CONCENTRATION OF THE LASER-RADIATION ENERGY IN THE CIRCLE OF IRRADIATION WITH SHARP EDGES

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A theoretical model for a beam-forming device, based on the ''lens-axicon" pair with a truncated central part is constructed. The device is distinguished by the fact that it gives a more uniform distribution of energy in the irradiation circle. Analytical relations, describing the form of the distribution as a function of the radius of the central part and the other parameters of the apparatus, are derived. The relations obtained pertain to both the cases of truncated and untruncated central part of the system, and reduce to known limits as the radius of the central truncated part approaches zero. Nomograms of the energy distribution over the irradiation circle are presented.

Analysis of the results obtained shows that the relations derived are useful for solving applied problems that require highly uniform energy distribution in the irradiation circle.

1. Optical systems with a lens and an axicon, which make it possible to concentrate laser-radiation energy in a narrow ring on the surface of the part being processed, are used in the technology for processing materials with a laser beam.[1] The industrial implementation of such technologies and systems is being facilitated by the development of the theory and elemental base for Besselian optics — motion-picture axicons, aspherical lenses, wavefront correctors, and other thin-film phase-optic elements.[2–5]

In the energy concentrator of the type studied here the lens plays the role of a quadratic, and the axicon plays the role of a linear spatial modulator of the phase. The combined effect of these two elements on the wavefront, described by a transmission function of the form

$$\Gamma(r) = \exp\left[ik\left(r - r_0\right)^2/2f\right], \quad k = 2\pi/\lambda, \tag{1}$$

gives the necessary redistribution of the laser-radiation energy over the cross section of the beam and concentrates the laser energy in a ring at the output of the system. In Eq. (1) the parameter f corresponds to the focal length of the lens; r_0 is the radius of the ring formed on the surface of the part being processed; r is the radial coordinate in the plane transverse to the beam axis; and, λ is the wavelength of the radiation. The ring is formed in the focal plane of the lens and the radius r_0 of its central line is determined by the characteristics of the lens and the axicon. For a linear conical axicon made of a transparent material with index of refraction n the equation[6]

$$r_0 = (n - 1)\alpha f, \tag{2}$$

where α is the angle at the base of the axicon, is satisfied. This angle usually does not exceed 0.1—0.15 rad. The working relations are derived and the diffraction fields in the focal plane of the system with the transmission function (1) are calculated in many works; see, for example, Refs. 6—10.

2. Modulators with the transmission function (1) are used in those cases when the energy of the beam is concentrated in a narrow ring on the surface of the material in order to obtain an equally narrow ring of heating. In most traditional applications of lasers in technology, however, it is necessary ultimately to have not a ring, but rather a circle of uniform heating with sharp edges. The axicon is employed in this case not to replace the circle with a ring but rather to increase the uniformity of the heating in the ring by redistributing the energy of the beam from the central part of the irradiation circle to the edges. This can be achieved in the scheme studied above by displacing the focal plane of the system from the surface of the part being processed or by not focusing the radiation with the lens at all. In this case a wide ring with a gap and not very sharp edges on the outside is created on the surface of the part. Because of heat flow the ring transforms after a short time into a circle with a uniform temperature distribution at the center.

The idea for this application of axicons was proposed in Ref. 11 in application to problems of surface heat hardening of materials. The experimental data presented confirm that the method is effective: it is possible to obtain a uniform temperature distribution in the central part of an irradiation circle with a diameter of up to 7 mm. However the "tails" of the distribution of the spot still remain wide.

The best results are obtained with a different method for forming beams, when the central part of the initial Guassian beam remains undisturbed while its peripheral part is contracted, by means of modulation of the phase with the help of the axicon and

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lens, into a ring around the central irradiation spot formed on the surface of the part. To implement this method it is of interest to calculate the ring diffraction field in the "lens-axicon" optical system with a truncated axial part that does not disturb the field of the beam. A modulator of this type is described by a transformation function of the form

$$\Gamma(r) = \mathrm{circ}(b/a) + \exp\left[ik\left(r - r_0\right)^2/2f\right] \times$$
$$\times\left[1 - \mathrm{circ}(b/a)\right], \tag{3}$$

where

$$\mathrm{circ}(b/a) = \begin{cases} 1, & b \le a, \\ 0, & b > a. \end{cases} \tag{4}$$

This is a more general modulator than the one described by Eq. (1). This is the modulator we shall study below.

3. A beam of monochromatic spatially coherent radiation with a Gaussian intensity distribution $I_0(r)$ over the cross section of the beam

$$I_0(r) \equiv |\psi_0(r)|^2 = \exp\left(-2r^2/a^2\right). \tag{5}$$

is directed into the modulator (4). The distribution of the complex amplitude of the field at the output of the modulator $\psi_{01}(r)$ and in the focal plane of the lens $\psi(r)$, in this case, are given by the formulas

$$\psi_{01}(r) = \exp\left(-r^2/a^2\right)\Gamma(r), \tag{6}$$

$$\psi(r) = \psi_1(r) + \exp\left(-r^2/a^2\right)\mathrm{circ}(r/b). \tag{7}$$

Here $\psi_1(r)$ is the field of the wave perturbed by the modulator. On the basis of the paraxial Fourier optics[12] the field of the wave is represented by the Fourier-Bessel integral of the function $\psi_0(r)\ \exp(-ikrr_0/f)\ [1 - \mathrm{circ}(b/a)]$:

$$\psi_1(r) = a^2\,\frac{k}{f}\int\limits_0^\infty \varphi(\rho)J_0(\omega\rho)\rho\ d\rho, \tag{8}$$

where J_0 is a Bessel function of order zero, and the parameters ω and ω_0 and the function $\varphi(\rho)$ are determined by the expressions

$$\varphi(\rho) = \exp\left(-\rho^2\right)\ \exp\left(-i\omega_0\rho\right)\left[1 - \mathrm{circ}(b/a)\right], \tag{9}$$

$$\omega = 2\pi\nu = kar/f,$$
$$\omega_0 = 2\pi\nu_0 = kar_0/f. \tag{10}$$

In writing down the second term in Eq. (7) we neglected the divergence of the beam in free space and the diffraction of radiation by the edges of the opening with radius $b \gg \lambda$. The distribution of the intensity in the focal plane of the lens is determined by the formulas

$$I(r) = I_1(r) + I_0(r)\ \mathrm{circ}(b/a), \tag{11}$$

$$I_1(r) \equiv |\psi_1(r)|^2. \tag{12}$$

Here we neglected the interference of the fields $\psi_0(r)$ and $\psi_1(r)$, making the assumption that $r_0 > b$ and taking into account the fact that the first field is localized in the region $r < b$, and the second field is localized in a narrow ring around the central line $r = r_0$.

To calculate $\psi_1(r)$ we shall rewrite the integral (8), taking into account Eqs. (9) and (10), in the form

$$\psi_1(r) = \int\limits_{-\infty}^{\infty} s(x)\ \exp\left(-i2\pi\nu_0x\right)\ dx \equiv F\{s(x)\}, \tag{13}$$

corresponding to a one-dimensional Fourier transform of the function

$$s(x) = \left(ka^2/f\right)xJ_0(2\pi\nu x)\ \exp\left(-x^2\right)H(x - b). \tag{14}$$

The function $H(x)$ in Eq. (14) is the Heaviside function. Separating $s(x)$ into the factors $s_1(x)$ and $s_2(x)$, the integral in Eq. (13) over the spatial variable x can be replaced by a convolution integral over the frequency variable ν_0:

$$\psi_1(r) = S_1\left(\nu_0\right) \bullet S_2\left(\nu_0\right) \equiv \int\limits_{-\infty}^{\infty} S_1(\xi)\ S_2\left(\nu_0 - \xi\right)\ d\xi, \tag{15}$$

$$S_1\left(\nu_0\right) \equiv F\left\{s_1(x)\right\},\ S_2\left(\nu_0\right) \equiv F\left\{s_2(x)\right\}. \tag{16}$$

The separation of $s(x)$ into the factors $s_1(x)$ and $s_2(x)$ makes sense in the case when one of the Fourier transforms, for example, $S_2(\nu_0)$, can be approximated by a Dirac delta function $\delta(\nu - \nu_0)$:

$$S_2\left(\nu_0\right) \simeq A\delta\left(\nu - \nu_0\right). \tag{17}$$

Then we obtain from Eq. (15)

$$\psi_1(r) = AS_1\left(\nu - \nu_0\right) = AS_1\left[\frac{a}{\lambda f}\left(r - r_0\right)\right], \tag{18}$$

which corresponds to the expected form of the distribution of the complex amplitude of the field $\psi_1(r)$ in the irradiation ring at the output of the lens-axicon system.[13]

Choosing for the second factor the function

$$s_2(x) = xJ_0(2\pi\nu x) \tag{19}$$

and Fourier transforming it, we obtain[14]

$$S_2\left(\nu_0\right) = i\nu_0/2\pi^2\left[\nu^2 - \nu_0^2\right]^{3/2}$$
$$\simeq \frac{1}{2\pi}\sqrt{\lambda f/2ar_0}\ \delta\left(\nu - \nu_0\right). \tag{20}$$

The Fourier transform of the first factor can be written in the form

$$S_1(\nu_0) = \frac{ka^2}{f}\frac{\sqrt{\pi}}{2}\exp\left(-\pi^2\nu_0^2\right)\operatorname{erfc}\left(b/a + i\pi\nu_0\right). \quad (21)$$

We obtain from Eqs. (20) and (21), in accordance with Eq. (15),

$$\psi_1(r) = \psi_{10}\exp\left(-t^2\right)\operatorname{erfc}(b/a + it), \quad (22)$$

where the function ψ_{10} is given approximately by

$$\psi_{10} \simeq \sqrt{\pi a^3/8\lambda f r_0}\ . \quad (23)$$

In Eq. (22) we introduced the dimensionless variable t given by

$$t = \pi\left(\nu - \nu_0\right) = \frac{\pi a}{\lambda f}\left(r - r_0\right); \quad (24)$$

the function $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$ is the error function with complex argument z.

To derive the second equality in Eq. (20) we employed the formulas for expansion of the generalized functions of the form $(x \pm i0)^\lambda$, regarded as entire functions of the parameter λ, in a Taylor series in powers of $(\lambda - \lambda_0)$ (for $\lambda = -3/2$ and $\lambda_0 = -1$).[15] We are justified in neglecting the regular part of the function $S_2(\nu_0)$ in calculating the integral (15) in the asymptotic approximation of the theory for values $\nu \geq 100$, characteristic for the real conditions under which axicons are used.[1]

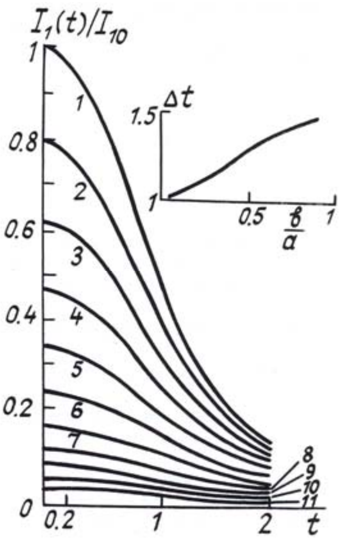

FIG. 1. The energy distribution in the ring bounding the central part of the irradiation spot for different values of b/a: $b/a \neq 0$ (1), 0.1 (2), ... 1 (11). The width of the ring Δt as a function of the parameter b/a is shown in the inset.

4. The formula (22), together with Eq. (12), leads to the following working relations for the distribution of the intensity of the modulated radiation in a ring bounding the central irradiation spot at the output of the system:

$$I_1(r) = I_{10}\exp\left(-2t^2\right)\left|\operatorname{erfc}(b/a + it)\right|^2, \quad (25)$$

$$I_{10} = \frac{\pi}{8}a^3/\lambda f r_0. \quad (26)$$

Plots of the normalized distribution function $I_1(r)/I_{10}$, constructed with the help of Eq. (25), are presented in Fig. 1. The width of the ring as a function of the parameter b/a is shown separately in the inset. With the help of these plots it is possible to select parameters of the modulator so as to optimize the intensity distribution $I(r)$ in the irradiation spot.

In the limiting case $b = 0$ Eq. (25) assumes the form

$$I(r) = I_1(r) = I_{10}\left[\exp\left(-2t^2\right) + \frac{4}{\pi}F^2(t)\right]. \quad (27)$$

The formula (27) approximates well the expression derived in Ref. 7 for the intensity $I(r)$

$$I(r) = I_{10}\left|{}_1F_1\left(3/4;\ 1/2;\ -t^2\right) - i2\frac{\Gamma(5/4)}{\Gamma(3/4)}\,t\,{}_1F_1\left(5/4;\ 3/2;\ -t^2\right)\right|^2. \quad (28)$$

In the interval $0 \leq t \leq 1$ the error of such an approximation does not exceed 7%. Here the function $F(t)$ is Dawson's integral[14]

$$F(t) = \exp\left(-t^2\right)\int_0^t \exp\left(y^2\right)dy, \quad (29)$$

$\Gamma(r)$ is the gamma function, and ${}_1F_1(\alpha, \beta, z)$ is Kummer's confluent hypergeometric function. The formula (28) can by derived directly from Eq. (15) by choosing in the following form the second factor in the separation of $s(x)$ into $s_1(x)$ and $s_2(x)$:

$$s_2(x) = x^{1/2}J_0(2\pi\nu x). \quad (30)$$

For the functions $S_2(\nu_0)$ and $S_1(\nu_0)$ we obtain

$$S_2(\nu_0) \simeq \frac{1}{2\pi}\sqrt{\lambda f/a r_0}\ \delta\left(\nu - \nu_0\right), \quad (31)$$

$$S_1(\nu_0) = \sqrt{\frac{\pi}{8}a^3/\lambda f}\left\{{}_1F_1\left(3/4;\ 1/2;\ -\pi^2\nu_0^2\right) - i2\pi\nu_0\frac{\Gamma(5/4)}{\Gamma(3/4)}\,{}_1F_1\left(5/4;\ 3/2;\ -\pi^2\nu_0^2\right)\right\}. \quad (32)$$

Using Eqs. (31) and (32), after simple transformations we obtain Eq. (28) from Eq. (15).

The formulas (27) and (28) are practically equivalent for modeling the radiation fields in the lens-axicon system. For the particular case $b = 0$ they give an asymptotic approximation to the theory based on the equations of the Fourier optics (8) and (9). The extension of the formula (28) to the case $b \neq 0$ is not so obvious as in the case of the formula (27) and in so doing great mathematical difficulties must be overcome.

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