MATHEMATICAL MODELING OF THE CLIMATIC DISTRIBUTION OF AEROSOLS

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This paper presents an approach to description of distribution of atmospheric aerosols which uses stochastic models that are based on solution to a boundary–value problem with random coefficients. Behavior of the coefficients is described by a multidimensional function of the probability density. This function is chosen so that it provides the closest approximation of the distribution laws established based on the results of long–standing observations. The diagnostic and prognostic (change of configurations of flooded and recultivated areas) versions of the calculations of the ash emission power of ash receptacles at heat and power plants are presented.

Methods of mathematical modeling of the processes of atmospheric pollution are usually used for calculation of absolute concentrations of ingredients under specific meteorological conditions. These methods include numerical, analytical and standard procedures. At the same time, it is of interest to look at the ecological situation in an economical region as a whole taking into account statistically stable specific features of its climate. Great computer resources are required for numerical solution to this problem.[1–4] Different versions of standard procedures[5,6] are not able to solve this problem. It seems to be optimal in this case to use stochastic models based on solution to boundary–value problem with random factors. The random factors are determined by a multidimensional function of probability density that, in its turn, is determined by minimum mismatch with empirical distribution laws.

The problem can be simplified when using analytical solutions to differential equations for transfer and turbulent diffusion of admixtures. Although these analytical solutions are obtained by certain simplification of the atmospheric processes, they can be utilized if distribution of admixtures is related, in a single–valued manner, to integral and differential probability functions of the distribution of meteorological parameters.

Let us consider this approach in more detail. Let certain limit for concentration of light admixture in the atmosphere be established (for instance, maximum permissible concentration (MPC): average daily MPC, peak MPC, MPC for industrial region, etc.). When analyzing analytical solutions obtained from differential equations for transfer and turbulent diffusion of light passive admixture[7,8] produced by elevated sources, it can be seen that the higher is the absolute value of wind velocity V, the lower is the concentration q. This means that if the wind speed V is higher than certain critical value V_c, any source of pollution produces surface concentration of admixtures lower than MPC. Let us assume that concentration of admixtures produced by an arbitrary system of sources is a superposition of concentration fields produced by each source separately. Then, using analytical solutions to differential equations,[7,8] we find that critical value of the wind velocity at some point is a function of polar coordinates (ρ, φ) and parameters of sources F contributing into the pollution of air at this point:

$$V = f(\rho, \varphi, F) .$$

After integration of the two–dimensional probability density function of wind velocity at each point of the region over wind directions ($0 < \varphi < 360$) and wind speed ($0 < V < V_c$) we obtain the distribution function of MPC excess. In other words, after solution to such a problem for light passive admixtures the probability space, where the integral function has values lower than the chosen criteria at any point within a certain time period, is omitted. Moreover, the model operates with a space, where function of provision plotted in Borel set serves as a probability measure.

Normal distribution law, Laplas––Sharley approximation or Waybull distribution law may be chosen as theoretical probability function depending on their affinity to empirical distribution law according to long–term climatic observations at meteorological points of this region. Thus, we find zones where the established norm of pollution is broken and put into consideration a new parameter – frequency of MPC excess. Simultaneously, we have concentration of admixtures averaged over the total number of samplings at each point of the region. It should be pointed out that in analytical solutions abscissa coincides with the direction of the averaged wind. Hence, calculation of pollution at each point is carried out in rotating polar coordinate system.

Calculation of the frequency of peak MPC for sulphur dioxide (0.5 mg/m^3) excess in December in Irkutsk (from long–term observations) is presented in Fig. 1 as an illustration of our discussion. The distribution law of wind velocity was a generalized normal law, analytical solution obtained in Ref. 7 was used, the horizontal factor of turbulent diffusion was chosen according to Monin––Obukhov scale, the vertical factor was approximated by power law. The exponent of power law was found for every season by the least squares method according to data observed at high platforms of a TV tower. For convenience, the frequency is standardized by the number of hours in a month: isoline *1* relates to 72 hours and corresponds to probability of MPC excess being

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equal to 0.1, isoline *2* relates to 144 hours (probability equals 0.2), and so on. The origin of coordinates coincides with the city center. The regions of ecological risk indicated in Fig. 1 are useful information to come to rational decisions.

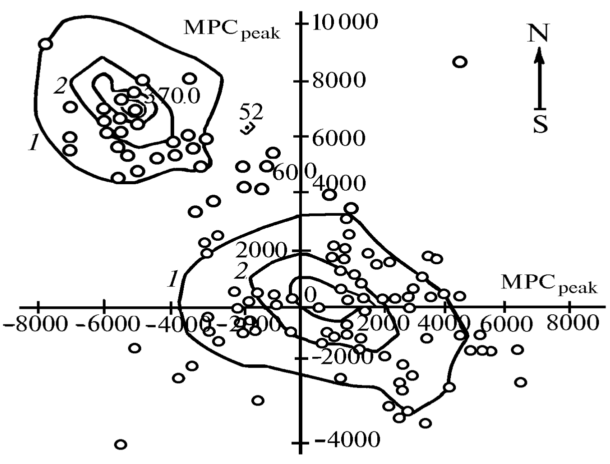

FIG. 1. Frequency of MPC excess for sulphur dioxide in Irkutsk in December (É indicates point sources).

Basic principles of this method are also valid for description of distribution of heavy particles that appear in the atmosphere from elevated sources[9] and ash receptacles at heat and power stations and mines. In this case, besides the characteristics that are described above, it is possible to obtain the accumulation of solid particles on underlying surface for a certain period of time and to identify zones of ecological emergency. However, dust emissions from stocks and ash receptacles are different. Indeed, the maximum concentration of solid particles near elevated sources occurs under calm weather conditions, whereas the dust emission from ash receptacles is minimum in this case. Hence, taking into account polydispersity of dust we have to note that the problem of emission from ash receptacles is more complicated. In fact, it is first necessary to find the critical speed of dust taking off and, second, to evaluate the intensity of emission.

Verification of the model presented here was carried out for ash receptacles at heat and power plants No. 4 and 5 in Omsk and at a heat and power plant in Kamensk settlement. Diagnostic versions of calculations were checked in direct experiments. This enabled us to use our calculations for forecasting the versions for reconstruction of ash receptacles, changes in their emitting surface, inundated zones, marks of comb, height of dams, and partial and complete recultivation of sections.

Let us illustrate our discussion by specific fragments of calculation of dust emission from ash receptacle at the heat and power plant No. 5 in Omsk. The size–spectrum of dust particles is presented in Table I; the average covering density is 900 kg/m^3.

TABLE I. Granularity composition of ash receptacle.

Diameter of the particles, mm	Content of the particles, %
2.000–1.000	1
1.000–0.500	2
0.500–0.250	4
0.250–0.100	25
0.100–0.050	58
0.050–0.010	5
0.010–0.005	1
<0.005	4

Calculations were done taking into account critical speeds of the displacement of the particles of each size[10–12] for different versions of inundating the open surfaces of ash receptacles. Two operating sections of ash receptacles at the heat and power plant No. 5 in Omsk are shown in Fig. 2. The blocking dams emit ash. This is diagnostic version of calculation. Isoline *1* corresponds to 0.01 of MPC value (0.5 mg/m^3). This isoline contours a zone where up to 6 000 mg/m^2 (or kg/km^2) of ash is accumulated during the period of time, when the emitting surface is open. Isoline *2* represents 0.1 of MPC value (60 000 mg/m^2), isoline *3* is plotted for 0.5 of MPC value (300 000 mg/m^2), isoline *4* corresponds to MPC value (600 000 mg/m^2), isoline *5* relates to 2 MPC value (1 200 000 mg/m^2), and, finally, isoline *6* contours the 3 MPC value zone (1 800 000 mg/m^2). Isoline *4* contours zone of ecological emergency. Figure 3 is forecasting one. Two sections emitting at present (see Fig. 2) will be recultivated. New sections with emitting beaches of 50–70 m wide at altitude of 8.5 m and mark of comb of 122.6 m along the dams are planned. The values of isolines are the same as that in Fig. 2.

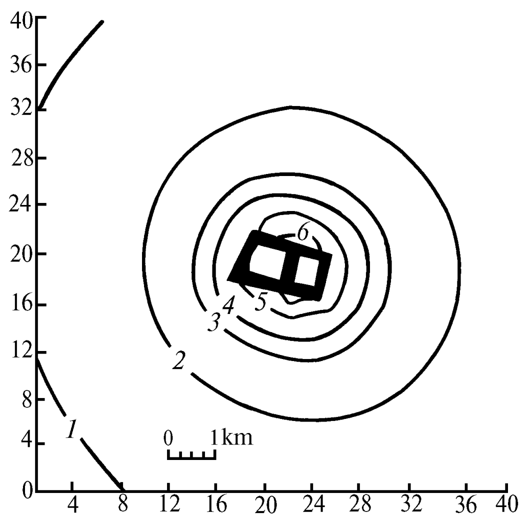

FIG. 2. Diagnostic version of dust emission of ash receptacle at the heat and power plant No. 5 in Omsk: black region corresponds to dust emitting beaches and white region relates to surface covered with water.

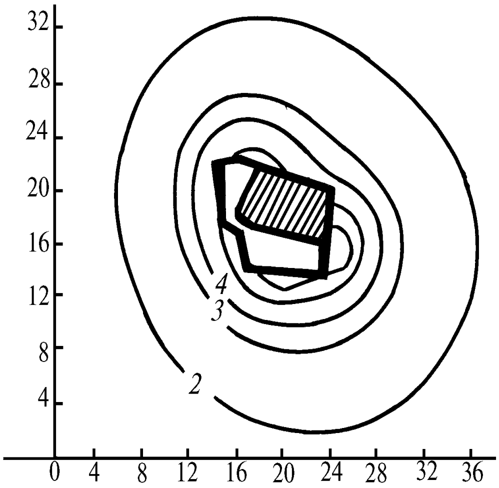

FIG. 3. Forecasting version of dust emission of ash receptacle at the heat and power plant No. 5 in Omsk: black band relates to dust emitting beaches, white region relates to surface covered with water, and shaded region relates to recultivated surface.

Since the particles in the ash receptacle are relatively large, the majority of dust covers the underlying surface not far from it. The long–dinstance transfer of the particles is possible in the following cases: 1) small size of particles (but their content in ash receptacles is negligible) and 2) high wind speed, but probability of this event is low in Omsk. (Indeed, according to observations at the meteorological station of Omsk the wind speed of 21 m/s is possible once a year, 23 m/s is possible once a five years period, 24 m/s – once a ten years period, 25 m/s – once a fifteen years period, and 26 m/s – once a twenty years period).

Thus, mathematical models developed and presented here enable one to evaluate the ecological situation in the region and to come to optimum decisions.

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