HYBRID SCHEME OF FORMING A LASER REFERENCE STAR

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Received May 19, 1997

The results are presented connected with the construction of an algorithm for total wave-front tilt correction of a real star image based on the data of measuring the angular position of a laser reference star. A hybrid scheme of formation of the laser reference star is used, because to measure the position of this star, three telescopes are used: one of them operates in the regime of monostatic star and two others in the regime of bistatic star.

This article is logical continuation of a number of publications.[1–5] In addition, there exists a definite connection of the main idea of this paper and publications of Ragazzoni[6] and especially Belen'kii.[7] However, in spite of its similarity, especially to the results of Ref. 7, there are significant differences which are discussed at the end of the paper. For correct comparison of our results with the data obtained in Ref. 7, the same designations are used, where possible.

To implement the proposed correction algorithm based on the hybrid scheme of forming a laser reference star, three telescopes should be used: principal and two auxiliary telescopes placed so that their configuration forms an isosceles rectangular triangle. The following scheme of forming the laser reference star is realized: a wide Gaussian laser beam is focused with the principal telescope at the distance X. The star is formed solely by the central part of the principal telescope (it is assumed that the initial laser beam diameter $a_0 < a_{\text{т}}$, where $a_{\text{т}}$ is the aperture diameter of the principal telescope).

In the focal plane of the principal telescope, the angular jitter in the image center of gravity of the laser reference star is measured along the OY and OZ axes. Simultaneously, in the focal planes of two auxiliary telescopes the angular shifts of the image along one of the two axes are measured in the direction transverse to the corresponding direction of separation of the axes of principal and auxiliary telescopes.

The laser reference star formed by focusing of the laser radiation represents a long cylinder with diameter a_{m} and length a_{b}, that is, $a_{\text{b}} >> a_{\text{m}}$. Suppose that the separations of principal and auxiliary telescopes are such that for the auxiliary telescopes the laser reference star is formed by the bistatic scheme.[т,6,7] In this case, the size of a laser beacon $\hat{a}_{\text{b}}$ (connected with a_{b}, altitude of star formation X, and separations between the axes of auxiliary telescopes and the principal telescope), seen from the points of location of the auxiliary telescopes is much greater than the beacon size seen from the point of the principal telescope location ($\hat{a}_{\text{b}} \gg a_{\text{m}}$).

Thus, we obtain that for the principal telescope, the formed star can be considered as monostatic. Then instantaneous position of its image (on the OY and OZ axes) is

$$\begin{aligned} \varphi_{\text{m,y}} &= \varphi_{\text{lb,y}} + \varphi_{\text{ps,y}} , \\ \varphi_{\text{m,z}} &= \varphi_{\text{lb,z}} + \varphi_{\text{ps,z}} , \end{aligned} \qquad (1)$$

where ($\varphi_{\text{lb,y}}$; $\varphi_{\text{lb,z}}$) specify the instantaneous angular positions (on the axes) of the gravity center of the laser beam focused at the distance X into the turbulent atmosphere; ($\varphi_{\text{ps,y}}$; $\varphi_{\text{ps,z}}$) specify the instantaneous angular positions of the image of the focused laser beacon considered as a point source. The auxiliary telescopes measure only one component of the image jitter of the laser reference star, that is, finally we have the following pair of measurable angles:

$$\begin{aligned} \varphi_{\text{b,y}} &= \varphi_{\text{lb,y}} + \varphi_{\text{ss,y}} , \\ \varphi_{\text{b,z}} &= \varphi_{\text{lb,z}} + \varphi_{\text{ss,z}} , \end{aligned} \qquad (2)$$

where ($\varphi_{\text{ss,y}}$; $\varphi_{\text{ss,z}}$) characterize the instantaneous angular positions of the image formed by an extended incoherent source, most correctly calculated in Ref. 7. Further, we calculate the corresponding differences:

$$\begin{aligned} \varphi_{\text{m,y}} - \varphi_{\text{b,y}} &= \varphi_{\text{ps,y}} - \varphi_{\text{ss,y}} , \\ \varphi_{\text{m,z}} - \varphi_{\text{b,z}} &= \varphi_{\text{ps,z}} - \varphi_{\text{ss,z}} . \end{aligned} \qquad (3)$$

Because the auxiliary telescopes operate in the regime of the bistatic reference star, corresponding variances of differences (3) are expressed as

$$\begin{aligned} &<(\varphi_{\text{ps,y}} - \varphi_{\text{ss,y}})^{\text{т}}> = <(\varphi_{\text{ps,y}})^{\text{т}}> + <(\varphi_{\text{ss,y}})^{\text{т}}> = \\ &= <(\varphi_{\text{ps,y}})^{\text{т}}> \{1 + (\hat{a}_{\text{b}}/a_{\text{aт}})^{-1/3}\}, \end{aligned} \qquad (4)$$

where $a_{\text{aт}}$ is the size of the auxiliary telescope.

Now let us formulate the problem on optimal correction (decrease) for the angular jitter in the real star $\vec{\varphi}_{\text{ns}}(\varphi_{\text{ns,y}}, \varphi_{\text{ns,z}})$ on the basis of measured angles (1)–(3) and necessary calculations. In fact, we should

0235-6880/97/08 609-03 $02.00

minimize the variance of the residual angular shifts of the real star through the correction based on the measurements, namely:

$$\beta_y^{\mathrm{T}} = <[\varphi_{ns,y} - A\,(\varphi_{m,y} - \varphi_{b,y})]^{\mathrm{T}}>,$$
$$\beta_z^{\mathrm{T}} = <[\varphi_{ns,z} - A\,(\varphi_{m,z} - \varphi_{b,z})]^{\mathrm{T}}>. \quad (5)$$

Taking the advantage of the results obtained in Refs. 2–5, we have (for the isotropic spectrum of turbulence)

$$\beta_y^{\mathrm{T}} = \beta_z^{\mathrm{T}} = <(\varphi_{ns,y})^{\mathrm{T}}> \left\{1 - \frac{2^{1/3} f\,(X,\, C_n^{\mathrm{T}})}{[1 + (\hat{a}_b/a_{a\mathrm{T}})^{-1/3}]}\right\}, \quad (6)$$

where

$$f(X, C_n^{\mathrm{T}}) = \frac{\left(\int\limits_0^X \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)\,(1-\xi/X)\,[1+b^{\mathrm{T}}(1-\xi/X)^{\mathrm{T}}]^{-1/6}\right)^2}{\int\limits_0^X \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)\,(1-\xi/X)^{5/3} \int\limits_0^\infty \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)}, \quad (7)$$

$<(\varphi_{ns,y})^{\mathrm{T}}>$ is the variance of the angular shift of the real star image (along one axis); $b = a_0/a_{a\mathrm{T}}$. Optimal value of the correcting coefficient A, minimizing functionals (5), is calculated for the average model vertical profiles of the structure parameter for the reflective index in the atmosphere $C_n^{\mathrm{T}}(\xi)$, characterizing the turbulent intensity

$$A_{opt} = \frac{2^{1/6}\int\limits_0^X \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)\,(1-\xi/X)\,[1+b^{\mathrm{T}}(1-\xi/X)^{\mathrm{T}}]^{-1/6}}{(1+(a_b/a_{a\mathrm{T}})^{-1/3})\int\limits_0^X \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)\,(1-\xi/X)^{5/3}}. \quad (8)$$

Let us estimate numerically the efficiency of this correction for the real parameters of the experiment. Let the principal telescope have the diameter varying between 3 and 10 m. The auxiliary telescopes we select from one-meter telescopes. Let the diameter of the laser beam forming the star be $a_0 = 1$ m. The wave parameter for the focused laser beam Ω ($\Omega = ka_0^{\mathrm{T}}/X$) is in the interval 10–100 for altitudes X varying from 10 to 100 km. Hence, in the focal waist the laser beacon size is $a_m = 1$–10 cm. Thus, the laser star cross section is seen by the principal telescope at angles $\theta \le 0.1''$, which practically can be considered as a point source. At the same time, the length of the laser star is a_b and hence for proper separation of the auxiliary telescope axes, the visible size of the star $\hat{a}_b$ may be several minutes of arc, that is, the laser star can be considered as an extended incoherent source in the image planes of the auxiliary telescopes. The real ratio is $\hat{a}_b/a_{a\mathrm{T}} \approx 10^3$, $b = 1$. In calculations, we used the average model $C_n^{\mathrm{T}}(\xi)$ suggested in Ref. 8.

Summarizing these data and making calculations, we obtain for Eqs. (6)–(8)

$$\Delta = \frac{\beta_y^{\mathrm{T}}}{<(\varphi_{ns,y})^{\mathrm{T}}>} = \frac{\beta_z^{\mathrm{T}}}{<(\varphi_{ns,z})^{\mathrm{T}}>} = \left(1 - \frac{2^{1/3} f(X,\, C_n^{\mathrm{T}})}{(1 + 0{,}1)}\right), \quad (9)$$

$$A_{opt} = \frac{2^{1/6}\int\limits_0^X \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)\,(1-\xi/X)\,[1+(1-\xi/X)^{\mathrm{T}}]^{-1/6}}{1.1\int\limits_0^X \mathrm{d}\xi\, C_n^{\mathrm{T}}(\xi)\,(1-\xi/X)^{5/3}}. \quad (10)$$

Results of numerical calculations are tabulated. The data for the case of nonoptimal correction (that is, for $A = 1$) are also given in Table I. In this case,

$$\Delta = \frac{\beta_y^{\mathrm{T}}}{<(\varphi_{ns,y}^{\mathrm{T}})^{\mathrm{T}}>} = 1 + [1+(\hat{a}_b/a_{a\mathrm{T}})^{-1/3}]\frac{\int\limits_0^X \mathrm{d}\xi\; C_n^{\mathrm{T}}(\xi)\;(1-\xi/X)^{5/3}}{\int\limits_0^\infty \mathrm{d}\xi\; C_n^{\mathrm{T}}(\xi)} - 2^{7/6}\frac{\int\limits_0^X \mathrm{d}\xi\; C_n^{\mathrm{T}}(\xi)\;(1-\xi/X)\;[1 + b^{\mathrm{T}}(1-\xi/X)^{\mathrm{T}}]^{-1/6}}{\int\limits_0^\infty \mathrm{d}\xi\; C_n^{\mathrm{T}}(\xi)}. \quad (11)$$

Thus, from the table it can be seen that already for the altitude of laser reference star formation higher than 10 km, this algorithm effectively corrects for the jitter in the real star image on the basis of measuring (two components) of the jitter in the monostatic star image in the principal telescope and individual components of the jitter in two perpendicular separated telescopes.

TABLE I.

X, km	A_{opt}	Δ from Eq. (9)	Δ from Eq. (11)
1	1.22	0.509	0.5139
10	1.096	0.1799	0.1802
100	1.019	0.0866	0.0927

It should be noted that in practice there is no need to optimize the correction by this scheme (that is, specially calculate the parameter A); nonoptimal correction (for $A = 1$) also highly efficiently corrects for the angular shift of the real star within the limits of isoplanatic angles with the use of the laser reference star.

Apparently, our results should be compared with the data of Ref. 7. First of all, we note that the final result is that the control signal so obtained, in contrast with the results of Belen'kii,[7] is completely independent of the laser beam characteristics. In Ref. 7, the useful signal for the correction is φ_{fa}, that is, the wave-front tilt on the entire aperture. At the same time, the total tilt of the beam φ_{lb} (in the beacon plane) is

$$\varphi_{lb} = \varphi_{fa} + \varphi_{lt} , \quad (12)$$

where φ_{lt} is the local tilt of the beam.

In this treatment, φ_{fa} is determined **by integration over the path in the upward direction** (although from Ref. 7 it is not clear, what wave: plane? laser beam? spherical wave?). However, on the backpath (in formulation of Belen'kii) this term is compensated in Eq. (12); therefore, in the principal telescope we measure

$$\varphi_m = \varphi_{lb} - \varphi_{fa} = \varphi_{lt} .$$

It is well known that in the monostatic scheme the variance of the angular jitter

$$\langle\varphi_m^{\text{T}}\rangle = \text{const}\,(a_0^{-1/3} + a_{\text{T}}^{-1/3} - 2^{7/6}\,(a_0^{\text{T}} + a_{\text{T}}^{\text{T}})^{-1/6}) \times$$

$$\times \int\limits_0^X d\xi\; C_n^{\text{T}}(\xi) \left(1 - \frac{\xi}{X}\right)^{5/3},$$

therefore, the complete correction can be realized only for the case $a_0 = a_{\text{T}}$. In this case, jitter in the beam of diameter a_0 is compensated by the point source jitter on the aperture a_{T} (when $a_0 = a_{\text{T}}$). Thus, the useful signal for the correction is the angle φ_{fa} (in Belen'kii's opinion, this is a portion of the jitter in the beam propagating **upward** caused by the entire aperture of the telescope). However, this is not the case. Most probably, separation of the term φ_{fa} from sum (12) is a far-fetched maneuver in the chain of explanation.

In my reasoning, the useful signal (for the correction) is the difference $\varphi_{ps} - \varphi_{ss}$, representing the image jitter of the point source without the average jitter of the secondary incoherent sources, being the difference of two measurements $\varphi_m - \varphi_b$; in this case, the useful correction signal represents the results of integration (practically for the point source) over the **upward** propagation path, whereas the signal obtained by Belen'kii (φ_{fa}) is the portion (see formula (12)) of the beam jitter φ_{lb}, connected with the integration of the turbulence over the **upward** propagation path.

I would say that the method suggested by Belen'kii is true in the essence of its operations, but was explained incorrectly; as a result, this complicates the understanding of actual operation of this algorithm.

And finally, the main point is that the angle φ_{lb} from (12) does not comprise the term, which would be useful for efficient correction for the tilt angle of the real star φ_{ns}. In my opinion, Belen'kii offers very strange explanation for each term of sum (12). According to his treatment, the total tilt of the laser beam φ_{lb} (in the plane X) does not depend on the size of the principal mirror, but each term of sum (12) depends on this size. I am convinced this is not the case: the total tilt of the beam φ_{lb} cannot comprise any components that depend on the characteristics of foreign objects, namely, of the principal telescope in this case, rather than on the characteristics of the beam itself (beam diameter and distance of focusing) and the propagation medium.

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