Parameterization of aerosol scattering phase functions

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We present the method developed for analytical approximation of aerosol scattering phase functions by using natural characteristics of aerosol microstructure (indices of refraction and disperse composition of aerosol) as the approximation parameters. Errors of such an approximation are shown to be within the limits of the errors of other methods available for measurements of the scattering phase functions. A nomogram for determining the lidar ratio from data on the aerosol refractive indices and disperse composition is presented. Possibilities of estimating these indices from the data on the origin and characteristics of air masses in the regions of lidar sensing are considered. The possibilities of solving the inverse problem based on reconstructed values of the aerosol microstructure parameters by use of inverse transformation method for parametric approximation of the experimental phase functions are investigated.

In solving problems on aerospace monitoring of the ground and ocean surface, as well as in the studies of directional atmospheric radiation, light regime of the scattered illumination of the Earth, the difficulties usually arise caused by the absence of adequate data on the aerosol scattering phase functions as referred to the whole depth of the atmosphere. G. Zimmerman, M. Shenemark et al, in their papers[1,2] devoted to the sounding of the Earth atmosphere from space, consider the possibility of using the aerosol scattering phase functions obtained from the data on the sky brightness measured at the solar almucantar.[3] The correctness of using such scattering phase functions is doubtless. But now there are only little data obtained by this technique[3] available in literature.

At different time, a number of relationships were proposed in literature that approximate the aerosol scattering phase function $\gamma_a(\theta)$ (formulas proposed by E. Shenberg, V.G. Fesenkov, V.A. Krag, G.S. Isaev, G.I. Gorchakov and other). Recently the three-parameter superposition of the Henyey–Greenstein functions has been often used (especially abroad) for these purposes:

$$\gamma(\theta) = \sum_{i=1}^{2} K_i(1 - a_i^2)/(1 + a_i^2 - 2a_i\cos\theta)^{3/2},$$

where a_i and $K_2 = 1 - K_1$ are the parameters.

Reynolds and McCormic[4] proposed a two-parameter formula for approximation of the strongly forward-peaked scattering phase functions

$$\gamma(\theta) = \frac{4\alpha(1 - a^2)^{2\alpha}}{(1 + a^2 - 2a\cos\theta)^{\alpha+1}[(1 + a)^{2\alpha} - (1 - a)^{2\alpha}]},$$

where $|\alpha| < 1$; $\alpha > -0.5$. This formula is reduced to the Henyey-Greenstein formula at $\alpha = 0.5$. The Henyey-Greenstein formula does not provide approximation of the scattering phase functions close to the Rayleigh shape.

Since the shape of the aerosol scattering phase function is principally determined by the size spectrum and the refractive index of the aerosol particles, it is natural to use these characteristics as the parameters for approximation of the scattering phase functions. However, we did not find any relationship between the parameters of the available approximations of the scattering phase functions (including the parameters a_1, a_2 and K in the Henyey-Greenstein formulas) and the aerosol microstructure characteristics. And we could not find any information about it in the literature.

Of course, it is expedient to accept the refractive index n as one of the approximation parameters, and the index of disperse composition ν^* can be taken as a characteristic of the particle size spectrum, at least for the particle size distribution close to the inverse degree law:

$$dN/(d \lg r) \sim r^{-\nu^*}.$$

Really, the index ν^* unambiguously determines the shape of the aerosol scattering phase function at the constant refractive index n. But direct determining of the index ν^* by means of counting the number of particles of different size is technically very difficult. Therefore, it is not casual that there is only little data of that type available in the literature.

At the same time, there are some papers containing large bulk of data on the indices of the spectral selectivity of aerosol extinction ω_a (Angstrom index) assessed from the measurement data on the spectral aerosol optical thickness $\tau_a(\lambda)$ or from the data of long-path measurements of the aerosol extinction coefficient $\beta_{a\text{ex}}(\lambda)$ (see references in Ref.5). The Angstrom index ω_a in the cases with the particle size-distributions close to the inverse-degree law and quite wide particle size spectrum ($0.025 \le r \le 25$ μm) correlates with the index of disperse composition ν^* by the relationship $\omega_a = \nu^* - 2$.

On this basis, we used the Angstrom index as the second natural parameter (the index of disperse composition) for the approximation of scattering phase functions.

Such a parameterization was presented for the first time in Ref. 5. This paper continues this study.

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Parametric approximation of the aerosol scattering phase functions

One can easily see that in relating the Rayleigh scattering phase function $\gamma_{\rm m}(\theta)$ to the scattering angle θ in a comparatively wide angular range one obtains the curve $\gamma_{\rm m}(\theta)/\theta$ which is similar to a part of the aerosol scattering phase function. Convolution of the obtained curve with a model scattering phase functions $\gamma_a(\theta)$ of a polydisperse ensemble different asymmetry is reached by raising the denominator (the angle θ) to a power s. The greater is the exponent s, the steeper is the curve $\gamma_{\rm m}(\theta)/\theta^s$. Normally an additional factor $(1 + t\sin K\theta)$ is introduced into the exponent, so that such curves have minimum like the model scattering phase functions (usually near 110–120°). Let us denote the obtained function as $\tilde{\gamma}(\theta)$:

$$\tilde{\gamma}(\theta) = \gamma_{\rm M}(\theta)\theta^{-s(1+t\,\sin\,K\theta)}.$$

Two disadvantages of such an approximation can easily be seen. First, the function $\tilde{\gamma}(\theta) \to \infty$, at $\theta \to 0$, that is unrealistic. In this connection, one should restrict the applicability of the approximation function $\tilde{\gamma}(\theta)$ to some minimum scattering angle $\theta_{\rm min}$. Second, it occurs in all cases at $\theta = 1$ that the function $\tilde{\gamma}(1)$ is equal to the Rayleigh scattering phase function $\gamma_{\rm m}(1)$. However this never occurs in natural and model scattering phase functions. This disatvantage is removed by replacing the angle θ in the denominator of the function $\tilde{\gamma}(\theta)$ by an angle $\theta' = \theta(1 + \varepsilon)$. The correction ε is assessed from the scattering angle θ_0 at which the functions $\tilde{\gamma}(\theta_0)$ and $\tilde{\gamma}_{\rm m}(\theta_0)$ are equal to each other. In this case

$$\theta_0(1 + \varepsilon) = 1\ . \qquad (1)$$

As a result, we obtain the following formula for the function $\tilde{\gamma}(\theta)$:

$$\tilde{\gamma}(\theta) = \gamma_{\rm M}(\theta)[\theta(1 + \varepsilon)]^{-s(1+t\,\sin\,K\theta)}. \qquad (2)$$

At first,[5] to establish the statistical relationships among the parameters s, t, K, ε and the model indices n and ω_a one used the data of model calculations of the scattering phase functions $\gamma_{\rm a}(\theta)$ obtained by E. Bari, B. Brown, K. Bullrich, K.S. Shifrin and T.P. Toropova. When preparing this paper, we have performed more detailed calculations, by the specrosonal method, of the aerosol scattering phase functions $\gamma_a(\theta)$, the scattering coefficients β_{as}, the extinction coefficients $\beta_{a{\rm ex}}$, and the Angstrom indices ω_a for the refractive indices of the particulate matter of $n = 1.34$, 1.43, 1.50 and 1.53, and absorption indices $\kappa = 0$, 0.005 and 0.05, indices of disperse composition $\nu^* = 2.0$, 2.5, 3, 4 and 5, and the wavelengths $\lambda = 400$, 800 and 1600 nm in the particle size range r from 0.025 to 25 μm and four types of the particle size-distribution functions.

Using thus obtained array of aerosol characteristics, we picked up the following regression relationships for the characteristics s, t, ε and K:

$$s = (6 - \omega_a)/[10.2(n - 1)]\ ; \qquad (3)$$

$$t = (0.72 + \sqrt{s})(n^2 - 1.5)\ ; \qquad (4)$$

$$\varepsilon = (s - 0.512 - \sqrt{0.15\omega_a})/3\ ; \qquad (5)$$

$$K = \left|\begin{array}{ll} 0.865 & \text{at}\ \ \theta \le 120°, \\ \dfrac{0.96}{n} + \left(3\sqrt{\omega_0 + \sin^2\theta}\right)^{-1} & \text{at}\ \ \theta > 120°. \end{array}\right.$$

As is seen from the relationships (2)–(5) the function $\tilde{\gamma}(\theta)$ is in fact determined only by two natural parameters n and ω_a, and the scattering angle θ. The values of s, t, and K are intermediate parameters used only for reduction of Eq. (2). The problem of natural parameterization of the aerosol scattering phase functions is formalized by formulas (1)–(5).

One should pay attention to one interesting peculiarity of the aerosol light scattering. Whith the increasing index of disperse composition up to $\nu^* = 6$ the Angstrom index ω_a becomes equal to the Angstrom index of Rayleigh scattering $\omega_{\rm m} = 4$. However, judging from the calculations and Eq. (3), the polydisperse scattering phase function of the system of such particles remains noticeably asymmetric because of the effect of coarse particles contained in such a system, although their amount is not significant.

As was mentioned above, Eq. (2) can be used for approximation of the aerosol scattering phase functions $\gamma_a(\theta)$ only starting from some minimum scattering angle $\theta_{\rm min}$. The applicability of this formula to size spectra strongly different from the inverse power law is also restricted to some maximum angles $\theta_{\rm max}$.

Let us accept the condition that the limiting errors of approximation should not exceed the limiting errors in experimental measurements of the aerosol scattering phase functions within the applicability limits $\theta_{\rm min} \div \theta_{\rm max}$ as a criterion for determination of the angular boundaries for the applicability of Eq. (2).

As was shown in Ref. 6, the limiting errors in assessing the aerosol scattering phase functions from the sky brightness in solar almucantar are about ±30%. The rms errors in nephelometric measurements of the scattering phase functions are about 10%.[7,8] Therefore, one can take the value $\delta\gamma_a = \pm 30\%$ as the limiting error. Thus, the scattering angles $\theta_{\rm min}$ and $\theta_{\rm max}$, within which the errors of approximation do not exceed ±30%, are accepted as the angular boundaries of the applicability of Eq. (2).

The errors from using the Eq. (2) were determined by comparison of the obtained approximations with the data of exact model calculations of the scattering phase functions of a polydisperse ensemble with the inverse-power particle size-distribution. The absorption of radiation by particles was not taken into account. The results of calculations are shown in Table 1.

Table 1. The errors $\delta\gamma_a$, % in calculating the aerosol scattering phase functions $\gamma_a(\theta)$ by Eq. (2).

θ°	ω_a	Refractive index n												$\bar{\delta}\gamma_a$, %	$\delta\gamma_a$, %
		1.34			1.43			1.50			1.53				
		0.546	1.008	1.983	0.541	1.006	1.982	0.537	1.005	1.981	0.533	0.983	1.98	–	–
10	γ_a	15.5	14.6	9.98	12.9	12.1	8.35	11.6	10.7	7.36	11.2	10.2	6.99	–	–
	$\delta\gamma$	25.5	17.6	23.9	12.5	6.1	12.8	5.1	1.2	9.5	2.2	0.4	9.0	10.5	13.4
15	γ_a	9.57	9.71	7.69	8.30	8.44	6.66	7.57	7.62	6.01	7.31	7.31	5.76	–	–
	$\delta\gamma$	7.5	−1.1	0.9	7.3	−2.4	−0.8	7.0	−1.8	−0.2	7.1	−0.9	0.5	1.9	4.3
20	γ_a	6.42	6.02	5.90	5.89	6.18	5.31	5.48	5.71	4.92	5.32	5.54	4.77	–	–
	$\delta\gamma$	−1.0	2.8	−7.5	3.3	−5.7	−5.6	6.8	−2.3	−3.5	−3.2	−0.9	−2.5	−1.6	4.7
30	γ_a	3.17	3.46	3.44	3.22	3.47	3.37	3.16	3.40	3.29	3.11	3.35	3.25	–	–
	$\delta\gamma$	−6.7	−11.0	−9.6	10.0	−5.7	−6.2	5.4	−1.5	−3.9	8.3	0.6	−2.8	−1.9	7.2
40	γ_a	1.63	1.83	2.08	1.81	2.01	2.18	1.87	2.07	2.22	1.88	2.09	2.22	–	–
	$\delta\gamma$	−5.0	−6.7	−7.3	0.3	−3.2	−4.6	4.9	0.3	−2.9	7.1	1.2	−1.9	−1.5	4.9
60	γ_a	0.494	0.604	0.851	0.632	0.740	0.944	0.724	0.828	1.01	0.759	0.862	1.03	–	–
	$\delta\gamma$	1.1	0.2	−3.0	−13.6	−0.7	0	−3.0	−1.3	0.3	−3.5	−2.4	0.6	−2.1	4.7
80	γ_a	0.187	0.247	0.409	0.266	0.324	0.468	0.325	0.382	0.511	0.349	0.406	0.529	–	–
	$\delta\gamma$	6.6	5.6	1.5	−5.6	−1.0	3.1	−11.9	−5.3	2.7	−14.0	−8.1	2.0	−2.0	7.3
100	γ_a	0.102	0.140	0.261	0.147	0.186	0.306	0.174	0.219	0.334	0.189	0.236	0.349	–	–
	$\delta\gamma$	8.5	9.7	5.6	−5.5	1.5	5.1	−8.8	−2.4	5.1	−12.0	−6.8	3.5	0.3	7.2
120	γ_a	0.0896	0.119	0.244	0.110	0.148	0.271	0.133	0.174	0.292	0.143	0.186	0.301	–	–
	$\delta\gamma$	−2.5	4.7	−0.7	−0.9	4.1	5.1	−6.0	0.7	6.9	−8.2	−3.1	6.8	0.7	5.0
140	γ_a	0.151	0.155	0.273	0.139	0.174	0.299	0.159	0.198	0.318	0.169	0.209	0.326	–	–
	$\delta\gamma$	−22.7	−5.6	−4.2	6.0	2.4	1.3	1.9	−1.4	2.3	−1.4	−5.1	1.9	−2.1	7.8
150	γ_a	0.172	0.177	0.245	0.230	0.229	0.330	0.221	0.246	0.349	0.224	0.253	0.356	–	–
	$\delta\gamma$	−15.3	−6.3	−5.7	−0.2	−10.1	−1.8	3.9	−8.8	−0.9	−3.0	−10.0	−1.1	−4.9	7.8
180	γ_a	0.327	0.260	0.340	0.544	0.361	0.390	0.623	0.495	0.455	0.752	0.592	0.491	–	–
	$\delta\gamma$	−2.1	1.1	−0.4	−0.8	3.2	6.6	5.3	−16.6	−2.8	−8.6	−28.3	−9.0	−4.2	11.2

As follows from this Table, the values $\theta_{\min} = 10°$ and $\theta_{\max} = 180°$ can be accepted as the angular boundaries of applicability of Eq. (2), because the errors, in this case, do not exceed ±30%. The mean $\overline{\delta\gamma_a}$ and rms $\sigma\gamma_a$ errors in calculating the scattering phase functions $\gamma_a(\theta)$ by Eq. (2) in the angular range 10 to 180° are the following: $\overline{\delta\gamma_a}$ == −0,7%; and $\sigma\gamma_a$ = 7,3%.

But the actual particle size-spectra at optical sounding of the atmosphere are usually unknown, and, in general case, can significantly differ from the inverse-power ones. Obviously, the question arises on the applicability of Eq. (2) for calculating the scattering phase functions using the data on the indices n and ω_a at different particle size-distribution functions. To answer this question, we have calculated the spectral scattering coefficients β_{as}, the Angstrom indices ω_a, and the scattering phase functions $\gamma_a(\theta)$ of polydispersions for four aerosol models with different particle size-distributions $f^*(r) = dN/d\lg r$:

I. Inverse-power distribution

$(\nu^* = 3;\ 0.025 \le r \le 25$ м*м)

$$f^*(r) = \frac{A_{\mathrm{I}}}{r^{\nu^*}}\ .$$

II. Weighted-mean distribution[9]:

$(r_0 = 0.03\ \mu\mathrm{m},\ \bar{\mathrm{n}} = 2.87,\ \tilde{\mathrm{n}} = \bar{\mathrm{n}} - 0.1r$; where r is in μm)

$$f^*(r) = \frac{A_{\mathrm{II}}}{r_0^{\tilde{\nu}} + |r - r_0|^{\tilde{\nu}}}\ .$$

III. Lognormal distribution:

$(r_0 = 0.05\ \mu\mathrm{m};\ \sigma = 1.095)$

$$f^*(r) = A_{\mathrm{III}} \exp\left[-\frac{\ln^2(r/r_0)}{2\sigma^2}\right].$$

IV. Bimodal distribution (superposition of two lognormal distributions $r_{01} = 0.072\ \mu\mathrm{m}$, and $r_{02} = 3.26\ \mu\mathrm{m}$, $\sigma = 0.5$)

$$f_{\Sigma}^*(r) = \sum_{i=1}^{2} A_i \exp\left[-\frac{\ln^2(r/r_{0i})}{2\sigma^2}\right].$$

Calculations of the optical characteristics were carried out for each model of the size-spectrum by the spectrozonal method[5] for the refractive index values $n = 1.43$ and 1.5 in three wavelength ranges of $\lambda = 400$, 800 and 1600 nm, in the particle size range from 0.025 to 25 μm. All values of the distribution function $f^*(r)$ were normalized assuming the condition that the scattering coefficient $\beta_{as} = 0.127\ \mathrm{km}^{-1}$ at $\lambda = 800$ nm (according to Elterman). The calculated values of the distribution function $f^*(r)$ for the models I–IV are shown in Fig. 1.

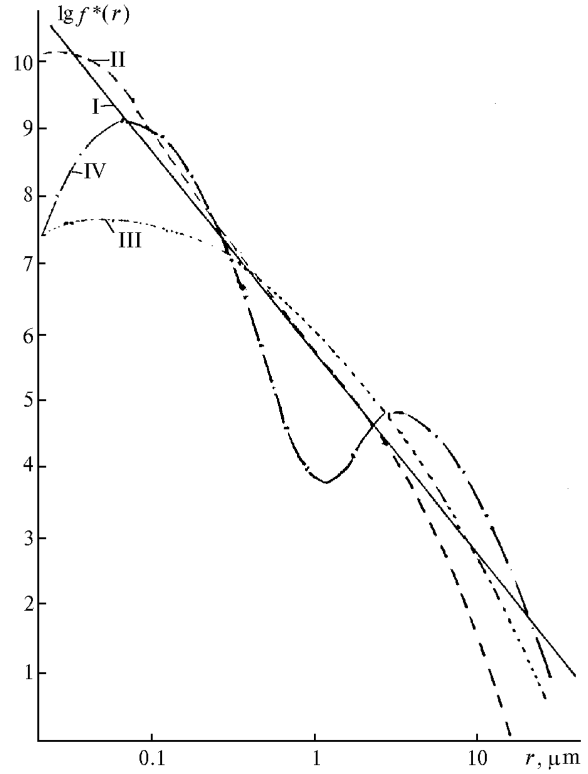

Fig. 1. Models of the aerosol particle size distribution.

The results of calculation of the Angstrom indices ω_a and the aerosol scattering phase functions $\gamma_a(\theta)$ for each model are shown in Table 2. The rms errors $\delta\gamma_a$, mean $\overline{\delta}\gamma_a$ and rms $\sigma_{\gamma a}$ errors in determining the scattering phase functions $\gamma_a(\theta)$ by Eq. (2) for each model are also shown in this Table.

It is seen from Table 2 that the mean and rms errors in calculating the scattering phase functions by Eq. (2) for the weighted-mean particle size distribution (model II)

$$\overline{\delta}_{\rm II}\,\gamma_a = -\,2.9\%,\ \sigma_{\rm II}\,\gamma_a = 7.5\%$$

are practically the same as for the inverse-power law distribution (model I). The applicability limits of Eq. (2) from 10 to 180° also do not change. The upper boundary of the angles of applicability for the lognormal distribution (model III) decreases to $\theta_{\max} = 150°$. The errors in calculating slightly increase (in the limits of the scattering angle θ = 10-150° $\overline{\delta}_{\rm III}\,\gamma_a = -5.3\%$; $\sigma_{\rm III}\,\gamma_a = 15.5\%$). The angles of applicability for the pronounced bimodal distribution (model IV) are reduced to the values of 20–150°. The errors in calculating the scattering phase functions $\gamma_a(\theta)$ within these limits are:

$$\overline{\delta}_{\rm IV}\,\gamma_a = 9.5\%,\ \sigma_{\rm IV}\,\gamma_a = 14.7\%.$$

The data presented evidence of the fact that in all cases of calculating the scattering phase functions $\gamma_a(\theta)$ by Eq. (2) in the angular range 20 to 150° the accuracy obtained ($|\delta\gamma| \le 30\%$) is quite acceptable. As it follows from the available data,[5,9] in the majority of cases the particle size-distribution functions are grouped about certain (weighted-mean) function, the shape of which is more close to the inverse-power than to the lognormal distribution function. Hence, one can expect the results obtained by Eq. (2) to be more correct than for the models III–IV.

The absorption of radiation by aerosol is determined by the absorption coefficient κ of the complex refractive index $m = n - \kappa i$. From the recent data,[10,11] the absorption index κ of the "dry matter" of natural aerosol in the visible wavelength range in most cases is not beyond the limits 0.005–0.007. The absorption coefficient significantly decreases at the presence of water vapor in the atmosphere because of moistening the aerosol particles. Actual values of the absorption coefficient of natural aerosol measured *in situ*[8] do not exceed κ = 0.002.

Table 2. The errors $\delta\gamma_a$, % in calculating the aerosol scattering phase functions $\gamma_a(\theta)$ by Eq. (2) for I–IV models of the particle size-distribution (λ = 0.8 μm).

Model	n	ω_a	γ, (σ, %)	Scattering angle θ°								$\overline{\delta}\gamma_a$, %	$\delta\gamma_a$, %
				10	20	40	60	90	120	150	180		
I	1.43	1.006	γ	12.1	6.18	2.01	0.740	0.236	0.148	0.229	0.361	−2.6	6.9
			δ	6.1	−5.7	−3.2	−0.7	−0.2	4.1	−10.1	3.2		
	1.50	1.005	γ	10.7	5.71	2.07	0.828	0.280	0.174	0.246	0.495		
			δ	1.2	−2.3	0.3	−1.3	−4.9	0.7	−8.8	−16.6		
II	1.43	1.234	γ	11.5	6.04	2.04	0.786	0.267	0.178	0.252	0.349	−2.9	7.5
			δ	4.6	−6.1	−2.2	0.2	0	1.4	−10.5	−3.4		
	1.50	1.247	γ	10.1	5.56	2.10	0.872	0.312	0.205	0.275	0.486		
			δ	0.2	−3.1	0.5	−0.2	−3.7	−0.2	−10.8	−19.4		
III	1.43	0.207	γ	15.2	6.18	1.72	0.578	0.174	0.099	0.230	0.430	−5.3	15.5
			δ	1.4	−0.9	−2.6	−7.5	−19.5	−17.8	6.2	226.8		
	1.50	0.160	γ	13.9	5.75	1.75	0.665	0.216	0.119	0.240	0.707		
			δ	−5.7	3.1	3.4	−10.3	−27.7	−23.8	27.1	243.9		
IV	1.43	1.127	γ	6.77	4.42	2.07	0.825	0.262	0.158	0.256	0.682	9.5	14.7
			δ	83.1	30.0	−3.8	−7.5	−3.8	6.2	−15.8	−46.7		
	1.50	1.230	γ	6.34	4.31	2.14	0.914	0.304	0.181	0.204	0513		
			δ	60.3	25.4	−1.5	−5.2	−2.0	11.9	19.6	−23.2		

Table 3 shows the Angstrom indices ω_a and the scattering phase functions $\gamma_a(\theta)$ that we calculated by the spectrozonal method assuming the values of the complex refractive index $m = 1.43-0.005i$ and $m = 1.50-0.005i$ (the value of the index $\kappa = 0.002$ is absent in the Tables of spectrozonal functions) for the inverse-power particle size-distribution. The errors $\delta\gamma_a$ in calculating these scattering phase functions $\gamma_a(\theta)$ by Eq. (2) are also shown in the Table.

As is seen from Table 3, the boundaries of applicability of Eq. (2) under these conditions are the same (10–180°) as in the case with pure scattering, and the rms error increases up to 11.3%. under real conditions, at the absorption coefficient $\kappa \le 0.002$, the effect of aerosol absorption is weaker.

One can conclude, on the basis of the results presented, that the scattering phase functions of natural aerosol that does not contain strongly absorbing and well pronounced specific fractions can be calculated by Eq. (2) using the data on the indices n and ω_a. As a rule, the errors in calculating will not exceed the measurement errors in the scattering phase functions ($|\delta\gamma_a| \le 30\%$).

Natural parameterization of the lidar ratio

When lidar investigations of the Earth atmosphere from space has started, the problems have arisen in the interpretation of the recorded lidar returns. Determination of the so called "lidar ratio" is one of these problems.

The lidar return $N(h)$ is related to the atmospheric light scattering characteristics by the laser sounding equation

$$N(h) = KT^2(h)\alpha_\pi(h)/(\Delta h)^2, \qquad (6)$$

where K is the lidar calibration constant, Δh is the distance from the atmospheric volume sounded, or the underlying surface, to the lidar, $\alpha_\pi(h)$ is the volume backscattering coefficient, $T(h)$ is the transmission of the atmosphere along the sounding path.

In their turn, the coefficients $\alpha_\pi(h)$ and $T(h)$ can be represented in the form

$$\alpha_\pi(h) = g_\pi(h)\beta_{\text{ex}}(h); \qquad (7)$$

$$T(h) = \exp\left[-\int\limits_{\Delta h}\beta_{\text{ex}}(h)dh\right], \qquad (8)$$

where $g_\pi(h)$ is the lidar ration, and $\beta_{\text{ex}}(h)$ is the volume extinction coefficient.

If the ratio between the coefficient $\alpha_\pi(h)$ and the $\beta_{\text{ex}}(h)$, i.e., the lidar ratio $g_\pi(h)$, is known, the problem should be reduced to solving the integral equation (6) with only one unknown parameter, $\beta_{\text{ex}}(h)$. A number of methods have been proposed to date for numerical and analytical solution of such of equations (see, for example, Ref. 12). All these methods assume that the lidar ratio $g_\pi(h)$ is known. In particular, Rocar accepted $g_\pi = 0.038$, Chesterman and Styles took $g_\pi = 0.040$, Foitzik took it to be 0.032, Gershun took the value of 0.050 for clear air and 0.029 for a turbid air. Russel et al accepted $g_{a\pi} = 0.0132$ for aerosol. The lidar ratio $g_{a\pi} = 0.035$ characteristic of aerosol in the near-ground layer was taken in Ref. 13.

As is seen, when solving Eq. (6), different authors took the values of the lidar ratio different by 2–3 times and more. Our investigations showed that the value $g_{a\pi}$ can vary, depending on the atmospheric optical conditions, within the limits from 0.008 to 0.065, i.e., up to 8 times. At the same time, it follows from Eqs. (6)–(8) that the solutions of the laser sounding equation are critical to the choice of the lidar ratio value, hence, to $g_{a\pi}(h)$.

In this connection, some Russian and foreign scientists (R.T. Kovalev, V.I. Khalturin, O.D. Barteneva, G.I. Gorchakov, R.V. Fenn, D.D. Klett etc.) proposed different empirical formulas for determining the lidar ratio from the extinction coefficient β_{ex}, optical thickness τ, meteorological visual range S_{m} and so on.

The existence of many different dependences which do not agree with each other is explained by the fact that, in fact, there is no unambiguous dependence of the lidar ratio on the extinction β_{ex}, τ and S_{m}, although certain correlation is observed.

As was mentioned above, the lidar ratio $g_{a\pi}$, as well as the backscattering phase function $\gamma_{a\pi}$ (in the absence of absorption $\gamma_{a\pi} = 4\pi g_{a\pi}$), is determined not only by the characteristics of extinction, but by the aerosol microstructure, i.e., should be parameterized by the indices n and ω_a.

Table 3. The errors $\delta\gamma_a$ in calculating the aerosol scattering phase functions $\gamma_a(\theta)$ at the presence of aerosol absorption of radiation ($\varkappa = 0.005$, model I).

m	$\omega_{a\text{ex}}$	γ, (δ, %)	Scattering angle, degrees								$\bar{\delta}\gamma_a$, %	$\delta\gamma_a$, %
			10	20	40	60	90	120	150	180		
$1.43-0.005i$	0.992	$\gamma_a(\theta)$	10.76	5.22	2.08	0.815	0.268	0.175	0.241	0.315	−0.4	11.3
		$\delta\gamma_a$	19.9	12.2	−6.5	−10.1	−12.7	−12.6	−14.8	18.8		
$1.50-0.005i$	0.994	$\gamma_a(\theta)$	10.78	5.59	2.03	0.826	0.284	0.182	0.246	0.420		
		$\delta\gamma_a$	0.9	0	2.1	−1.5	14.7	−4.7	−9.4	−1.5		

To provide for a possibility of operatively determining the lidar ratio $g_{a\pi}$ from the data on indices n and ω_a we have calculated the lidar ratio values $g_{a\pi}$ on a sufficiently detailed grid of the refractive and Angstrom indices. The calculations were performed by the spectrozonal method assuming the inverse-power size-distribution of particles within the Mie parameter, $\rho = 2\pi r/\lambda$, range from 0.1 to 409 without the account of radiation absorption by aerosol. The calculated results are shown in Fig. 2 as a nomogram.

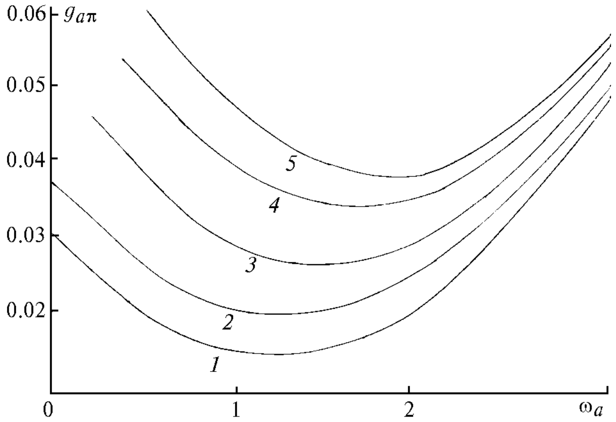

Fig. 2. The lidar ratio $g_{a\pi}$ as a function of the Angstrom index ω_a at the values of the refractive index $n = 1.25$ (*1*), 1.33 (*2*), 1.43 (*3*), 1.50 (*4*), and 1.53 (*5*).

As in the case with the approximation of scattering phase functions, there arises a question on the applicability of such a nomogram to interpretation of the results of sounding of atmospheric aerosol from space, when the particle size-distribution and the absorption of radiation by aerosol are unknown, and, in general case, can significantly differ from the conditions assumed when constructing the nomogram.

Partially the answers to these questions can be found in the previous section where tables 2 and 3 are analyzed. Besides, the lidar ratio values $g_{a\pi}$ were calculated for aerosol with the complex refractive indices $m = 1.43{-}0.005i$; $1.50{-}0.005i$ and $1.53{-}0.005i$ for the models I and II of the particle size-distribution. The calculated results are shown in Table 4. The lidar ratio values for pure scattering (without absorption) are also shown in this Table.

Table 4. The effect of aerosol absorption on the lidar ratio g_{as} at $\varkappa = 0$ and $g_{a\text{ex}}$ at $\varkappa = 0.005$.

Parameter	Model of distribution					
	I			II		
n	1.43	1.50	1.53	1.43	1.50	1.53
$g_{as}\ 10^4$	287	394	471	287	387	457
$g_{a\text{ex}}\ 10^4$	251	334	401	252	351	411
δg, %	−12.7	−15.2	−14.9	−9.2	−9.3	−10.1

Analysis of the data given in Tables 2–4 shows that the lidar ratios $g_{a\text{ex}}$ at the particle size-distributions close to the inverse-power and weighted-mean ones, at the absorption characterized by the index $\kappa = 0.005$ differ from that shown in the nomogram, g_{as}, by no more than 20%. The difference between the lidar ratios $g_{a\text{ex}}$ and g_{as} at the absorption index $\kappa = 0.002$ characteristic of natural aerosol are no greater than 8-10%. Significant deviations of $g_{a\text{ex}}$ and g_{as} from that shown in the nomogram can appear at the particle size-distributions significantly different from the models I–II. It is not excluded that such distributions can really occur, but according to available information[5,9], they are very rare.

Determination of the under-satellite values of the parameters *n* and ω_a

Obviously, it is impossible to operatively determine the values of the indices n and ω_a from the data obtained at underflights by the routine processing the data of optical sounding of the atmosphere from space by traditional methods. The regression methods presented in the literature for determining the scattering phase functions and lidar ratios from the data on the meteorological visibility range along the sounding path from satellite, the extinction coefficient or the optical thickness of the atmosphere are little suitable under these conditions.

The approach that was proposed in Ref. 5 to solving this problem is based on the following suppositions:

a) the following meteorological and synoptical data can be more or less available in the regions of sounding from space:

– origin and type of air mass (marine, continental, arid, arctic, etc.);

– humidity, pressure, temperature;

– type and state of the underlying surface;

– synopticl conditions (cyclone, anticyclone, etc.);

b) certain relation is known between the atmospheric conditions and the optical characteristics of the atmosphere, including the refractive index and the Angstrom index.

According to the published data, aerosol of different origin is characterized by its own values of the indices n and ω_a. The refined (in comparison with the data from Ref. 5, Table 4.4) characteristic values of the indices n and ω_a of aerosol of different types are shown in Table 5.

However, one should keep in mind that the relation between the type of air mass and the indices n and ω_a is ambiguous. It strongly depends on the landscape and climatic conditions, humidity, synoptic conditions, the height of the boundary layer of the atmosphere, etc. In this connection, compilation of Tables similar to Table 5 and additionally taking into account the effect of the aforementioned factors seems to be very urgent.

Table 5. Characteristics values of the indices n and ω_a for aerosol of different types.

Type of aerosol	$m = n - \kappa i$	ω_a
Arctic:		
winter	1.50	1.0
summer	1.45	1.3
Marine (far from land):		
north latitudes	$1.52 - 10^{-4}\,i$	1.0
midlatitudes	$1.52 - 10^{-4}\,i$	0.6
tropics	$1.52 - 10^{-4}\,i$	0.3
Marine (coastal):		
λ = 550 nm	$1.5 - 0{,}002\,i$	0.36
λ = 1060 nm	$1.47 - 0.002\,i$	0.75
Continental	$1.525 - 0.002\,i$	1.25
Rural:		
λ = 550 nm	$1.53 - 0.006\,i$	1.44
λ = 1060 nm	$1.52 - 0.014\,i$	1.34
Urban:		
λ = 550 nm	$1.48 - 0.075\,i$	1.19
λ = 1060 nm	$1.47 - 0.06\,i$	0.78
Arid:		
summer – fall	$1.58 - 0.008\,i$	0.60
winter – spring	$1.50 - 0.005\,i$	1.15
Sahara dust	$1.53 - 0.003\,i$	0.60
Stratospheric	1.43 – 1.47	1.20
Volcanic	$1.50 - 0.005\,i$	0.65

Inversion of the aerosol scattering phase functions

The aerosol microstructure characteristics are reconstructed *in situ* by the method of inverting the aerosol scattering phase functions without changing the aerosol structure. Nevertheless, the majority of data on the microstructure of atmospheric aerosol are obtained by contact methods. One of the reasons for this is difficulty and inconvenience of calculation procedures for inverting optical data into the aerosol microstructure parameters.

The analytical approximation of the scattering phase functions in the form of Eqs. (2)–(4), where the characteristics of aerosol microstructure n and $\omega_0 = \nu^* - 2$ are used as the parameters of approximation, allows one to obtain the explicit form of the relationships for determining the indices n and ω_a by means of the inverse transformation.

By substituting the measured values of the scattering phase function $\gamma_a(\theta)$ at two angles, θ_1 and θ_2, as well as the correction $\varepsilon = \theta_0^{-1} - 1$ to Eq.(2), we obtain the system of two equations. Solving it relative to the intermediate values t and s, we obtain

$$t = (1 - U)/(U\sin K\theta_1 - \sin K\theta_2);$$

$$s = P_i/\big[(1 + t\sin K\theta_i)\ln(\theta_i/\theta_0)\big],\ \ i = \overline{1,2}\ ,$$

where

$$U = \frac{P_2\ln\dfrac{\theta_1}{\theta_0}}{P_1\ln\dfrac{\theta_2}{\theta_0}};\quad P_i = \ln\frac{\gamma_{\text{M}}(\theta_i)}{\gamma_a(\theta_i)};\quad K = 0.865.$$

The values of the indices, n and ω_a, are determined by the obtained values t and s, using Eqs. (3) and (4)

$$n = \sqrt{[t/(0.72 + \sqrt{s}]) + 1.5}; \qquad (9)$$

$$\omega_a = 6.0 - 10.2s(n - 1). \qquad (10)$$

To determine the errors in reconstructing the indices n and ω_a of the aerosol microstructure by Eqs. (9) and (10) we inverted 12 model scattering phase functions from Table 1. The values $\theta_1 = 20°$ and $\theta_2 = 120°$ were taken as the angles θ_1 and θ_2. The inversion results are shown in Table 6. As follows from Table 6, the mean and rms errors in reconstructing the refractive index of particles are: $\overline{\Delta n} = -0.001$; and s(D$n$) = 0.036.

The Angstrom indices ω_a are reconstructed with the errors $\overline{\Delta\omega_a} = -0.008$ and $\sigma(\Delta\omega_a) = 0.16$.

Undoubtedly, it is the idealized accuracy, because the polydisperse scattering phase functions calculated for exactly inverse-power size-distributions without taking into account the absorption and without random measurement errors were used in inverting.

It was necessary to check the applicability of the proposed method to inversion of the experimentally measured scattering phase functions. In so doing we used the scattering phase functions obtained by statistical processing of the "universe" of nephelometric data on the aerosol scattering phase functions at the wavelength $\lambda = 0.55$ μm measured in Karadag (Crimea) and in Tomsk in 1975.[8]

The scattering phase functions $\gamma_a(\theta)$ are represented in the form of the regression relationship:

$$\gamma_a(\theta) = C_a(\theta)\ \beta_{as}^{K_a(\alpha)-1}\ . \qquad (11)$$

Taking the values of the volume aerosol scattering coefficients β_{as} to be 0.15, 0.20, and 0.30 km^{-1}, we calculated, by Eq.(11), the values of the scattering phase functions $\gamma_a(\theta)$ at the angles $\theta_1 = 20°$ and $\theta_2 = 120°$ and the correction $\varepsilon = \theta_0^{-1} - 1$ for the conditions of Karadag and Tomsk. Resulting from the solution of the system of two equations, the intermediate values t and s were found, from which the values of the indices n and ω_a were then determined by Eqs. (9) and (10) (see Table 7).

It is seen from Table 7 that:

– the index ω_a at Karadag is relatively small (0.34–0.38). This is an evidence of the fact that the microstructure of coastal aerosol is similar to that of marine aerosol that is characterized by small values of the index ω_a;

– the refractive index decreases from $n = 1.59$ to 1.41 as the aerosol scattering coefficient β_{as} increases. This is indicative of the fact that the increase of β_{as} in coastal area occurs mainly due to moistening of particles;

– the aerosol turbidity in Tomsk increases without a noticeable change of the nature of particles ($n \simeq 1{,}4$), but the relative number of coarse particles increases, and the index ω_a decreases from 1.78 to 1.25.

Table 6. Inverting the aerosol scattering phase functions. Model I. $\varkappa = 0$.

		Refractive index n											
		1.34			1.43			1.50			1.53		
initial data	ω_a	0.546	1.008	1.983	0.541	1.006	1.982	0.537	1.005	1.981	0.533	0.983	1.980
	ε	0.258	0.180	0.034	0.149	0.080	−0.047	0.092	0.026	−0.090	0.072	0.011	−0.104
	$\gamma_a(20°)$	6.42	6.02	5.90	5.89	6.18	5.31	5.48	5.71	4.92	5.32	5.54	4.77
	$\gamma_a(120°)$	0.0896	0.119	0.344	0.110	0.148	0.271	0.133	0.174	0.292	0.143	0.186	0.301
Rasults of inverting	$\tilde{s}$	1.583	1.490	1.251	1.232	1.174	0.964	0.996	1.00	0.802	0.960	0.942	0.749
	$\tilde{t}$	0.558	0.500	0.421	0.994	0.978	0.890	1.427	1.249	1.279	1.450	1.351	1.429
	$\tilde{n}$	1.335	1.326	1.315	1.429	1.429	1.422	1.526	1.492	1.514	1.534	1.516	1.550
	Δn	−0.005	−0.014	−0.025	−0.001	−0.001	−0.008	0.026	−0.008	0.014	0.004	−0.014	0.020
	$\tilde{\omega}$	0.59	1.05	1.982	0.604	0.86	1.85	0.65	0.98	1.80	0.77	1.04	1.80
	$\Delta\omega$	0.044	0.042	−0.001	0.063	−0.146	−0.132	0.113	−0.015	−0.181	0.237	0.057	−0.18

$\overline{\Delta n} = -0.001$; $\sigma(\Delta n) = 0.036$; $\overline{\Delta\omega}_a = -0.008$; $\sigma(\Delta\omega_a) = 0.16$.

Table 7. Examples of inverting the measured aerosol scattering phase functions.

Parameters		Region						Fog[14]	
		Karadag[8]			Tomsk[8]			class 8	class 9
initial data	β_{as}	0.15	0.20	0.30	0.15	0.20	0.30	1.75	3.50
	$\gamma_a(20°)$	6.27	6.7	7.36	5.51	5.80	6.24	7.38	7.77
	$\gamma_a(120°)$	0.146	0.134	0.119	0.247	0.221	0.190	0.114	0.0741
	ε_0	−0.0017	0.031	0.077	−0.012	0	0.023	0.184	0.246
Rasults of inverting	$\tilde{s}$	0.943	1.098	1.332	1.051	1.090	1.174	1.696	1.808
	$\tilde{t}$	1.729	1.347	0.935	0.773	0.822	0.812	0.382	0.480
	$\tilde{n}$	1.59	1.50	1.41	1.39	1.40	1.40	1.30	1.316
	$\tilde{\omega}_a$	0.34	0.36	0.38	1.78	1.53	1.25	0.81	0.53

However, one should note that the values of indices n and ω_a obtained in such a numerical experiment for the conditions of Karadag and Tomsk do not allow one to estimate their reality (the values n and ω_a are not presented in Ref. 8).

One can draw a more definite conclusion about the correspondence of the refractive index n to reality by inverting the scattering phase functions obtained in the near-ground layer.[14] Two flat scattering phase functions of 8-th and 9-th class characteristic of the fogs with the mean meteorological visual range $S_m = 2$ and 1 km, respectively, are presented in this paper. One can assume for the mentioned conditions that the scattering phase functions were formed exclusively by the fog particles (one can ignore the Rayleigh component), and the refractive index n should be practically equal to the refractive index of water $n = 1.33$.

The values of t and s and indices n and ω_a obtained from the numerical experiment on inverting the fog scattering phase functions are shown in Table 7. The indices $n_8 = 1.30$ and $n_9 = 1.316$ are very close to the refractive index of fog droplets $n = 1.33$. The decrease of the Angstrom index ω_a from 0.81 to 0.53, as the meteorological range decreases, is an evidence of the relative increase in the number of coarse particles that is in a good agreement with the conclusions of the fog physics.

However, one should note that the experimentally measured scattering phase functions[8,14] were obtained by averaging a large bulk of data. So one can assume that they do not contain random errors of measurement. The problems of inverting the experimental scattering phase functions measured with an error by the proposed method should be studied.

We have carried out model calculations of the indices n and ω_a from the data on "measured" scattering phase functions at six scattering angles $\theta = 15$, 20, 30, 110, 120, and 130° at two wavelengths $\lambda = 400$ and 800 nm. We obtained 3×3×2 pairs of equations and, hence, 18 pairs of the values n_i and ω_{ai}. Supposing that the indices n and ω_a do not significantly change in the limits of spectral measurements, the weighted-mean values $\bar{n}$ and $\overline{\omega_a}$ were determined by averaging the values n_i and ω_{ai}.

The total errors in reconstructing the indices n and ω_a were $\sigma(\Delta n) = 0.004$ and $\sigma(\Delta\omega) = 0.19$ at the error in "measuring" the scattering phase functions $\sigma\gamma = 3.8\%$, at the scattering angles 15–30°, and $\sigma\gamma = 9.35\%$ at $\theta = 110–130°$.

The model estimates show that the accuracy of reconstruction of the indices n and ω_a by the proposed method is as good as that of the methods for experimentally measuring these characteristics. The method is also attractive because it allows one to promptly obtain the data on the characteristics of aerosol microstructure *in situ*, including the indices n and ω_a in the atmospheric column from the data of optical measurements of the scattering phase functions.

At the same time, one should keep in mind that the aforementioned estimates of the capabilities of the proposed method in inverting the aerosol scattering phase functions are obtained from the data of model calculations at a number of simplifying assumptions (there is no significant absorption of radiation by particles, all particles are homogeneous, spherical and have the same refractive index, the particle size distribution is close to the inverse-power law and can be characterized by the index of the disperse composition $\nu^* = \omega_a + 2$, etc.).

In this connection it seems to be very urgent to carry out further experiments on the studying capabilities and applicability of the proposed method.

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