MODELING OF ERRORS IN THE MANUFACTURE OF OPTICAL SURFACES

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Methods for modeling the most characteristic errors in the manufacture of optical surfaces, i. e., astigmatism and local errors, are presented. The derivation of formulas for the passage of a ray through such surfaces which take the above defects into account is based on the Herzberger technique.

In the manufacture of optical parts certain surface errors are inevitable. The most characteristic of these are astigmatic error (the surface loses its axially symmetric shape and the radii of curvature in the meridional and sagittal cross-sections differ from each other) and local error (axial symmetry is not affected in this case). Therefore, in the design of lidar optical components it is necessary to have estimates of the effect of these errors on image quality and determine their maximum permissible values. This is of especial importance in the use of large mirrors or lenses as receiving antennas, and in the design of optical transmitters for high altitude sensing, i.e., when strict requirements are imposed on the divergence of the collimated sounding beam. To make such estimates, it is necessary to create a mathematical model of a real surface (i.e., a surface with manufacturing errors), and then to calculate the beam path through the investigated system, replacing the theoretical surfaces by real ones, and thus to assess the image quality on the basis of known criteria.

MODEL OF SURFACE ASTIGMATISM

To model an optical surface with astigmatism, we will use the equation

$$F(x^*, y^*, z) = 2z + Cz + Cz^2 - Ax^{*2} - By^{*2} = 0, \qquad (1)$$

written in the coordinate system whose origin coincides with the apex of the surface and whose z-axis coincides with the line of intersection of the symmetry planes. Here A and B are the curvatures in the meridional and sagittal cross-sections; C is a constant which determines the shape of the surface. Equation (1) is a general expression which describes surfaces of the second order. For $A = B$, Eq. (1) is transformed into the equation of a surface of revolution; for $A = 0$ or $B = 0$, Eq. 1 describes cylindrical surfaces. Unlike Eq. 1, the modeling of the astigmatic surfaces by the toroid equation[1] is of particular character because the generatrix of the toroid surface is the circular arc described by Eq. 1 at $B = 0$ and $C = -1$.

We will use the Herzberger method to calculate the passage of a ray through the surface described by Eq. (1).[2] Let the incident ray be described by the vectors $\mathbf{A}(x, y)$ and $\varphi(\varepsilon, \eta)$ (x and y are the coordinates of the point of intersection of the incident ray and the XOY plane; ε and η are the optical direction cosines of the ray with respect to the axes X and Y, respectively). It can be shown that the corresponding vectors of the refracted (reflected) rays $\mathbf{A}'(x', y')$ and $\varphi'(\varepsilon', \eta')$ (x' and y' are the coordinates of the point in the XOY plane, and ε' and η' are the optical direction cosines of the beam with respect to the X and Y axes, respectively) are related to the Incident ray vectors by the following matrix equation:

$$\begin{pmatrix} A' \\ S' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} A \\ S \end{pmatrix}, \qquad (2)$$

where

$$\alpha = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix},$$

$$\gamma = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}, \quad \delta = \begin{pmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{pmatrix},$$

and the matrix elements are given by the following formulas: $\zeta^2 = n^2 - \varphi^z$, where n is the refractive index of the first medium;

$$X = \xi x, \quad Y = \xi y, \quad D = \xi^2 - AX\xi - BY\eta;$$

$$q^2 = D^2 - (AX^2 + BY^2)(A\xi^2 + B\eta^2 - C\zeta^2), \quad q > 0;$$

$$z = \frac{D - q}{A\xi^2 + B\eta^2 - C\xi^2} = \frac{AX^2 + BY^2}{D - q};$$

$$X^* = X + z\xi, \quad Y^* = Y + z\eta;$$

$$R = A(A + C)X^{*2} + B(B + C)Y^{*2} + \xi^2;$$

$q'^2 = (n'^2 - n^2)R + q^2$, n' is the refractive index of the second medium; $q' > 0$, $q' = -q$ for the case of reflection.

$$\Psi = (q'-q)/R,\ \ t = 1+\psi(1+Cz),\ \ \xi' = \xi t;$$

$$\gamma_1 = -A\psi\xi,\ \ \delta_1 = 1-A\psi z;$$

$$\gamma_2 = -B\psi\xi,\ \ \delta_2 = 1-B\psi z;$$

$$\alpha_1 = (\xi t-\gamma_1 z)/\zeta',\ \ \beta_1 = z(t-\delta_1)/\zeta;$$

$$\alpha_2 = (\zeta t-\gamma_2 z)/\zeta',\ \ \beta_2 = z(t-\delta_2)/\zeta.$$

MODEL OF LOCAL ERROR

According to Ref. 3 the local error is a deviation of the real surface from the theoretical one that does not violate the axial symmetry of the surface. Most typical is the local error at the edge, i.e., the edge of the surface is either lowered or raised. Occurrence of errors in the form of a prominence or a pit is also fairly frequent at the center of the surface or in any zone lying between the center and the edge. As a rule, the local errors worsen image quality considerably.

The authors of Refs. 4 and 5 suggest that real errors of manufacturing, including local ones, be modeled by polynomial approximation based on the use of the results of measurements. It is pertinent to use this method at the manufacturing stage for technological and quality control but not in the design stage. In the latter case it may be more reasonable to use a simple function to model local error, for example, the following function:

$$f(\rho) = \frac{A}{2}\left[1+\cos\frac{2\pi}{T}(\rho-\rho_0)\right],\ \ \rho = (x^{*2}+y^{*2})^{1/2}, \quad (3)$$

assigned on the interval $\rho_1 \le \rho \le \rho_2$ and vanishing outside it, where A is the error amplitude; T is the error period; and, ρ_0 is a shift of the function from the origin.

Using Eq. (3), we may write the equation of a surface of revolution of the second order with local error in the following form:

$$F(z,\rho) = z + \frac{r-\mathrm{sign}\ r\cdot(r^2+C\rho^2)^{1/2}}{C} - f(\rho) = 0,\ \ C \ne 0;$$

$$F(z,\rho) = z-\rho^2/(2r)-f(\rho) = 0,\ \ C = 0, \quad (4)$$

where r is the radius of curvature of the apex.

By varying the boundaries ρ_1 and ρ_2 of the interval we can model surfaces with errors in any given zone. The ray path through the surface is calculated by the Hertzberger formulas.[2]

The programming package developed on the basis of the above-presented algorithms allows one to model astigmatic and local errors on the surfaces of an optical system effectively and to calculate tolerances for their values within the framework of the suggested models already in the design stage.

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