DETERMINATION OF LASER BEAM STRUCTURE WITH THE ACOUSTIC METHOD UNDER KINETIC AIR COOLING CONDITIONS

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Received April 16, 1990

The possibility of restoring the cross-sectional energy density distribution of a laser beam from the air pressure measurements in a divergent sound wave produced by a laser pulse in the atmosphere is discussed taking into account the gradual transfer of the absorbed radiation energy into heat and kinetic cooling. Some results of the experiments conducted with CO_2-laser radiation in the atmosphere are presented.

Measurements of sound pulses generated during laser radiation absorption in liquids and gases and the processing of these data with the help of computational-tomography methods enable one to reconstruct the energy distribution over the cross section of a laser beam. The reconstruction method was described in Ref. 1. Experimental tests were reported in Refs. 2 and 3. In the experiment on sound generation induced by laser radiation in the visible spectral range in liquid[2] the energy distributions over the beam, reconstructed from acoustic measurements, are in good agreement with those made photometrically. The applicability of the acoustic method to the real atmosphere was also confirmed by measurements of CO_2 -laser radiation along the atmospheric path in summer.[3] Winter measurements, however, showed that the requirement of a small thermalization time (the time necessary for transforming laser radiation energy into heat) compared with the smallest characteristic time of variation of the sound pressure, which was imposed while confirming the acoustic measurement method,[1] is not valid in a number of cases. This requirement is not valid for conditions of low air humidity[3] when absorption of the 10.6-μm wavelength radiation by water vapor and CO_2 in the air can be comparable in magnitude, or when the absorption due to CO_2 greatly exceeds the absorption due to water vapor. In this case a kinetic air cooling effect can exist[4–7] when the transformation of the absorbed energy into heat occurs within a time of the order of 10^{-3} s, which is equal to or exceeds the duration of the sound pulses generated by a beam with transverse dimensions less than 1 m.

In this paper we propose a method for retrieving the beam structure, taking into account the finite thermalization time and the kinetic cooling effect (KC). Its applicability was confirmed by simultaneous acoustic and bolometric measurements carried out during studies of the propagation of 10.6 μm radiation along an atmospheric path.

MATHEMATICAL STATEMENT OF THE PROBLEM

Variation of the density ρ, velocity $\vec{V}$, pressure P, air enthalpy h, and internal vibrational energy of the molecules E_v during the absorption of optical radiation with power density J is described by the equations

$$\frac{\partial\rho}{\partial t} + V\nabla\rho + \rho\ \mathrm{div}V = 0; \tag{1}$$

$$\frac{\partial V}{\partial t} + (V\nabla)\ V = -\frac{\nabla P}{\rho}; \tag{2}$$

$$\rho\left(\frac{\partial h}{\partial t} + V\nabla h\right) - \left(\frac{\partial P}{\partial t} + V\nabla P\right) = \alpha_t J; \tag{3}$$

$$\rho\left(\frac{\partial E_v}{\partial t} + V\nabla E_v\right) = 2.44\ \alpha_{CO_2} J - \rho\frac{E_v}{\tau}; \tag{4}$$

$$h = \frac{\gamma}{\gamma - 1}\frac{P}{\rho} + E_v, \tag{5}$$

where α_t is the total absorption coefficient of the laser radiation by water vapor, carbon dioxide, and other absorbing species in air; α_{CO_2} is the coefficient of radiation absorption by CO_2; and τ is the thermalization time.

A physical picture of the radiation absorption process and its effect on the thermodynamic characteristics of air are as follows.[4–7] The absorption of 10.6 μm radiation causes the vibrational-rotational modes in H_2O and CO_2 to be excited. Transformation of the energy of the excited mode into heat is brought about by collisions with other molecules. The energy absorbed by water vapor is transformed into heat during a very short period of time[5] equal to ~ 10^{-7} s. The energy absorbed by CO_2 takes much longer to be transformed into heat. This absorption, to start with, leads to an inverse laser transition from the state (10°0)

0235-6880/90/08 792-07 $02.00

with energy equal to 1390 cm^{-1} with respect to the ground state into the state (00°1) with energy equal to 2350 cm^{-1}. In a time of the order of 10^{-7} s vibrational energy is transferred from a CO_2 molecule in the state (00°1) to a N_2 molecule, which has the same transition energy from the ground state to the excited state as the energy of the CO_2 level (00°1).

The internal vibrational energy of the N_2 molecules (the ratio of the energy of the CO_2 level (00°1) to the energy of the transition (10°0)–(00°1), equal approximately to 960 cm^{-1}) is 2.44 times higher than the absorbed radiation energy. The CO_2 mode (10°0), depleted by absorption, is replenished as a result of molecular collisions, cooling the air. The cooling time τ is determined by the lifetime of the excited N_2 molecule. Since the transition from this level to the ground level is forbidden, excitation deactivation runs through a complicated system of transitions including the excitation of O_2, H_2O, and CO_2 molecules. According to the estimates in Ref. 5 the value of τ at sea level is equal to 0.005 and 0.003 s at 30% and 100% humidity, respectively. Note that theoretical estimates of τ are given only for the case of monochromatic radiation propagation. Powerful lasers generate radiation with a complex frequency spectrum and in this case it is impossible to obtain reliable theoretical calculations of the τ values. All the more valuable and informative then would be an experimental determination of the relaxation time.

Linearization of Eqs. (1)–(5) with respect to the initial values ρ_0, P_0, and h_0, assuming $\vec{V}_0 = 0$ and $E_{v,0} = 0$, leads to an equation which describes the variation of the pressure under the action of radiation

$$\frac{\partial^2 P}{\partial t^2} - U^2 \Delta_\perp P = \alpha_t (\gamma - 1) \frac{\partial}{\partial t}\left[J - \frac{\partial E}{\partial t}\right]; \tag{6}$$

$$\frac{\partial E}{\partial t} + \frac{E}{\tau} = 2.44 \frac{\alpha_{CO_2}}{\alpha_t} J, \tag{7}$$

where $U^2 = \gamma \dfrac{P_0}{\rho_0}$ is the sound velocity in the medium and $E = \rho_0 E_v$.

Replacing $2.44 \dfrac{\alpha_{CO_2}}{\alpha_t}$ by the parameter c we rewrite Eq. (7) in the following form:

$$E(t) = c \int\limits_0^t J(t') \exp\left[-\frac{t - t'}{\tau}\right] dt'. \tag{8}$$

Then, the solution of Eq. (6) in the polar coordinate system (r, φ) with the origin at the test point reads

$$P(t) = \frac{1}{2\pi U} \int\limits_0^t dt' \int\limits_0^{U(t-t')} \frac{\partial F(r, t')}{\partial t'} \times$$

$$\times \frac{r dr}{\sqrt{U^2(t - t')^2 - r^2}}, \tag{9}$$

where

$$F(r, t) = \alpha_t(\gamma - 1) \int\limits_0^{2\pi} \left[(1 - c) J(r, \varphi, t) + \right.$$

$$\left. + \frac{c}{\tau} \int\limits_0^t J(r, \varphi, t') \exp\left[-\frac{t - t'}{\tau}\right] dt'\right] d\varphi. \tag{10}$$

If the duration τ_p of the acting laser pulse is much less than the characteristic time needed for the sound to cross the beam $\tau_s \approx \dfrac{d}{u}$, where d is the characteristic spatial distance of the intensity distribution, then the distribution $J(r, t)$ can be taken as a delta-function of the time

$$J(r, \varphi, t) = Q(r, \varphi)\, \delta(t), \tag{11}$$

where $Q(r, \varphi)$ is the energy density over a beam cross section and the δ-function is defined such a way

$$\int\limits_0^t \delta(t') f(t - t') dt' = \Theta(t) f(t),$$

$$\Theta(t) = \begin{cases} 0, & t \le 0 \\ 1, & t > 0. \end{cases}$$

Consequently

$$F(r, t) = \alpha_t(\gamma - 1) W(r) \left[(1 - c)\, \delta(t) + \right.$$

$$\left. + \frac{c}{\tau} \exp(-t/\tau)\, \Theta(t)\right], \tag{12}$$

$$W(r) = \int\limits_0^{2\pi} Q(r, \varphi)\, d\varphi. \tag{13}$$

We define the function

$$P_0(t) = \frac{\partial}{\partial t} \int\limits_0^{2\pi} \frac{W(r) r\, dr}{\sqrt{U^2 t^2 - r^2}}, \tag{14}$$

where $P_0(t)$ is the pressure at the point $r = 0$ in the case of instantaneous (at $\tau = 0$) transformation of the absorbed laser radiation energy into heat. Then Eq. (9) can be written in the form

$$P(t) = (1 - c)\, P_0(t) + \frac{c}{\tau} \times$$

$$\times \int\limits_0^t P_0(t') \exp\left[-\frac{t - t'}{\tau}\right] dt' \tag{15}$$

or

$$\tau \frac{dP}{dt} + P = (1 - c)\tau \frac{\partial P_0}{dt} + P_0. \quad (16)$$

If we consider Eqs. (15) and (16) as equations in $P_0(t)$, their solution will have the form

$$P_0 = \frac{1}{1-c}\left[P(t) - \frac{c}{\tau(1-c)} \times \right.$$

$$\left. \times \int_0^t P(t') \exp\left[-\frac{t-t'}{\tau(1-c)}\right] dt'\right], \quad \text{at } c < 1,$$

$$P_0(t) = P(t) + \tau \frac{dP}{dt}, \quad \text{at } c = 1, \quad (17)$$

$$P_0(t) = \frac{1}{1-c}\left[P(t) - \frac{1}{\tau(c-1)} \times \right.$$

$$\left. \times \int_t^\infty P(t') \exp\left[-\frac{t-t'}{\tau(c-1)}\right] dt'\right], \quad \text{at } c > 1.$$

If the values of the parameters c and τ are known it is possible with the help of Eq. (17) to retrieve the pressure $P_0(t, M_1)$ from the measured values $P(t, M_1)$ and the function

$$W(r_1, M_1) = B \int_0^{r_1/U} \frac{P_0(t, M_1)}{\sqrt{r_1^2 - U^2 t^2}} dt, \quad (18)$$

where M_1 are the points at which the pressure $P(t)$ is measured (see Fig. 1) and $B = \frac{2\pi u}{\alpha_t(\gamma - 1)}$. Then, it is possible to retrieve the distribution $Q(x, y)$ by applying the methods of computational tomography.

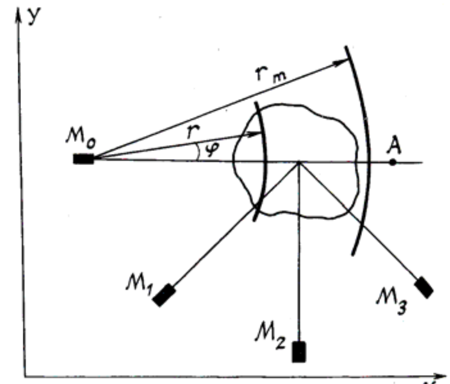

FIG. 1. Coordinate system and location of the acoustic detectors M_0 –M_3.

The parameter $c = 2.44 \frac{\alpha_{CO_2}}{\alpha_t}$ can be calculated if the temperature, humidity, and air pressure are known.[8,9] The thermalization time τ, except in the case in which it is evaluated theoretically, can be determined from acoustic measurements. To this end, it is sufficient to use *a priori* information about the finite beam aperture. If it is known when measuring the pressure at a point M_0 that a point A (Fig. 1) is located outside a circle with radius r_m, within which the beam passes, then the function $W(M_0, r)$ must be equal to zero for $r > r_m$. In the retrieval of this function using formulas (17) and (18), as numerical simulation shows, $W(r - \infty)$ can take both positive and negative values if the time τ, chosen during the retrieval process, is not equal to the actual thermalization time. The iteration procedure for determining the time τ can be constructed on the basis of the above-described property of the function W. In the presence of noise and errors in the measurements of the pressure $P(t)$, the relative error in determining τ stands at 14% at the 5% level of the acoustic noise and 30% at the 10% level. The error in determining τ associated with errors in c does not exceed 10% if the meteorological data are evaluated with the standard accuracy.

EXPERIMENTAL PROCEDURE

The experiment was conducted along a near-ground atmospheric path. A CO_2 laser with pulse duration much less than the time needed for the sound to cross the beam was used. Acoustic signals were recorded by a MK 102 detector with a PSI 0017 amplifier and a C8-13 recording oscilloscope. The detectors were located at a distance from the beam many times greater than its diameter. After recording and processing the acoustic signals the obtained functions $W(r)$ were compared with the results of direct bolometric measurements carried out simultaneously with the acoustic measurements. The bolometric detector is a grating with uniformly spaced thin wires, placed across the beam perpendicular to its axis. The resistance of each wire due to heating caused by the laser radiation was measured. The bolometric measurements were used to determine the function $W(x)$ by the relationship

$$W(x) = \int_{-\infty}^{\infty} Q(x, y)\, dy. \quad (19)$$

The functions $W(r)$ and $W(x)$, obtained under different conditions of radiation propagation, and the initial acoustic signals are plotted in Fig. 2. As can be seen from the figure, the acoustic data agree well with the bolometric measurements in reproducing the large-scale features of the energy distribution but differ somewhat in reproducing the small-scale inhomogeneities. The accuracy in the determination of the function $W(r)$ was approximately the same for the signals recorded under KC conditions as for the signals recorded without KC. The air temperature during the experiments varied between 7° and 25°C, the humidity varied from 2 to 15 g/m^3, and the cooling parameter c varied from 0.62 to 1.88. Thus, the application of the heat-release model to the problem under discussion proved to be valid for a wide variety of propagation conditions in the real atmosphere.

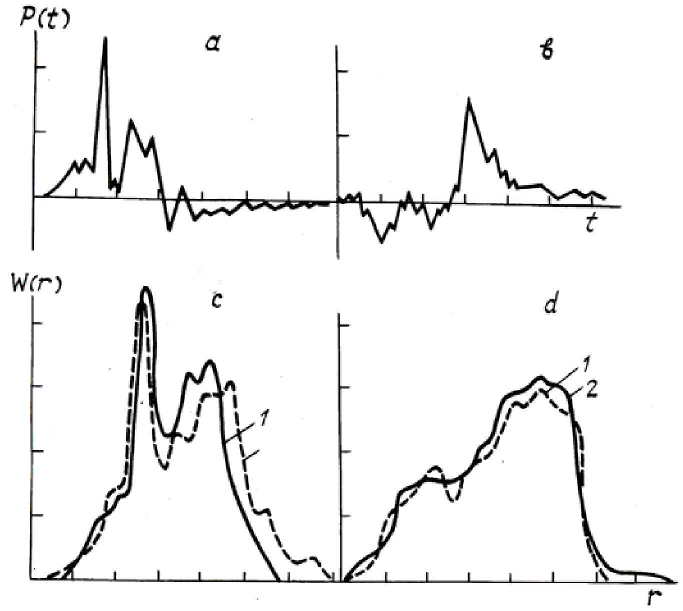

FIG. 2. Acoustic signals recorded at $T = 12.3°C$, $\rho = 10.5$ g/m^3, $c = 0.71$ *(a) and at* $T = 0.5°C$, $\rho = 2.8$ g/m^3, $c = 1.85$ *(b). The W(r) distributions were retrieved with the kelp of the above parameters (1) and the bolometric distributions (2). The heat-release time was equal to* 0.4 ms *(a) and* 1.6 ms *(b).*

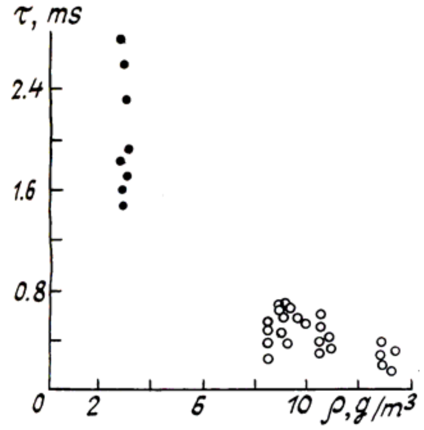

FIG. 3. Heat-release time as a function of the absolute air humidity: points — under KC conditions; circles — without KC.

Figure 3 shows the experimental dependence of τ on the water vapor content of the atmosphere. Despite the significant spread of the points one can observe a substantial decrease in the duration of the heat-release process with increase in the air humidity.

To determine the two-dimensional structure of the beam, the acoustic signals were recorded simultaneously from four directions (Fig. 1). The angle between two adjacent directions was equal to $\pi/4$. After passing through the acoustic measurement zone the radiation fell on photosensitive paper and left its trace on it. This trace characterized the qualitative energy distribution over the beam cross section. The beam was shaped so that its trace should consist of clearly distinguished separate spots. This experiment was carried out at $T = 12°C$, $\rho = 8.5$ g/m^3, $C = 0.9$.

The average calculated value of τequaled 0.4 ms. After processing all four signals according to the above-mentioned method, we obtained the functions (projections) $W_i(r)$ ($i = 1, 2, 3, 4$), from which the distribution $Q(x, y)$ was retrieved. The method of convolution and inverse projection, which is well known in tomography, was used.[10] To smooth out spurious structures in the retrieved distribution which occur due to the small number of the projections available, twelve additional projections of $W_k(r)$ (three between every two adjacent initial projections), calculated by linear interpolation from the initial ones, were used in the algorithm. Then, following Ref. 10, the distribution $Q(x, y)$ was calculated at the points arranged in the form of a 30×30 matrix and isolines were obtained.

The obtained tomographic image (b) and its corresponding trace (a) are represented in Fig. 4.

As can be seen from the figure, the intense and large features of the energy distribution, their relative dimensions, the distances between them, and their orientation relative to one another are sufficiently well retrieved by the acoustic method. Distortion in the reconstructed image is evidently caused by an insufficient number of projections as well as measurement errors.

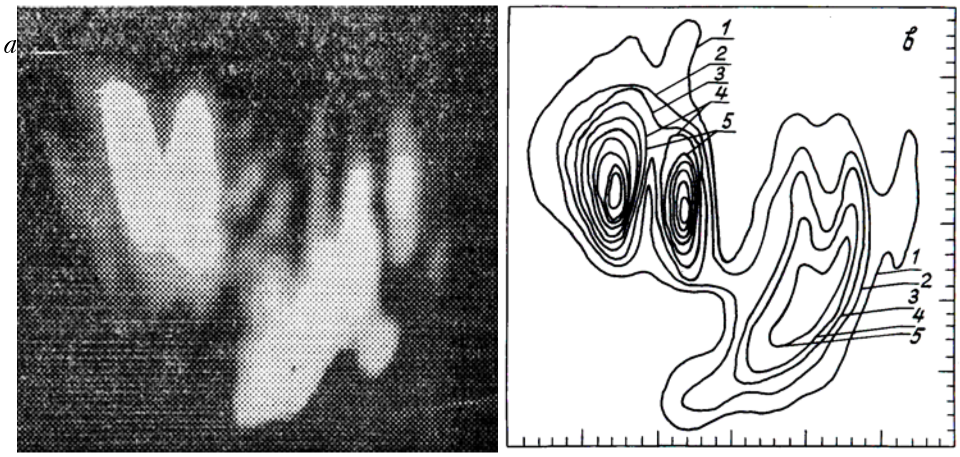

FIG. 4. The trace on photopaper (a) and the tomographic image (b): The isocontours of the tomogram correspond to different values of the function (K/10) Q_{max} for K = 1, 2 ... 10, Q_{max} is the maximum energy density.

CONCLUSIONS

We have shown that measurements of sound pulses during the absorption of 10.6 μm radiation in air allow one to determine the energy distribution of a laser beam if the kinetics of radiation absorption are accounted for within the framework of a simple model with one relaxation time. It is also possible to evaluate the relaxation time for this model from these measurements. In more detailed studies of the absorption kinetics and the relative contribution to the absorption coefficient from carbon dioxide, the use of laser beams with a known and stable cross-sectional energy distribution is promising.

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