AEROSOL TRANSFER IN THE ATMOSPHERE: SELECTION OF A FINITE DIFFERENCE SCHEME

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In this paper we present a comparison of schemes from a wide class of monotonic and quasi-monotonic schemes of the approximation order not higher than 2, that are used for solving problems in meteorology, gas dynamics, and physics of plasma. Such a choice of the approximation order is caused by the fact that it is used in calculations of wind velocity components in the majority of atmospheric models. Numerical experiments have shown the Bott scheme to fit best of all the solution of the transfer equation for non-negative characteristics as compared to other investigated schemes of the second order.

1. INTRODUCTION

In this paper we describe the MAR system that is being developed at ICT SB RAS. The system simulates atmospheric processes and admixture transfer for expert estimations by calculational experiments to be performed using observational data on meteorological conditions and atmospheric pollution (see Ref. 1). The problems to be solved relate to the transfer of mass fraction of water and water vapor, the air density and intensity of aerosol substance migrated together with the air flow. As was pointed out in Ref. 2, requirement for monotony of the finite difference schemes used and the condition of non-negative values of the solution are basic for these problems since the error in the sign of these functions can result in absolutely incorrect description of the atmospheric processes estimated using these functions. The paper by S.K. Godunov (see Ref. 3) is the fundamental study of the monotonic schemes. Further development of the schemes and description of their applications can be found in Refs. 2, 4, and 5.

For the problems in weather forecasting, ecology and climate theory two schemes (variants of Van Lire scheme described in Ref. 7 and that of the flux correction described in Ref. 4) were compared in Ref. 6, where the most appropriate scheme (a variant of Van Lire scheme described in Ref. 8) was chosen and using ecological problems as an example it was shown numerically, using actual information that the requirement of monotony in transfer problems is of fundamental importance. Besides, the authors have pointed out that application of the monotonic schemes is very promising when making progressively increased body of calculations in weather forecasting, ecology and climate theory. This scheme was used in the investigation of an admixture transfer on the global scale (see Ref. 9).

In this paper we compare a wider class of monotonic schemes than that used in Ref. 6, including the schemes used in meteorology (see Refs. 10 and 13) as well as in gas dynamics and plasma physics. The class of schemes compared is broadened by including those used in gas dynamics and plasma physics and careful study of practically all existing monotonic schemes (see Refs. 14 and 17). Besides, in our numerical experiments we have used as the initial data not only the function with high gradient, but relatively smooth ones. A wider class allows us to choose a more efficient method as compared to that suggested in Ref. 6.

2. STATEMENT OF THE PROBLEM

For a comparison, the following groups of schemes of calculations with the approximation order no higher than the second one are considered. Such a choice of the order is made because the values of the velocity vector components in most models of the atmosphere are correct to no higher than the second order.

The first group includes the monotonic schemes used in meteorology. There are modified Van Lire scheme in the form described in Ref. 6, Bott scheme (see Ref. 10), and Smolarkevitch scheme (see Refs. 11 and 13).

The second group consists of the schemes from a relatively wide class which according to Ref. 16 demonstrate the best efficiency. There are the schemes with Van Lire limitation (see Ref. 17), and with the UNO (Uniformly Nonoscillatory Scheme) limitation (see Ref. 18), with SUPERBEE limitation (see Ref. 19), with Harten compression (see Ref. 20), respectively.

The third group includes for completeness of the comparison two schemes which already became

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classical. There are a monotonic explicit scheme and a non-monotonic Lax-Vendorf one (see Ref. 21).

Estimation of the approximation order of the scheme used in this paper has been made by the authors of the above references.

The comparison is carried out using a group of tests which are model for meteorological problems. The group includes solution of the following equation:

$$\frac{\partial \varphi}{\partial t} + u\frac{\partial \varphi}{\partial x} = 0, \qquad (1)$$

$$\varphi(x, 0) = \varphi_0(x), \qquad (2)$$

where $u = \text{const} > 0$, $a \le x \le b$, $0 < t < T$.

As is known, solution of Eq. 1 has the form:

$$\varphi(x, t) = \varphi_0(x - ut). \qquad (3)$$

The condition of periodicity is used as the side boundary condition.

3. BRIEF DESCRIPTION OF THE SCHEMES COMPARED

Expression (1) is a special case of the following expression:

$$\frac{\partial \varphi}{\partial t} + \frac{\partial F(\varphi)}{\partial x} = 0, \qquad (4)$$

($F(\varphi) = u\varphi$ in Eq. (1)).

Let us consider the following grids and grid functions:

$$D_h^\varphi = \{x_j;\ x_j = a + (J + j)\,h,\ j = -J, \dots, J\},$$

$$D_h^F = \{x_{j+1/2};\ x_{j+1/2} = x_j + h/2,\ j = -J, \dots, J-1\},$$

$$D_\tau = \{t_n;\ t_n = n\tau,\ n = 0, \dots, N\},$$

$$D(\varphi^{h\tau}) = D_h^\varphi \times D_\tau,$$

$$D(F^{h\tau}) = D_h^F \times D_\tau,$$

where $D(\varphi^{h\tau})$ and $D(F^{h\tau})$ are the domains of definition of the grid functions $\varphi^{h\tau}$ and $F^{h\tau}$, respectively, $b = a + 2Jh$, $T = N\tau$.

If we designate

$$\lambda = \tau/h,\ \ c_j^n = \lambda\, u_j^n,\ \ \varphi_j^n = \varphi^{h\tau}(x_j, t_n),$$

$$F_{j+1/2}^n = F^{h\tau}(x_{j+1/2}, t_n),$$

where c_j^n is the Currant number, according to Ref. 17 the following form can be used as a difference scheme for Eq. (4):

$$\varphi_j^{n+1} = \varphi_j^n - \lambda(F_{j+1/2}^n - F_{j-1/2}^n), \qquad (5)$$

where

$$F_{j+1/2}^n = u_{j+1/2}\frac{\varphi_{j+1/2}^r + \varphi_{j+1/2}^l}{2} - |u_{j+1/2}|\frac{\varphi_{j+1/2}^r - \varphi_{j+1/2}^l}{2}$$

and

$$\varphi_{j+1/2}^r = \varphi_{j+1}^r - \frac{1}{2}(1 + c_{j+1}^n)\,L_{j+1}^n,$$

$$\varphi_{j+1/2}^l = \varphi_j^n + \frac{1}{2}(1 - c_j^n)\,L_j^n.$$

Values of L_j^n and L_{j+1}^n of grid function $L^{h\tau}$ (this function is called to be a limiter) depend on the increment of $\varphi^{h\tau}$ in the ranges (x_{j-1}, x_j) and (x_j, x_{j+1}), respectively.

Using the following designations:

$$\Delta\varphi_{j+1/2}^n = \varphi_{j+1}^n - \varphi_j^n,\quad \Delta\varphi_j^n = \varphi_{j+1}^n - \varphi_{j-1}^n,$$

$$\Delta^2\varphi_j^n = \varphi_{j+1}^n - 2\varphi_j^n + \varphi_{j-1}^n,$$

$$\text{MINMOD}\,(x, y) = \text{sgn}(x)\max\{0, \min(|x|, y\,\text{sgn}(x))\},$$

$$\text{MAXMOD}\,(x, y) = \text{sgn}(x)\max(|x|, |y|)$$

and according to Ref. 17 one can write the limiter considered in the following form:

– the VL (Van Lire) limiter:

$$L_j^n = \begin{cases} \text{sgn}(|\Delta\varphi_{j+1/2}^n|)\min(2|\Delta\varphi_{j-1/2}^n|, |\Delta\varphi_j^n|/2, 2|\Delta\varphi_{j+1/2}^n|), \\ 0, \qquad \text{if } \Delta\varphi_{j+1/2}^n\,\Delta\varphi_{j-1/2}^n| \le 0; \end{cases} \qquad (6)$$

– the VLm (Van Lire modified):

$$L_j^n = \frac{1}{2}\Delta\varphi_{j+1/2}^n - \frac{1}{2}S_j\,\Delta^2\varphi_j^n, \qquad (7)$$

where $S_j = \dfrac{|\Delta\varphi_{j+1/2}^n| - |\Delta\varphi_{j-1/2}^n|}{|\Delta\varphi_{j+1/2}^n| + |\Delta\varphi_{j-1/2}^n|}$;

– the SB (Superbee):

$$L_j^n = \text{MAXMOD}\,\{\text{MINMOD}\,(2\Delta\varphi_{j+1/2}^n, \Delta\varphi_{j-1/2}^n),$$

$$\text{MINMOD}\,(\Delta\varphi_{j+1/2}^n, 2\Delta\varphi_{j-1/2}^n)\}; \qquad (8)$$

– the UNO limiter:

$$L_j^n = \text{MINMOD}\,(\Delta\varphi_{j+1/2}^n - \frac{1}{2}d_{j+1/2}^n, \Delta\varphi_{j-1/2}^n + \frac{1}{2}d_{j-1/2}^n), \qquad (9)$$

where $d^n_{j-1/2}$ = MINMOD $(\Delta^2\varphi^n_{j+1}, \Delta^2\varphi^n_j)$;

– Harten compression:

$$L^n_j = \text{MINMOD} \;\; (\Delta\varphi^n_{j+1/2}\,,\;\; \Delta\varphi^n_{j-1/2})(1 + w\,\theta_j), \quad (10)$$

where w is the compression parameter (as proposed in Ref. 20, in our experiments $w = 2$) and

$$\theta_j = \frac{|\Delta\varphi^n_{j+1/2} - \Delta\varphi^n_{j-1/2}|}{|\Delta\varphi^n_{j+1/2}| + |\Delta\varphi^n_{j-1/2}|}\,.$$

Let us now focus on Smolarkevitch (see Ref. 11) and Bott (see Ref. 10) schemes. The explicit scheme expressed by Eq.(5) and directed "counter the flux" with

$$F^n_{j+1/2} = \frac{u^n_{j+1/2} + |u^n_{j+1/2}|}{2}\,\varphi^n_j + \frac{u^n_{j+1/2} - |u^n_{j+1/2}|}{2}\,\varphi^n_{j+1}, \quad (11)$$

is chosen as a basic for constructing Smolarkevitch scheme.

This scheme is of the first order approximation it is monotonic, but, as known, it has high calculational viscosity (see Ref. 21). For the effect of the viscosity to be decreased we introduce additional correction step which is analogous to the first one but here "antidiffusion velocity" is used instead of the transfer velocity. To perform the step "antidiffusion velocity" is defined as

$$\tilde{u}_{j+1/2} = \frac{(|u_{j+1/2}|\,h - \tau u^2_{j+1/2})(\varphi^*_{j+1} - \varphi^*_j)}{(\varphi^*_j + \varphi^*_{j+1} + \varepsilon)h}, \quad (12)$$

where ε is a small parameter determined by a computer performance characteristics ($\varepsilon = 2^{-m}$, where m is the number of bits used to write mantissa of a real number, or the so called instrumental null).

Generally speaking similar idea was published in Ref. 4.

Bott scheme is a generalization of the scheme of integral fluxes (see Ref. 22) by introducing additional steps which allow one to retain non-negative solution (at non-negative initial solution values) and to reduce the phase error. Explicit scheme expressed by Eq. (5) with

$$F^n_{j+1/2} = \frac{i^+_{l,j+1/2}}{i_{l,j}}\,\varphi^n_j - \frac{i^-_{l,j+1/2}}{i_{l,j+1}}\,\varphi^n_{j+1}\,, \quad (13)$$

where $i^+_{l,j+1/2} = \max\,(0, I^+_l)$, $i^-_{l,j+1/2} = \max\,(0, I^-_l)$, $i_{l,j} = \max\,(I_{l,j},\, i^+_{l,j+1/2} + i^-_{l,j-1/2} + \varepsilon)$ forms the basis for Bott scheme.

Here ε has the same meaning as in Eq. (12).

$$I^+_l = \int_{1/2-c^+_j}^{1/2} \varphi^n_{j,l}(x')\,dx', \quad (14)$$

$$I^-_l = \int_{-1/2}^{-1/2+c^-_j} \varphi^n_{j+1,l}(x')\,dx', \quad (15)$$

$$I_l = \int_{-1/2}^{1/2} \varphi^n_{j,l}(x')\,dx', \quad (16)$$

where $c^{n\pm}_j = \pm(c^n_j \pm |c^n_j|)/2$.

For calculating integrals the function φ is expressed by the following polynomial:

$$\varphi^n_{j,l}(x') = \sum_{k=0}^{l} a_{j,k}\,x'^k\,, \quad x' = (x - x_j)/h$$

and $-1/2 \le x' \le 1/2$. (17)

Then

$$I^+_l = \sum_{k=0}^{l} \frac{a_{j,k}}{(k+1)\,2^{(k+1)}}\,[1 - (1 - 2\,c^+_j)^{k+1}], \quad (18)$$

$$I^-_l = \sum_{k=0}^{l} \frac{a_{j+1,k}}{(k+1)\,2^{(k+1)}}\,(-1)^k\,[1 - (1 - 2\,c^-_j)^{k+1}], \quad (19)$$

$$I_l = \sum_{k=0}^{l} \frac{a_{j,k}}{(k+1)\,2^{(k+1)}}\,[(-1)^k + 1]. \quad (20)$$

In our experiments $l = 4$ and

$$a_{j,0} = \varphi_j, \quad a_{j,1} = \frac{1}{12}\,(-\varphi_{j+2} + 8\varphi_{j+1} - 8\varphi_{j-1} + \varphi_{j-2}),$$

$$a_{j,2} = \frac{1}{24}\,(-\varphi_{j+2} + 16\varphi_{j+1} - 30\varphi_j + 16\varphi_{j-1} - \varphi_{j-2}),$$

$$a_{j,3} = \frac{1}{12}\,(\varphi_{j+2} - 2\varphi_{j+1} + 2\varphi_{j-1} - \varphi_{j-2}),$$

$$a_{j,4} = \frac{1}{24}\,(\varphi_{j+2} - 4\varphi_{j+1} + 6\varphi_j - 4\varphi_{j-1} + \varphi_{j-2}).$$

Note that for improving the Bott's scheme efficiency authors of Ref. 23 use the trapezium method instead of the exact integration.

4. NUMERICAL EXPERIMENT

All calculations were made using Currant number less than 1 at the following values of parameters: spatial step $h = 3750$ m, time $T = 24$ hours, transfer velocity $u = 5$ m/s, $a = -b = 25\,h$. The values of these parameters are equal to those used in Ref. 6 except for $T = 3$ h 20 min. Although the values of a and b are for a convenience changed as compared to that in Ref. 6, this distinction has not any effect on the calculation results because the interval length is the same.

The following functions were chosen as the initial in the numerical experiments:

$$\varphi_0^{(1)}(x_j) = 0.5 + 0.5 \sin [4\pi\, x_j/(b - a)]; \qquad (21)$$

$$\varphi_0^{(2)}(x_j) = \begin{cases} 1, & \text{if } x_{-10} < x_j < x_0, \\ 0, & \text{otherwise;} \end{cases} \qquad (22)$$

$$\varphi_0^{(3)}(x_j) = \begin{cases} 1, & \text{if } x_j = x_{-5}, \\ 0, & \text{if } x_j \neq x_{-5}; \end{cases} \qquad (23)$$

$$\varphi_0^{(4)}(x_j) = \begin{cases} 0.2\, x_j + 2, & \text{if } x_{-10} \le x_j \le x_{-5}, \\ -\,0.2\, x_j, & \text{if } x_{-5} \le x_j \le x_0. \end{cases} \qquad (24)$$

Note that setting the initial value of the solutions $\varphi_0^{(1)}$, $\varphi_0^{(2)}$, $\varphi_0^{(3)}$ and $\varphi_0^{(4)}$ one can simulate transfer of large scale variable cloudiness, large scale uniform cloud formations, emiss+of air pollution at a single point, and non-uniform distribution of a substance, respectively.

The following parameters were used to compare the results obtained:

– the mean absolute error of solution of the finite-difference problem

$$\varepsilon_a = \| (\varphi(x, T))_h - \varphi^h \|_1 / M, \qquad (25)$$

– the error in estimation of the maximum value

$$\varepsilon_{max} = \| \varphi(x, T) \|_C - \| \varphi^h \|_\infty , \qquad (26)$$

– the maximum absolute error of solution of the finite-difference problem

$$\varepsilon_\infty = \| (\varphi(x, T))_h - \varphi^h \|_\infty . \qquad (27)$$

Here $\varphi^h = \varphi^{h\tau}\,|^N$ is the solution of finite-difference equation at $t = T$; $M = 2J + 1$ is the number of the grid nodes D_h^φ; $(\varphi(x, T))_h$ is the projection of a solution of the initial differential equation at $t = T$ on the space of the grid functions φ^h with determination area D_h^φ. The projection is given by the following expression:

$$(\varphi(x, T))_h = \{\varphi(x_j, T); \quad j = -J, \ldots, J\}.$$

The following parameters are used as norms:

$$\| \varphi^h \|_1 = \sum_{j=-J}^{J} | \varphi_j^h | , \quad \|\varphi^h\|_\infty = \max_{-J \le j \le J} | \varphi_j^h | ,$$

$$\| \varphi(x, T) \|_C = \max_{a \le x \le b} | \varphi(x, T) | .$$

The following designations are used in the experiment description: E – the explicit scheme, LV – Lax-Vendorf scheme, S – Smolarkevitch scheme, VL – the scheme with Van Lire limiter, VLm – modified Van Lire scheme, UNO – the scheme with UNO limiter, B – Bott scheme, SB – the scheme with SUPERBEE limiter, H– the scheme with Harten compression.

The results of calculations with sine-shaped solution are presented in Table I and Fig. 1. As is seen from Table I, the explicit scheme demonstrates the worst results. High absolute error and significant deviation from the exact solution (the maximum is decreased by about one half) are evident. Small error in determination of the maximum value in Lax-Vendorf scheme demonstrates seemingly good results obtained using the scheme. However, significant mean absolute error shows that this is not the case because of a reasonably high phase error. Smolarkevitch scheme is also unsuccessful for the given initial field, as well. Perhaps, modified Van Lire scheme used in Ref. 6 well compares with the previous one. The scheme with Van Lire limiter gives reasonably good results. As to the scheme with UNO limiter, in spite of the fact that it estimates the maximum value with a small error, relatively high mean error shows that the scheme does not suit well that initial function. From the remaining schemes the Bott scheme demonstrates better results as compared to those obtained with SUPERBEE limiter and Harten compression and especially as compared to the above schemes. It should also be mentioned that the mean absolute error and error of determination of the maximum value for Bott scheme do not change first three significant figures at different Currant numbers.

TABLE I. Solution error for the initial sine-shaped field

Scheme	$c = 0.2$		$c = 0.4$		$c = 0.6$		$c = 0.8$	
	$\varepsilon_=$	ε_{max}	$\varepsilon_=$	ε_{max}	$\varepsilon_=$	ε_{max}	$\varepsilon_=$	ε_{max}
E	0.300	– 0.472	0.282	– 0.443	0.244	– 0.383	0.165	– 0.259
LV	0.077	– 0.006	0.065	– 0.011	0.046	– 0.011	0.019	– 0.007
S	0.092	– 0.098	0.076	– 0.067	0.059	– 0.041	0.039	– 0.015
Vlm	0.054	– 0.107	0.042	– 0.092	0.034	– 0.074	0.023	– 0.048
VL	0.038	– 0.072	0.030	– 0.062	0.024	– 0.051	0.019	– 0.034
UNO	0.039	– 0.019	0.023	– 0.016	0.013	– 0.010	0.011	– 0.006
B	0.016	– 0.002	0.016	– 0.002	0.016	– 0.002	0.016	– 0.002
SB	0.031	– 0.044	0.028	– 0.041	0.024	– 0.034	0.019	– 0.022
H	0.025	– 0.027	0.022	– 0.027	0.019	– 0.023	0.017	– 0.014

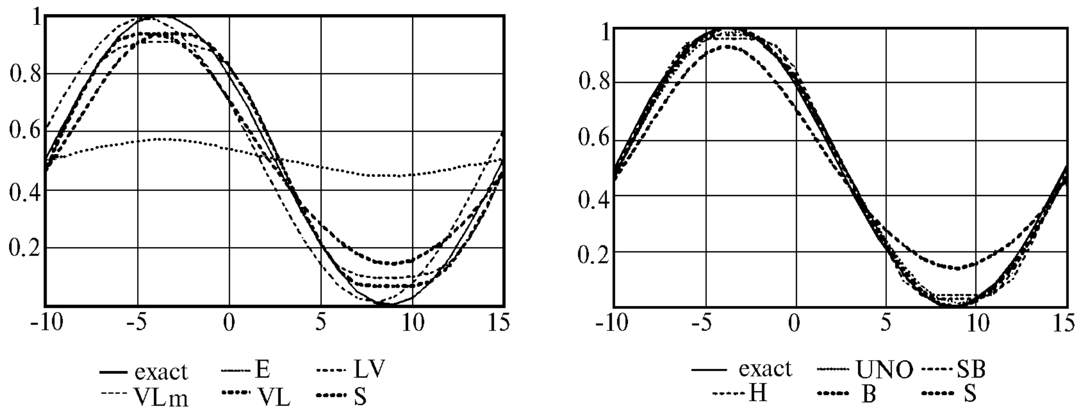

FIG. 1. Exact and approximate solutions at T = 24 hours *and c* = 0.4 *obtained using sine-shaped initial field.*

TABLE II. Solution error for the initial field in the form of rectangular step.

Scheme	c = 0.2		c = 0.4		c = 0.6		c = 0.8	
	ε_a	ε_{max}	ε_a	ε_{max}	ε_a	ε_{max}	ε_a	ε_{max}
E	0.232	–0.639	0.216	–0.588	0.191	–0.508	0.146	–0.350
LV	0.151	–0.016	0.132	–0.016	0.108	0.047	0.083	0.108
S	0.123	–0.022	0.110	–0.160	0.095	–0.091	0.076	0.020
VLm	0.088	–0.172	0.078	–0.135	0.067	–0.089	0.054	–0.033
VL	0.072	–0.096	0.063	–0.067	0.056	–0.039	0.047	–0.010
UNO	0.077	–0.037	0.070	–0.008	0.064	0.005	0.055	0.003
B	0.049	–0.114	0.046	0.098	0.041	0.072	0.035	0.069
SB	0.039	–0.023	0.036	–0.019	0.035	–0.012	0.032	–0.004
H	0.044	–0.006	0.042	–0.005	0.039	–0.002	0.037	0.005

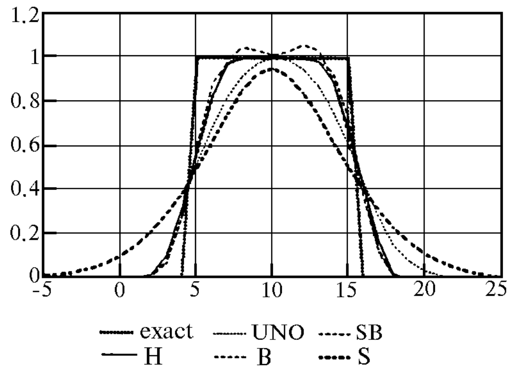

FIG. 2. Exact and approximate solutions obtained using the initial field with the step width of 9 points of the grid and UNO, SUPERBEE, Harten , Bott and Smolarkevitch schemes.

The results of solution of the transfer equation with the initial field in the form of rectangular step described by Eq. (22) are presented in Table II and Fig. 2. For this initial pulse the schemes considered could be subdivided into three groups. The explicit, Lax-Vendorf and Smolarkevitch schemes from the first group were most inefficient. The second group included Van Lire, modified Van Lire schemes, and that with UNO limiter which have demonstrated better results as compared to those obtained using schemes from the above group. Nevertheless, the schemes from the second group are less efficient than those from the third group which includes Bott scheme, and these with SUPERBEE limiter and Harten compression. The last group of schemes provides closely related mean average error of the difference solution, but the scheme with Harten compression gives higher accuracy of the maximum value. Besides, the experiment with the transfer extended up to 10 days was carried out. In that case the scheme with Harten compression was, as before, the best, while the schemes with SUPERBEE and Harten limiters provide much the same results. Figure 3 depicts the plots of the solutions obtained at different Currant numbers for the scheme with Harten compression. One can see that the scheme provides the error of approximately the same order (except for the calculation with Currant number c_j^n = 0.8). However the calculational results remain quite good in spite of such a long transfer time.

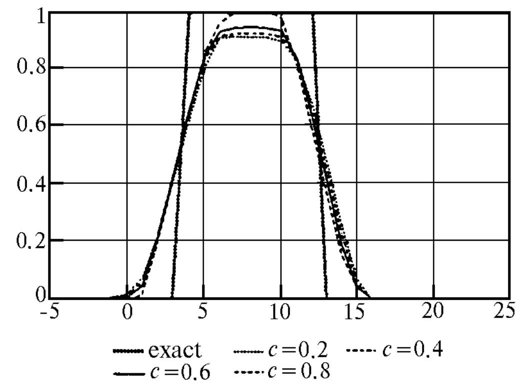

FIG. 3. Exact and approximate solutions at T = 240 h *and different Currant numbers obtained using the initial field with the step width of 9 points of the grid and Harten scheme.*

The results of experiments performed using the initial field with "narrowed" width of the step are presented in Fig. 4. One can see that all schemes give the solution profiles identical to those obtained at the "step" width of 9 points. However the maximum value is significantly lowered and the solution provided by Bott scheme is closer to the maximum of the exact solution as compared to those obtained using other schemes.

The initial field in the form of point sources described by Eq. (23) could be considered as limiting narrowing of the step. The solution error for this case is given in Table III.

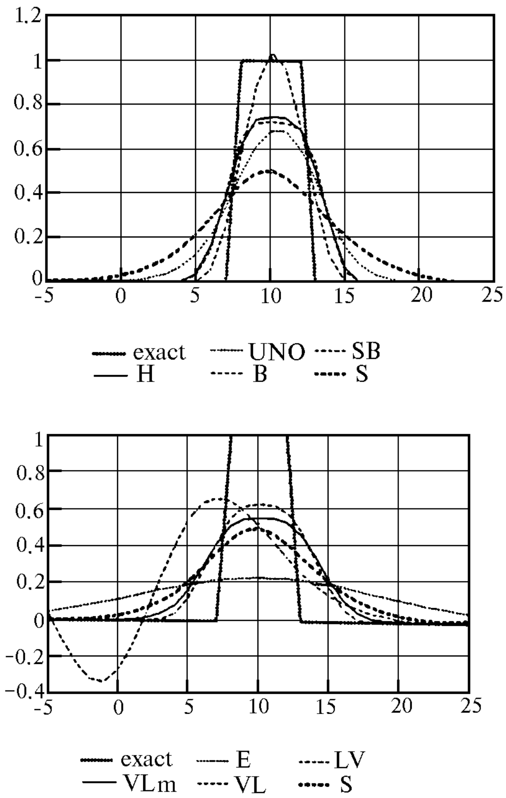

FIG. 4. Exact and approximate solutions at T = 24 h *and c =* 0.4 *obtained using the initial field with the step width of 5 points of the grid.*

TABLE III. Solution error for the initial field in the form of a point pulse.

Scheme	c = 0.2		c = 0.4		c = 0.6		c = 0.8	
	ε_a	ε_{max}	ε_a	ε_{max}	ε_a	ε_{max}	ε_a	ε_{max}
E	0.038	–0.958	0.038	–0.952	0.037	–0.941	0.036	–0.917
LV	0.064	–0.864	0.055	–0.856	0.049	–0.847	0.043	–0.818
S	0.036	–0.908	0.035	–0.897	0.035	–0.883	0.034	–0.854
VLm	0.035	–0.894	0.035	–0.885	0.034	–0.871	0.033	–0.841
VL	0.034	–0.878	0.034	–0.869	0.034	–0.855	0.032	–0.825
UNO	0.034	–0.087	0.034	–0.857	0.033	–0.839	0.032	–0.808
B	0.030	–0.0776	0.030	–0.766	0.029	–0.751	0.028	–0.725
SB	0.034	–0.855	0.033	–0.848	0.033	–0.835	0.032	–0.807
H	0.033	–0.851	0.033	–0.844	0.033	–0.831	0.032	–0.804

TABLE IV. Solution error for the initial field in the form of a triangle pulse.

Scheme	c = 0.2		c = 0.4		c = 0.6		c = 0.8	
	ε_a	ε_{max}	ε_a	ε_{max}	ε_a	ε_{max}	ε_a	ε_{max}
E	0.128	–0.797	0.119	–0.767	0.107	–0.719	0.083	–0.619
LV	0.131	–0.393	0.109	–0.328	0.088	–0.350	0.058	–0.287
S	0.072	–0.551	0.063	–0.506	0.053	–0.451	0.041	–0.340
VLm	0.052	–0.493	0.045	–0.457	0.036	–0.406	0.023	–0.319
VL	0.040	–0.427	0.034	–0.393	0.027	–0.347	0.015	–0.205
UNO	0.044	–0.381	0.033	–0.333	0.023	–0.272–	0.015	–0.205
B	0.010	0.0130	0.010	–0.123	0.010	–0.113	0.010	–0.100
SB	0.023	–0.320	0.021	–0.304	0.018	–0.273	0.015	–0.221
H	0.021	–0.303	0.018	–0.285	0.016	–0.256	0.012	–0.210

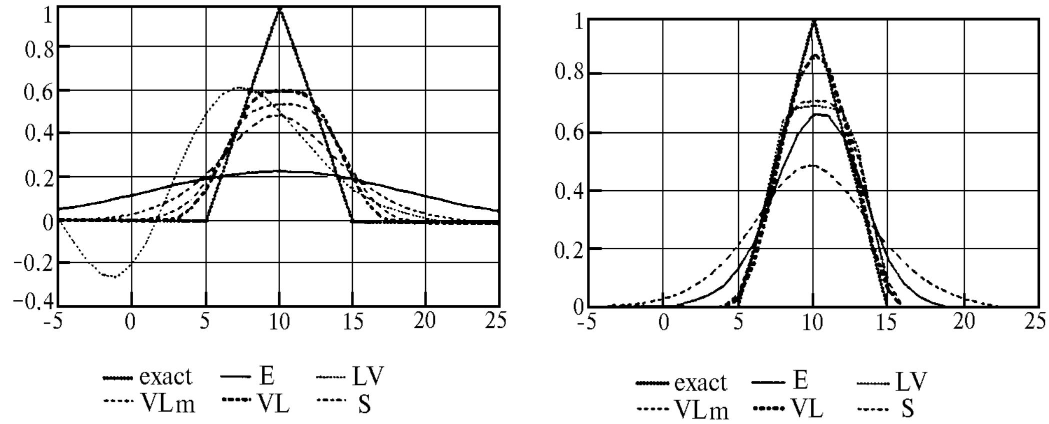

FIG. 5. Exact and approximate solutions at $T = 24$ h and $c = 0.4$ obtained using a triangle pulse as the initial field.

One can see that the schemes considered provide approximately the same error, nevertheless, Bott scheme demonstrates somewhat better results.

The estimations for the initial field in the form of a triangle pulse are listed in Table IV. Plots of the corresponding solutions are presented in Fig. 5. In that case unquestionable advantage should be given to Bott scheme. The rest schemes are comparable to each other. The schemes with SUPERBEE limiter and Harten compression demonstrate somewhat better results among these schemes.

5. CONCLUSION

The numerical experiments performed for different tests using one-dimensional transfer equation have shown that Bott scheme provides the most successive results. For the initial field in the form of a rectangular pulse with a relatively wide step the best results were obtained using the scheme with Harten compression. The fact of relatively poor efficiency of Smolarkevitch scheme is found to be unexpected. The other schemes (except of course the explicit and Lax-Vendorf schemes) have demonstrated relatively high performance for a part of the tests, while the modified Van Lire scheme have not advantages over the initial formulation.

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