kappa_z, \kappa_x) = \frac{1}{6\pi}\frac{\sigma_u^2(z)\,[8.43L_u(z)]^2}{\{1 + [8.43L_u(z)]^2(\kappa_z^2 + \kappa_x^2)\}^{4/3}} \times \left[1 + \frac{8}{3}\frac{[8.43L_u(z)\,\kappa_x]^2}{1 + [8.43L_u(z)]^2(\kappa_z^2 + \kappa_x^2)}\right]. \quad (11)$$

In the simulation by Eqs. (9)–(11), we used the parameters $\sigma_u^2(z)$ and $L_u(z)$ calculated by Eqs. (7) and (8).

The vertically inhomogeneous (along the coordinate $z$) random wind field $\tilde{V}(z)$ was simulated by the following scheme. First the array of $\tilde{V}_0(z_k)$ values, where $z_k = z_0 + \Delta z k$, $k = 0, 1, 2,\ldots, N_z$, was simulated for the constant values $\sigma_u^2(z) = \sigma_{u0}^2$ and $L_u(z) = L_{u0}$. Then it was corrected:

$$\tilde{V}(z_k) = [\sigma_u(z)_k/\sigma_{u0}]\ \tilde{V}(\hat{z}_k),$$

where

$$\hat{z}_k = (z_0 + \Delta z k)\ L_u(z_k)/L_{u0};$$

$\sigma_{u0} = \sigma_u(z_0)$; $L_{u0} = L_u(z_0)$; $z_0$ is the initial height.

## Effect of turbulent velocity fluctuations on the accuracy of the VSA method

The effect of wind field turbulence on the accuracy of the VSA method was analyzed for the case of a spaceborne lidar. The scanning geometry and lidar parameters can be found in Ref. 2.

Let $U$ and $\theta_V$ be the wind speed and direction. Then for the radial wind velocity component $V_r(\theta_i)$ measured with a spaceborne lidar at the azimuth scanning angle $\theta_i$ we have

$$V_r(\theta_i) = U \cos\alpha \cos(\theta_i - \theta_V), \qquad (12)$$

where $\alpha$ is the angle between the laser beam and the Earth's surface. The task is to assess $U$ and $\theta_V$ from Doppler spectra in the chosen sensing volume. For this purpose, the 2D array of values of the function $F(U, \theta_V)$ is calculated taking into account Eq. (12) for arbitrary $U$ and $\theta_V$ (Ref. 2):

$$F(U, \theta_V) = \sum_{i=1}^{n} \sum_{k''=k'-\Delta k}^{k'+\Delta k} W\left(\theta_i, \frac{k''}{MT_s}\right), \qquad (13)$$

where

$$k' = \left[\frac{U}{\Delta V} \cos\alpha \cos(\theta_i - \theta_V)\right], \qquad (14)$$

$U = \Delta U l$, $\theta_V = \Delta\theta_V m$, ($l = 1, 2, \ldots$, $m = 1, 2, \ldots$), $\Delta U$ and $\Delta\theta_V$ are the speed and angular resolutions, the square brackets in Eq. (14) denote the integer part of a quantity; $W(\theta_i, k''/(MT_s))$ is the spectrum of the Doppler signal; $T_s^{-1}$ is the data reading (discretization) frequency; $MT_s$ is the time of data reading for estimation of the speed ($M$ is the sample size); $n$ is the number of pulses in the sensing volume; $\Delta V = (\lambda/2)\ \Delta f$ is the velocity resolution, $\lambda$ is the wavelength, $\Delta f = 1/(MT_s)$. The wind speed $U$ and direction $\theta_V$ are determined from the position of maximum of the function $F(U, \theta_V)$. Thus, using Eq. (13), we estimate the wind speed from measurement data averaged over $n$ sensing pulses and over the spectral window with the width $(2\Delta k + 1)/(MT_s)$. At $M = 1024$, $\lambda = 2$ μm, and $T_s = 10$ ns the speed resolution is $\Delta V = 0.1$ m/s. Numerical experiments showed that at this resolution $\Delta k = 4$ is the optimal value in Eq. (13).

Turbulent fluctuations of the wind speed lead to random shifts in the position of maximum of the Doppler spectrum and to its broadening.[12] As was shown in Ref. 13, for a pulsed lidar the effective mean width of the Doppler spectrum is well approximated by the equation

$$w_{\rm eff}^2 = w^2 + 4\sigma_{\rm turb}^2/\lambda^2, \qquad (15)$$

where $w^2$ is determined by the shape of the sensing pulse,

$$\sigma_{\rm turb}^2 = 2\sigma_u^2 \int_0^1 (1 - s)\ \Lambda(s\Delta p/L_u)\ ds, \qquad (16)$$

$$\Lambda(x) = 1.0 - 0.59x^{1/3} K_{1/3}(x), \qquad (17)$$

$K_{1/3}(x)$ is the modified Bessel function, $\Delta p = MT_s c/2$. It follows from Eqs. (15)–(17) that turbulence increases the statistical uncertainty in the position of the maximum of the Doppler spectrum and must lead to larger errors in estimation of the mean wind speed by the VSA method.

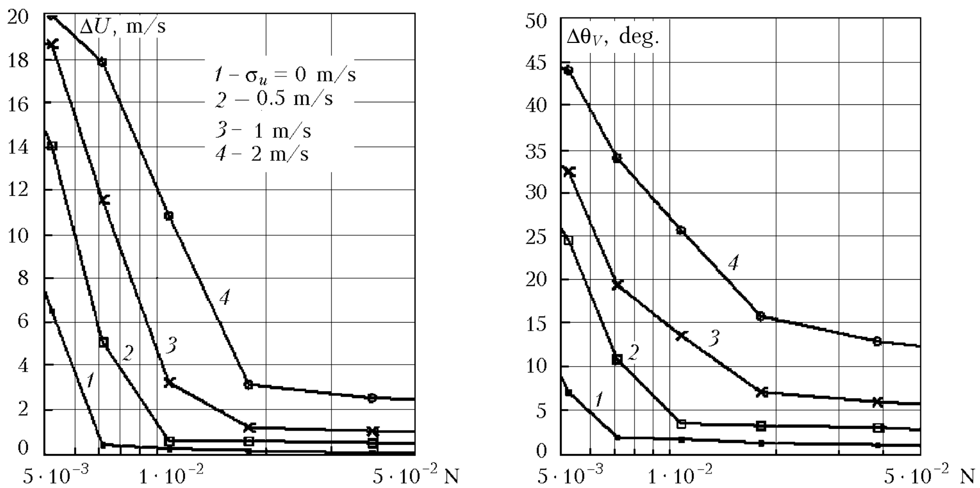


**Fig. 1.** Absolute error of wind velocity and direction retrieval from spaceborne Doppler lidar data vs. signal-to-noise ratio.

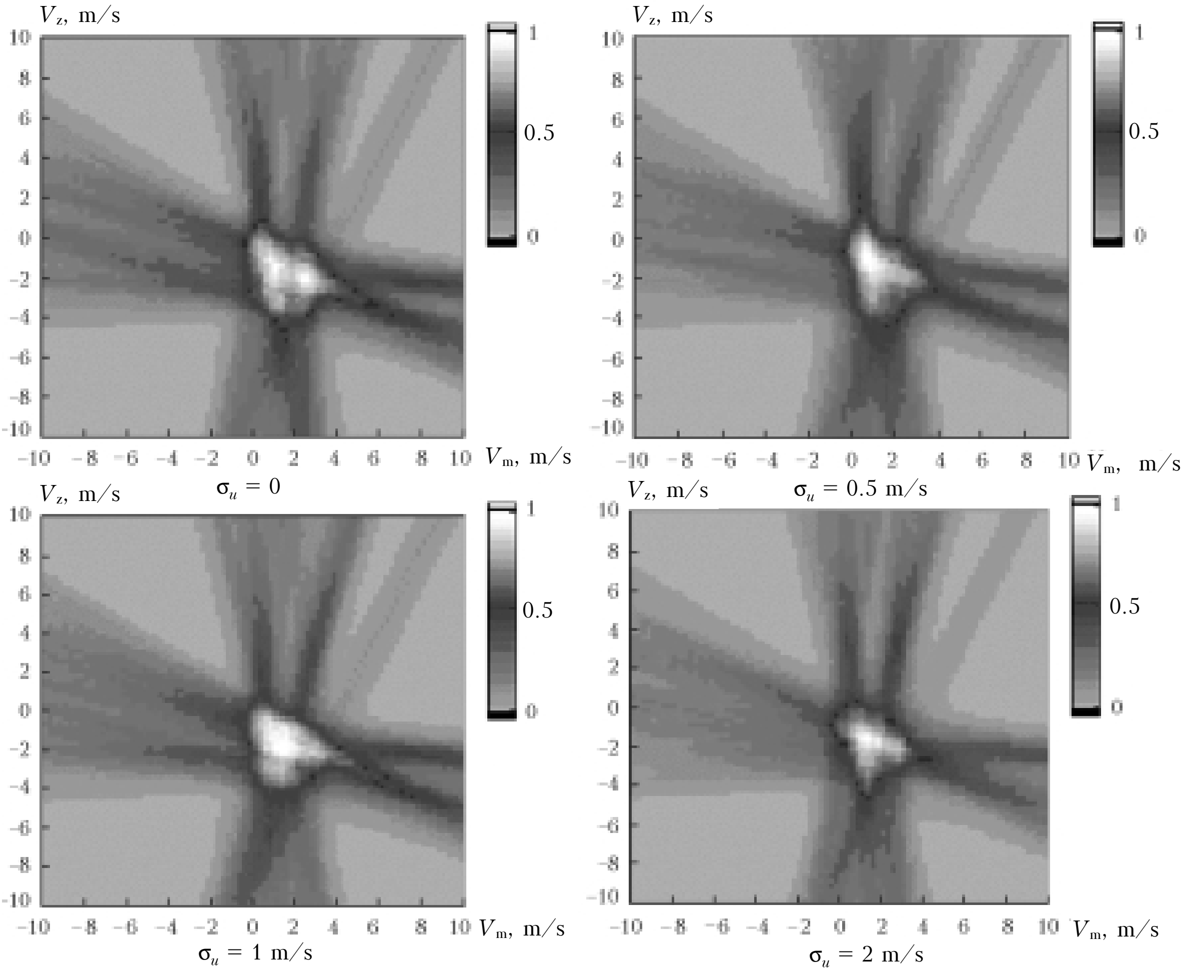


**Fig. 2.** Function $F(U, \theta)$ at different values of standard deviation of wind velocity. $V_m = U \cos \theta$, $V_z = U \sin \theta$, altitude $z = 7$ km.

As was shown in Ref. 2, if sensing is carried out with the frequency of 10 Hz, then in the case of scanning from a space platform, the maximum of 53 pulses fall within one spatial cell of the GWS meteorological network with the size of about 100 × 100 km. On the average, the number of pulses $n$ falling within a spatial cell and involved in summing in Eq. (13) is equal to 26.

Figure 1 depicts the calculated absolute errors of retrieval of the wind speed and direction at different values of the variance of velocity fluctuations vs. the signal-to-noise ratio for 20 pulses in one spatial cell. It follows from Fig. 1 that with no turbulence the retrieval by the VSA method with the accuracy no worse than 2 m/s is possible at the signal-to-noise ratio no lower than $7 \cdot 10^{-3}$. This corresponds to the altitude of about 8 km (see Fig. 5 in Ref. 2). (To retrieve the wind by the VSA method with good accuracy at higher altitudes, $n > 20$ is needed.[2]) At $\sigma_u^2 \neq 0$ the accuracy decreases, and at the velocity variance $\sigma_u^2 = 4$ m$^2$/s$^2$ the wind can be retrieved with the acceptable accuracy only for the signal-to-noise ratio no lower than $5 \cdot 10^{-2}$, which corresponds to the altitudes no higher than 4 km. Thus, turbulent velocity fluctuations can halve the accuracy of wind retrievals by the VSA method with the accuracy acceptable for using the obtained lidar data in the weather-prognostic models.

In some cases, the wind shifts are main sources of errors in determining the wind mean speed. The small-scale turbulence under these conditions may serve a smoothing factor, and in its presence the VSA method accuracy may be higher than in its absence. Just this situation is exemplified in Fig. 2, where two-dimensional distributions of the $F$ function at different values of the speed dispersion are presented. It is seen that thanks to the smoothing effect, the $F$ maximum at $\sigma_u \neq 0$ is less spread than at $\sigma_u = 0$; therefore, the determination of the mean speed and direction of wind at $\sigma_u = 2$ m/s can be much more accurate than at $\sigma_u = 0$.

Thus, the results presented suggest that turbulent fluctuations of the wind velocity lead to the increase of statistical uncertainty in the position of the maximum of the Doppler spectrum and the corresponding decrease of the accuracy of the VSA method. Under conditions of wind turbulence, the signal-to-noise ratio as if decreases, and the altitude, up to which the mean wind can be retrieved from the lidar data with the accuracy acceptable for use in weather forecast models, decreases as well. Nevertheless, the accuracy of the VSA method turns out to be much higher than that of the traditional method of mean wind determination with prior estimation of the radial velocity. Under conditions of wind shear, the small-scale turbulence may produce the smoothing effect. In this case, the accuracy of the VSA method in the presence of wind turbulence is higher than in its absence.

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