On evaluation of information content of observational experiments

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In this paper we discuss the methodological problems that arise in planning and conducting specialized large-scale observational experiments on investigating energy exchange processes in climatic systems. The investigations are based on application of the methods of direct and inverse modeling in the framework of the scenario approach. Based the aim of an experiment, a set of functionals is formulated, and the information content is evaluated from these functionals by the methods of theory of model sensitivity. The results of numerical simulation of observational experiments along the Moscow–Vladivostok main line are presented as an example.

Introduction

The main goal of this work is to develop the methodology and accumulate an experience in solving problems on informational support of the idea of “purposeful” monitoring. Traditionally, purposeful monitoring is aimed at solving preset problems, such as, for example, revealing of pollution sources, provision of mathematical models with factual evidences, detection of substances being secondary products of gaseous pollutants and predicted by models but absent in primary exhausts, etc. The approach combining mathematical modeling and purposeful monitoring has been developed within the framework of the Integration Project IG SB RAS–97 No. 30 (Ref. 1).

Numerous problems arise when conducting field experiments, and the problem of information content of the obtained experimental data is among them. Studies of the information content are based on the use of methods of direct and inverse modeling and solution of specific inverse problems in the framework of the scenario approach. On the basis of the aims of an experiment, a set of functionals that describe the results of atmospheric observations is formulated, and the information content of the observational experiments is evaluated from their behavior in the space of model parameters. For these functionals, sensitivity functions are calculated. These functions serve for seeking informative areas for every individual observation and for the whole totality of observations; then the relative hazard of a pollution source is estimated from the standpoint of the atmospheric quality in a detector zone.

The computational algorithms for sensitivity functions involve solutions of the corresponding conjugate problems. Various aspects of application of conjugate equations to analysis of complex systems are described in Ref. 2.

In this paper, we use the variational principle developed in Refs. 3–5 for numerical simulation and combined usage of models and measurements. This principle is based on the methods of classic theory of calculus of variations modified for operating in finite-dimensional spaces of discrete approximations of models. Application of this principle forms a unified constructive basis for the entire sequence of mathematical modeling: from construction of discrete analogs for the models of studied processes to systematic organization of algorithms for direct and inverse modeling and optimizing procedures. The conjugate problems for the models of processes arise therewith as a consequence of the variational principle when studying sensitivity of the model to variations of input data and when obtaining optimal estimates of the functionals defined in the set of the functions of state of these models.

Algorithms for estimation of functionals

Let us consider the problem of estimating the functionals defined in the set of the functions of state of a climatic system subjected to some natural and anthropogenic factors.

With the help of the functionals that are generalized characteristics of the system behavior, we can describe various aspects of the studied and modeled processes, as well as observations of these processes. Therefore, in practice, there can be a great deal of functionals with different information content.

To describe the behavior of the climatic system, we use the set of models of hydrothermodynamics, pollutant transport and transformation, and hydrologic cycle, which usually take part in related problems of ecology and climate.[6–7]

To avoid constructing algorithms for every type of functionals individually, we propose the universal scheme of algorithms for estimating the functionals of the general form and show the way of including any particular functional in this scheme. The functionals for which these methods are true must have the following properties.

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1. Let the mathematical models and state functions be defined in the space-time domain D_t, and their discrete analogs are defined in $D_t^h \in D_t$. Then the functionals must be limited and differentiable with respect to the state functions in D_t. In its turn, the continuous dependence of the state functions on the input parameters of the model is a consequence of well-posedness of the corresponding problems.

2. The functionals can necessarily be represented as integrals in the domain D_t with Radon or Dirac measures.[8] These measures are elements of the space conjugate to the space of state functions. The discrete analogs of the functionals are determined as cubature sums in D_t^h with the same digitization of the state functions as in mathematical models.

Thus, the functionals can be presented in the form

$$\Phi_k(\varphi) = \int F_k(\varphi)\chi_k(\mathbf{x},t)\,\mathrm{d}D\,\mathrm{d}t,\ k = \overline{1, K},K \geq 1. \quad (1)$$

Here $F_k(\varphi)$ are the characteristics of the state functions to be estimated. Their form is defined by an investigator based on the properties formulated above; $\chi_k(\mathbf{x},t)$ are the weight functions with carriers $D_k^0 \subset D_t$; $\chi_k(\mathbf{x}, t)\,\mathrm{d}D\,\mathrm{d}t$ are the Radon measures in D_t meeting the conditions of normalization $\int_{D_t} \chi_k(\mathbf{x},t)\,\mathrm{d}D\,\mathrm{d}t = 1$. If D_{tk}^0 is a set of points (more than one point), then $\chi_k(\mathbf{x}, t)\,\mathrm{d}D\,\mathrm{d}t$ are the Dirac measures in D_t (Ref. 8). Functionals (1) always have the form of scalar product, therefore the structures of the functions $F_k(\varphi)$ and $\chi_k(\mathbf{x}, t)$ must be coordinated, i.e., they both must be either scalar or vector.

Functionals (1) can be treated as generalized characteristics of the processes under study obtained from monitoring or mathematical modeling. Now we can refine the sense of the weight functions $\chi_k(\mathbf{x}, t)$ in Eq. (1) from the monitoring standpoint:

(1) They may be the distribution functions of the observational instruments (detectors) used for weighted estimation of the functions $F_k(\varphi)$ in D_t;

(2) The carrier of the weight function $\chi_k(\mathbf{x}, t)$, i.e., the set of points $(\mathbf{x}, t)$, at which the function is nonzero, describes arrangement of instruments in space and time; the domain D_{tk}^0 can be treated as the detector zone for corresponding functional;

(3) Values of $\chi_k(\mathbf{x}, t)$ determine the weight with which the function $F_k(\varphi)$ at the point $(\mathbf{x}, t)$ is taken into account in the final equation $\Phi_k(\varphi)$ for the estimated characteristic.

Our task is to link variations of functionals (1) directly to variations of the input parameters and external sources in the models in the optimal way. Here the optimal way means that variations of the characteristics to be estimated are independent of variations of the state functions inside the domain D_t. The needed estimates can be found by the methods of sensitivity theory from the conditions that the functionals (1) are stationary to variations of the state functions provided that the latter ones meet the equations of the models of processes or their discrete analogs. In other words, the models of processes serve as restrictions to the class of functions and as links between the state functions, parameters, and sources.

To construct the discrete analogs of the models and modeling algorithms, the variational formulations of the models in the form of integral identity

$$I(\varphi, \varphi^*, \mathbf{Y}) = 0\ ; \quad (2)$$

$$\varphi \in Q(D_t)\,,\ \ \varphi \in Q^*(D_t)\,;\ \ \mathbf{Y} \in R(D_t)\,,$$

are most suitable.[4,7] Here φ is the state function, $\mathbf{Y}$ is the vector of parameters of the model and external sources, $R(D_t)$ is the domain of their permissible values, φ^* is an arbitrary sufficiently smooth function from the space $Q^*(D_t)$ conjugate to the space of the state functions $Q(D_t)$. Note that the weight functions in Eq. (1) also belong to this conjugate space. It is important that the functional (2) is linear with respect to the vector φ^*.

If the model from the problem (2) is treated as a restriction, then, based on the technique of calculus of variations, the functionals (1) can be replaced by a family of extended functionals equivalent to the initial ones in the set of the state functions of the model

$$\tilde{\Phi}_k(\varphi) = \Phi_k(\varphi) + I(\varphi, \varphi^*, \mathbf{Y})\ . \quad (3)$$

In this case φ^* can be considered as a generalized Lagrange factor or as a weight function for taking into account the equations of the model.

The optimal scheme of estimates results from the conditions that variations of the discrete analogs of the functionals (1)–(3) are independent of variations of the functions φ and φ^* in D_t^h. These conditions lead to the set of equations

$$\frac{\partial}{\partial \varphi^*} I^h(\varphi, \varphi^*, \mathbf{Y}) = 0\ , \quad (4)$$

$$\frac{\partial}{\partial \delta\varphi}\left\{\frac{\partial}{\partial \xi}\left[\Phi_k^h(\varphi + \xi\delta\varphi) + I^h(\varphi + \xi\delta\varphi, \varphi^*, \mathbf{Y})\right]\Big|_{\xi = 0}\right\} = 0, \quad (5)$$

where the superscript h marks the discrete analogs, $\delta\varphi$ are variations of the state function, ξ is a real parameter. The functional $I(\varphi, \varphi^*, \mathbf{Y})$ is digitized by the methods of weak approximation, splitting, and decomposition.[4,6]

Equations (4) are the discrete analog of the models of processes, and Eq. (5) is a set of conjugate problems for the functionals (1). In the conjugate problems constructed in such a way, the functions

$$\eta_k \equiv \frac{\partial}{\partial \delta\varphi}\left\{\frac{\partial}{\partial \xi}\left[F_k(\varphi + \xi\delta\varphi)\right]\Big|_{\xi = 0}\right\}\chi_k(\mathbf{x},t) \quad (6)$$

generated by the functionals (1) play the role of sources. In Eq. (5) they always have the structure of a column vector chosen with regard for coordination of the cubature formulas for digitizing the summand functionals in Eq. (3). These problems are solved back in time under the initial conditions $\varphi_k^*(\mathbf{x}, t) = 0$ at $t \geq \overline{t}$, where $\overline{t}$ denotes the last value of time t at which the function η_k is nonzero at least at one point of the domain D_t^h.

The use of the solutions of the problems (4)–(6) in the optimal algorithm of estimation results in the following basic equations of the sensitivity theory:

$$\delta\Phi_k^h(\varphi) \equiv (\mathrm{grad}_{\mathbf{Y}}\, \Phi_k^h(\varphi),\, \delta\mathbf{Y}) =$$

$$= \frac{\partial}{\partial\xi} I^h\, (\varphi_k^*,\, \mathbf{Y} + \xi\delta\mathbf{Y})\Big|_{\xi = 0},\quad k = \overline{1, K}, \qquad (7)$$

where $\delta\mathbf{Y}$ is the vector of variations of the input parameters, $\mathrm{grad}_{\mathbf{Y}}\, \Phi_k(\varphi)$ is the set of functions of sensitivity of the functional to be estimated to these variations.

The second equality in Eq. (7) is a constructive transformation of the structure of the scalar product in Eq. (1) from the space of the state functions into the space of model parameters. The particular form of the sensitivity functions derives from equating the coefficients at the same variations $\delta\mathbf{Y}_i$, $(i = \overline{1, N})$ in the right-hand and left-hand sides of the equality or by differentiating the last expression in Eq. (7) with respect to $\delta\mathbf{Y}_i$ in $R^h(D_t^h)$.

The functions φ and φ_k^* $(k = \overline{1, N})$ are sought at the unperturbed input data $\mathbf{Y}$, and variations are estimated by Eqs. (7) also in the vicinity of these values. The values of variations (7) can be calculated, if at least one component $\delta\mathbf{Y}_i$ is nonzero. The relations for estimating the variations given by the algorithm (4)–(7) are of the second order of accuracy in terms of the variations $\delta\varphi$.

Let us note some special cases. If the functional and model are linear, model parameters do not vary, and we need to estimate the dependence of the functionals on only the sources and initial data, then the inherent structure of the algorithms becomes simpler. In this case, the operations of linearization in respect to φ are excluded in the conjugate problems (5) and (6), and there is no need to solve the basic problem (4). Only those summands remain in Eq. (7) which relate to the sources and initial data. Depending on the way of setting the input data on sources of inhomogeneities, the values of both the functionals themselves and their variations can be calculated by these equations. If other parameters vary in addition to the sources, then the algorithm follows the general scheme (4)–(7) even for the linear models and functionals.

Let us exemplify the sensitivity relation (7) for the model of atmospheric hydrothermodynamics with allowance made for the hydrological cycle and in combination with the model of pollutant transport and transformation[7] which is accepted as a basic one in the problems of monitoring and forecasting:

$$\delta\Phi_k^h(\varphi) = \Bigg\{ \int\limits_{D_t} \{c_3 \delta Q_T T_k^* + c_4 \delta Q q_k^* +$$

$$+ \sum_{\alpha=1}^{n} c_{\alpha+4} (\delta Q_{C\alpha} - \delta(B(\mathbf{C}))_\alpha C_\alpha^*\}\, \mathrm{d}D\, \mathrm{d}t +$$

$$+ \int\limits_D \sum_{i=1}^{4+n} c_i\, \delta\psi_i\, \varphi_i^*\Big|_{t=0}\, m\mathrm{d}D + \mathbf{R}_1(\varphi,\, \varphi_k^*,\, \delta\mathbf{Y}) +$$

$$+ \mathbf{R}_2(\varphi,\, \varphi_k^*,\, \delta\mathbf{Y}) + \mathbf{R}_3(\varphi,\, \varphi_k^*,\, \delta\mathbf{Y}) \Bigg\}, \qquad (8)$$

where $k = \overline{1, K}$, and $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ have the form

$$\mathbf{R}_1(\varphi,\, \varphi_k^*,\, \delta\mathbf{Y}) \equiv \int\limits_\Omega \Bigg\{ \delta U_n \sum_{i=1}^{4+n} c_i\, \psi_i\, \varphi_{ik}^* \frac{m^2}{\pi} +$$

$$+ \sum_{i=1}^{4+n} c_i\, U_n\, \delta\psi_i\, \varphi_{ik}^* \frac{m^2}{\pi} - \frac{\delta\pi}{\pi^2} \sum_{i=1}^{4+n} m^2\, c_i\, U_n\, \psi_i\, \varphi_{ik}^* \Bigg\}\, \mathrm{d}\Omega\, \mathrm{d}t,$$

$$\mathbf{R}_2(\varphi,\, \varphi_k^*,\, \delta\mathbf{Y}) \equiv$$

$$\equiv \sum_{i=1}^{4+n} c_i \Bigg\{ \int\limits_{D_t} \left[\delta\mu_i\, \mathrm{grad}_s\, \psi_i\, \mathrm{grad}_s\, \varphi_{ik}^* + \frac{\delta\nu_i}{m} \frac{\partial\psi_i}{\partial\sigma} \frac{\partial\varphi_{ik}^*}{\partial\sigma} \right] m^2 \mathrm{d}D\mathrm{d}t +$$

$$+ \int\limits_{\Omega_t} \delta r_i\, \varphi_{ik}^*\, m\, \mathrm{d}\Omega\, \mathrm{d}t + \int\limits_{S_t} \delta\tau_i\, \varphi_{ik}^*\, m\, \mathrm{d}S\, \mathrm{d}t \Bigg\},$$

$$\mathbf{R}_3(\varphi,\, \varphi_k^*,\, \delta\mathbf{Y}) \equiv \int\limits_{\Omega_t} \{G_k^* \delta U_n + U_{nk}^* \delta G +$$

$$+ (U_{nk}^* - U_n T_k^*)\delta\pi - \pi T_k^* \delta U_n\}\, m\, \mathrm{d}\Omega\, \mathrm{d}t -$$

$$- \int\limits_S T_k^*\, \delta(G_s\, \pi) + \pi_k^* \delta\pi\Big|_{t=0}\, \mathrm{d}S.$$

Here the notations from Ref. 7 are used: $\varphi = \{\varphi_i,\ (i = \overline{1, 4+n})\} \equiv \{u, \nu, T, q, C_\alpha(\alpha = \overline{1, n}),\ \dot{\sigma},\ G,\ \pi\}$ is the state function of the basic model; $\psi = (\pi/m)\varphi$, asterisks denote the corresponding components of the conjugate functions; U_n is the component of the velocity vector $\mathbf{U} = (\pi/m)\ (u, \nu, \dot{\sigma})$ normal to the boundary Ω_t of the domain D_t; S_t is the projection of the domain D_t on the Earth's surface; u, ν, and $\dot{\sigma}$ are the components of velocity vector along the coordinates x, y, and σ, respectively; T is temperature; q is the specific humidity; C_α is the concentration of pollutants; n is the number of different substances; G is geopotential; π is a function of pressure; m is the scale factor of the coordinate system; $\mathrm{d}D$, $\mathrm{d}\Omega$, and $\mathrm{d}S$ are the volume and area elements; c_i $(i = \overline{1, 4+n})$ are the weight coefficients for equalizing physical dimensions of summands in the integral identity of model (2); μ_i and ν_i are the coefficients of horizontal and vertical exchange for a substance of number i; and r_i and τ_i are the values of turbulent fluxes at boundaries Ω_t and S_t. The symbol δ denotes variations of the input (relative to the model) parameters: components of the state vector φ, ψ,

$(\delta\psi_i = \pi\delta\varphi_i + \varphi_i\delta\pi)/m$, $\delta\varphi_i = (m\delta\psi_i - \varphi_i\delta\pi)/\pi$, vector of parameters $\mathbf{Y}$, sources of heat Q_T, humidity Q_q, and pollution Q_c. The term $\delta(B(\mathbf{C}))$ describes the variation of the operator of pollutant transformation due to variations of the rate constants of reactions included in this operator. Summands including variations of heat influxes δQ_T depend on variations of the concentration of optically active gases. To calculate them, the complex basic model of hydrothermodynamics incorporating the radiation block with the model of pollutant transfer is used.

Let us comment this relation. What is most important, it demonstrates that all elements of the models are interrelated; therefore, possible perturbations of all the input parameters and external sources should be taken into account to estimate the functionals. The inner relations of the models are responsible for solution of the conjugate problems. As a result, only summands with variations of the input parameters remain in Eq. (8). The integrals over the boundaries of the domains D_t and D at $t = 0$ take into account the influence of the boundary conditions and initial data. For models on a sphere, some of these integrals are excluded because of the conditions of periodicity.

Three types of integrals are of particular interest, namely, the integrals including the sources of heat, humidity, and pollution. The factors near the variations of the sources are the sensitivity functions. They are the measures of the direct influence of variations of the sources on variations of the functional (in linear problems, the influence of the sources themselves on the functional). But these summands do not fully describe the influence of the sources on the functional. There exists one more indirect contribution described by the cooperative action of other summands with the sensitivity functions, whose equations include the components of the state function. These are, for example, summands including variations of coefficients of turbulence, functions of near-ground pressure, geopotential, pollutant transformation operator, and so on.

The sensitivity functions of the sources (depending on the aims of a study and for convenience of interpretation, they can be named a function of influence or hazard of the sources, information value, information content of the monitoring system, etc.) are defined in the domain $D_t = D \times [0, \bar{t}]$. They depend on the space and time coordinates $(\mathbf{x}, t) \in D_t$ and on five generalized parameters: the structure of the estimated functional of the atmospheric quality, Y_1; configuration and arrangement of detector zone in D, Y_2; time period of “observations” for estimation, Y_3; time interval of action of the sources, Y_4; and the characteristic of the operation mode of a source or parameter, Y_5.

The first three parameters depend on the form of the functional and the carrier of the weight function in it; the fourth and fifth ones are determined by the model parameters. The latter two parameters determine the spatiotemporal structure of the sensitivity function. Other sensitivity functions can be described similarly.

In terms of information, the hazard function for the functionals determining the atmospheric quality within the detector zone can be described as follows. The value of this function at the point $(\mathbf{x}, t) \in D_t$ is the relative contribution of pollutants emitted by the source at the point $\mathbf{x}$ for the period of its action to the total pollution (given by the value of the functional) of the atmosphere in the detector zone for the time of observation.

The concept of the information content of a monitoring system for estimation of the functionals (1) is closely connected with the concept of observability in the theory of optimal control. The general condition of observability for the models of the considered class is that the carriers of the sensitivity functions cover the ranges of the parameters or sources to be estimated for which these sensitivity functions are to be determined. On this basis, the level of values of the sensitivity function conceptually determines the possibility to estimate the sought characteristics based on the observed data by solving the corresponding inverse problems. The higher is this level, the stronger is the connection between the parameters to be estimated and observations, i.e., the functionals, and, consequently, the better posed is the inverse problem. Hence, it follows that only parameters that fall into the areas of sensitivity-observability can be found with certainty. Inclusion of the sensitivity functions and informative areas in planning observational experiments can significantly increase the efficiency of investigations. Note that the same is true when solving the problem of the use of observational data in reconstruction of the spatiotemporal structure of fields with deficient data.

Our experience shows that multifactor estimates of the information content with a set of models are more valuable as compared to those made with one factor and one model. This was distinctly shown, in particular, in estimates of the levels of anthropogenic impact on the region of Lake Baikal.[9] For solution of this problem, the set of models, including the regional model of hydrothermodynamics as well as regional and hemispheric models of pollutant transport in the atmosphere, was used. It is interesting to note that the hazard zones of pollution sources calculated by the sensitivity functions for the atmospheric quality above Lake Baikal coincided with the zones of influence of Baikal mesoclimates. The same can be said about estimates of the information content of observational data. If the observation system is chosen with regard for significance of the sensitivity functions, then the results of reconstruction of fields are acceptable for practical use even at deficient data,[10] because in this case the models serve as interpolants with a rather wide range of influence of every observation.

Let us comment the practical implementation of the algorithms from the viewpoint of computations. The complexity of the models themselves and algorithms as well as bulky information that must be accumulated and processed send us in search of efficient ways of obtaining results. The method of parallel computations has a potential for this. Analysis of the problem as a whole shows that the modeling process can be paralleled into several system levels, namely:

1. All basic models can operate in parallel until the preset moments of their meeting and informational exchange. Their numerical schemes are constructed using the identity (2) based on the principles of splitting and decomposing. Consequently, all the variety of elementary procedures and splitting stages at one step can be made in parallel.

2. All conjugate problems for a set of the functionals (1) can be solved in parallel. Their computational schemes are identical and differ only in the algorithms of calculation of the functions (6). In its turn, the structure of each conjugate problem is generated by summator analog of Eq. (2) and is paralleled by analogy with the algorithms of implementation of the basic models (4).

3. The algorithms of calculation of the sensitivity functions by Eqs. (7) and (8) can work independently and in parallel for all functionals and all model parameters.

4. Scenarios for different versions of the input data and hydrometeorological situations can be calculated in parallel with the hierarchic structure of parallelizing at the system levels 1–3.

Development of the parallel versions of the basic models of different system levels is now in progress with allowance made for the features of computers available at the supercomputer center of the Institute of Computational Mathematics and Mathematical Geophysics SB RAS.

Numerical experiments

To illustrate the basic statements of this paper, we have chosen the results of scenario calculations by the model of pollutant transfer for the functionals having dual interpretation. From the viewpoint of environmental protection, they give the weighted integral characteristic of the atmospheric quality $F_k(\varphi)$ in the detector zones determined by the carrier of the weight function $\chi_k(\mathbf{x}, t)$. In terms of monitoring, the same functionals can be treated as a mathematical description of the results of observation of the characteristic $F_k(\varphi)$ obtained with instruments located in the detector zone and performing observations in a preset regime. The results of observation are summed up with the weight determined by the values of function $\chi_k(\mathbf{x}, t) \geq 0$. In this sense, numerical experiments imitate the actions of an observer.

For simplicity, it was taken $F_k(\varphi) \equiv \varphi$ in all scenarios in order to avoid solution of the direct problem and to estimate the information content relative to the pollution sources only. The difference between scenarios was in setting different detector zones and weight functions in the functionals. Scenarios for weightless passive pollutants were considered.

The first example presents the calculations on imitation and interpretation of the observational experiment conducted by researchers of the Institute of Atmospheric Physics RAS. The essence of this experiment was as follows. A railway train including a car-laboratory capable to measure atmospheric characteristics went along the Moscow–Vladivostok main line on a certain schedule. Our task was to estimate the information content of these measurements. In other words, we had to find pollution sources contributing to the results of observations.

To formulate the task mathematically, we had to set the structure of the observational functional and to make calculations for the conjugate problem for this functional. We used the hemispheric model of pollutant transport in the hybrid "p-sigma" coordinate system (this model was one of the models described in Refs. 7 and 11). The field experiment was conducted in August 12–20, 1996. The hydrometeorological data for the Northern Hemisphere for that period were borrowed from the Reanalysis NCEP/NCAR database.[12] The spatiotemporal structure of the hydrometeorological fields with the time step of 30 minutes in the hybrid coordinate system was reconstructed with the use of the procedures described in Ref. 13. The weight function in the functional was assumed to be a nonzero constant along the phase trajectory of the train movement. The coordinates of its spatiotemporal carrier were set in accordance with the train schedule. So, the functional was a sum amount of pollutants measured for the whole time of movement along the trajectory.

The calculation was made in the regime of inverse modeling, i.e., the sources of inhomogeneities (7) generated by the functional of observations in the conjugate problem moved in the backward direction of time (relative to the train movement) from Vladivostok to Moscow. The time interval, in which the problem was being solved, was somewhat wider (from 20 to 5 of August in the backward direction) than the period of observations. This was needed to determine how long the sensitivity functions remain significant beyond the observation interval. In other words, we wanted to know how strongly events happened prior to the experiment can affect the functional.

To analyze variations of the sensitivity functions in space and time, the calculated results were visualized as computer animation. They pictorially showed the marked variability of the sensitivity functions in response to the variability of atmospheric hydrodynamic characteristics. It turned out that these functions retained their information content from two to four days beyond the period of observations. Consequently, events happened 1–100 hours prior to observations in the areas with significant levels of the sensitivity functions can contribute to the values of the observed parameters. The calculation gives the relative levels of significance for every area regardless of the power of pollution sources located in it. To obtain the absolute contribution of each source, we must know its emission volume.

Figure 1 shows the timely integrated function of the information content relative to pollution sources which can be located at two levels: on the Earth's surface and in the stratosphere at the height of 20 mbar. The 3D field whose 2D cross sections are shown in Fig. 1 was normalized to maximum value. Near the Earth's surface, areas adjacent to the main line or situated at the distance of 100–200 km from it have rather high level of significance.

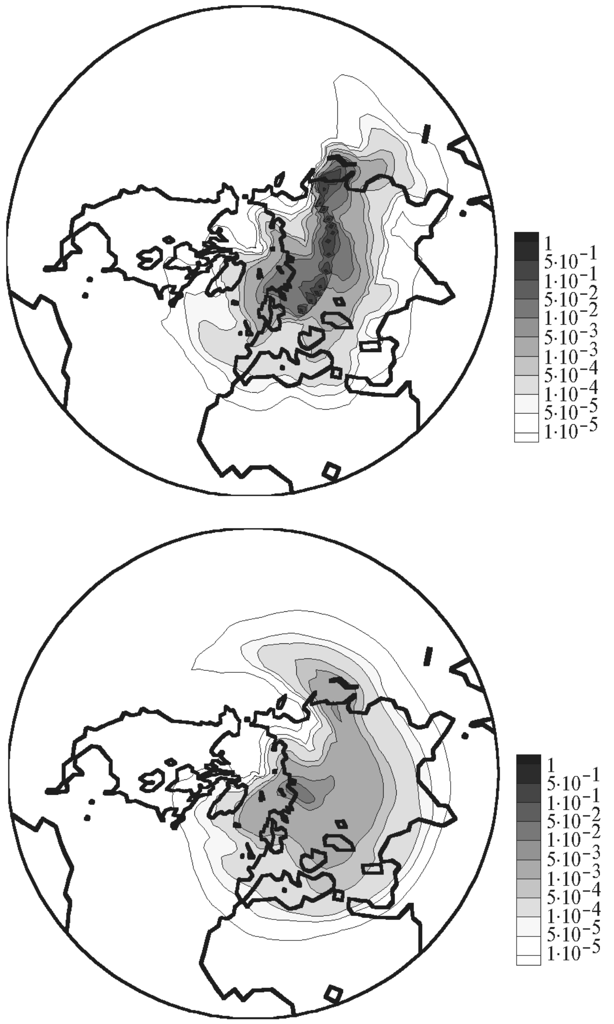

Fig. 1. Function of information content for moving observation system relative to ground-based sources (top panel) and relative to sources located at 20 mb level (bottom panel).

The second example differs from the first one in only the type of the observer's operating mode. It was supposed that a network of stationary stations was conducting observations along the path at the same time, i.e., measurements were being conducted along the whole path for the entire period of observations, and the results of these measurements were summed up. The computer animations have shown that in this case the variability manifests itself in a different way. Figure 2 shows the same characteristics as Fig. 1. Similarly, the field is normalized to its maximum value.

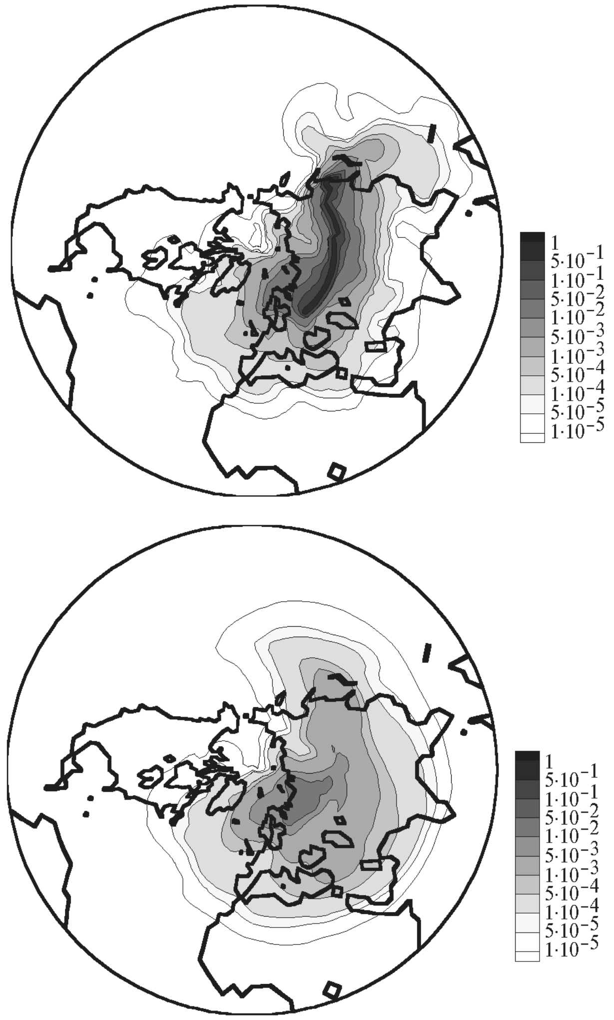

Fig. 2. The same as in Fig. 1 but for stationary observation system.

Conclusions

Comparison of experimental results obtained for stationary and moving observation systems under the same conditions shows that the sensitivity functions in their dynamics differently response to variability of atmospheric circulation. In the case of the moving system, the effect like a traveling wave in space in accordance with the passage of the carrier of the weight function along the phase trajectory manifested itself in the sensitivity functions.

The information content depends on the duration of measurements. The stationary system proved to be more informative as a whole because of longer periods of individual observations and their synchronism. In the both cases, the effect of the near zone predominates, although the influence of sources located at the distance about 500 km from the path remains significant. This means that some specific substances emitted by far sources (providing they are absent in near emission zones) can be identified with a certain degree of confidence by the measurements with time lag between the moments of emission and measurement about 5–7 days in the case of the stationary system and 2–4 days in the case of the moving system. These circumstances should be taken into account when planning phase trajectories for observations. Consequently, in the absence of a stationary monitoring systems, moving laboratories equipped for measuring a wide spectrum of substances can identify emissions from sources located within the range of sensitivity–observability along the phase trajectory of the moving laboratory. This is of prime importance in solving the problem of ecological safety of territories, especially, in the absence of stationary monitoring systems.

Mathematical models are necessary components of a monitoring systems, because they help to increase the information content of observations, combine dissimilar and nonsynchronous information, and obtain a comprehensive idea on evolution of the observed situation.

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