ENHANCEMENT OF THE SPEED OF SPECTRAL ANALYSIS OF THE ANALYTICAL SIGNALS

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The algorithms are presented which allow one to enhance the speed of spectral–correlation processing of stationary analytical signals, for purposes of acoustic sounding of the atmosphere.

Nowadays in different branches of science and technology the calculation techniques based on the complex representation of signals whose real and imaginary parts are connected by Hilbert transform[1–3] are used. The analytical signals (AS) are expected to be applied successfully to sodars also. In this case, in contrast to traditional real representation of the acoustic information[4] a possibility appears of correct studying the amplitude and phase–frequency modulation of signals received by sodar, that will allow sounding to be more informative and reliable. Together with the above–mentioned new possibilities, the ordinary spectral analysis, whose advantages are discussed in Ref. 4, also will not lose its significance for the measurement of wind velocity with a sodar. When developing this technique the enhancement of the processing speed is an urgent problem. One of the versions is proposed below for solution of this problem for the periodogram and correlation methods of obtaining spectral information. It stems from the fact that the discrete Fourier transform (DFT) of AS on the basis of Nyquist interval differs from zero only for the positive frequencies.[1,2]

The acoustic signal reflected from the atmospheric inhomogeneities is assumed to be converted by a sodar antenna system into the electrical one, amplified, subjected to the analog filtration, digitized, quantized, and then it arrives at an input of preliminary processing block, where AS $z(n\Delta t)$ is formed with the help of corresponding digital filters with the finite pulse characteristic.[1] Since useful acoustic information is sufficiently narrow–band,[4] then to decrease its volume and to improve the measurement effectiveness, it is appropriate to choose the digitization frequency f_d proceeding from the known conditions of subdigitization, i.e. connecting f_d with the necessary width of frequency range of the spectral measurements only.[3] Naturally, in this case the requirements to the degree of the amplitude–frequency response roll–of, of the input analog band–pass filter, outside its bandwidth are increased.

Further the real and imaginary components of $z(n\Delta t)$ must arrive at the input of the fast Fourier transform (FFT) processor which calculates DFT of a sample being analyzed:

$$Z\left(\frac{k}{N\,\Delta\,t}\right) = \sum_{n=0}^{N-1} z(n\,\Delta\,t)\,\exp(-\,j2\pi\,k\,n/N)\,, \qquad (1)$$

where $k = 0, 1, \ldots, N - 1$; N is the number of readouts, $\Delta t = 1/f_\mathrm{d}$ is the digitization interval. Then the readouts of a sampled energy spectrum (periodogram) are formed:

$$G(k) = (\Delta\,t/N)\,|Z(k)|^2, \qquad (2)$$

from which the wind velocity is determined in the atmospheric volume gated.[4]

However, the use of a standard FFT processor in the considered case is characterized by a nonoptimal employment of the algorithm because $Z(k)$ are calculated for the negative frequencies, i.e. $k = N/2$, $N/2 + 1, \ldots, N - 1$, where DFT of AS obviously equals to zero.[1,2] So, the calculation of DFT for a single AS realization is excessive. It can be avoided by processing two realizations $z_1(n\Delta t)$ and $z_2(n\Delta t)$ simultaneously. To do this, it is sufficient to perform a preliminary heterodyning of the spectrum of one of the realizations in the negative frequency region at the value $f_\mathrm{het} = f_\mathrm{d}/2$. For such a choice of f_het the overlapping of spectra will not be observed due to linearity of DFT, that will allow us to separate the spectra in future. As it follows from properties of DFT,[2] the spectrum shift by the value f_het for the complex signals is achieved through multiplication of the initial sample by a complex exponent $\omega(n\Delta t) = \exp(-\,j\,2\,\pi\,f_\mathrm{het}\,n\,\Delta\,t)$. For the required frequency $f_\mathrm{het} = f_\mathrm{d}/2$, $\omega(n\,\Delta\,t) = \exp(-\,j\,\pi\,n) = (-\,1)^n$. Hence, the necessary heterodyning is realized by simply inverting the sign of the every second reading, for example, the sample $z_2(n\Delta t)$. And finally the sum

$$z(n\,\Delta\,t) = z_1(n\,\Delta\,t) + (-\,1)^n\,z_2(n\,\Delta\,t),\quad n = 0, 1, \ldots, N - 1, \qquad (3)$$

arrives at the FFT input, moreover, the complex sequence $z(n\Delta t)$ is not a realization of AS because its spectrum is extended to the negative frequencies too.

After calculations made using Eq. (2), retrieval of the energy spectra of the separate realizations is carried out

$$G_1(k) = G(k),\quad G_2(k) = G(k + N/2),\quad k = 0, 1, \ldots, N/2 - 1.$$

Thus, simultaneous spectral processing of two sodar signals keeping the same volume of the initial input data for FFT in N complex readouts becomes possible. In this case the additional time required for computations by formula (3) are a small portion of the volume of calculational operations needed for the calculation of DFT (1). Therefore, in this case one can say about approximately twofold enhancement of the speed of the response as compared with the successive mode of the spectral processing.

In the correlation technique of calculation of energy spectra[1–3] the autocorrelation function of the initial sample is determined first as

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$$B(m) = \frac{1}{N}\sum_{n=0}^{N-m-1} z^*(n)\, z(n+m), \qquad (4)$$

where $m = 0, 1, \ldots, M-1$, $M \le N$, asterisk denotes the complex conjugation, $B(m)$ satisfies a condition of Hermite symmetry, i.e. $B(m) = B^*(-m)$, and its connection with the continuous sampling spectrum $G(f)$ is determined as[2]:

$$G(f) = \Delta t \sum_{n=-(M-1)}^{M-1} B(m) \exp(-j2\pi f m \Delta t),$$
$$-f_d/2 \le f < f_d/2. \qquad (5)$$

Then it can be shown that calculation by Eq. (5) with the standard FFT processor with the basis 2 is reduced to an addition of necessary odd number of the zero elements to $B(m)$ and forming of the autocorrelation sequence Hermite–symmetric about $m = L/2$ with the length of L readings, i.e.

$$G(k/L\, \Delta t) = \Delta t \sum_{m=0}^{L-1} B(m) \exp(-j2\pi\, k\, m / L), \qquad (6)$$

where $k = 0, 1, \ldots, L-1$;

$$B(m) = \begin{cases} B(m) & m = 0, 1, \ldots, M-1, \\ 0 & m = M, M+1, \ldots, L/2, \end{cases}$$

$B(L-m) = B^*(m)$, $m = 1, 2, \ldots, L/2-1$. Hereinafter $L \ge 2M-1$ is the nearest integer equal to 2 in a certain power. Moreover, the presented technique of calculation of the sampling spectrum of AS is more excessive as compared to the periodogram one. Here the three quarters of values at the FFT output are zero. One of the reasons was considered above, another reason consists in the fact that $G(k/L\,\Delta t)$ is purely real function.

To remove the mentioned excess, let us use the obvious relation following from the linearity of DFT:

$$\text{DFT}\{x(n) + j\, y(n)\} = \text{DFT}\{x(n)\} + j\, \text{DFT}\{y(n)\}.$$

Thus, the real DFT $X(k)$, $Y(k)$ of the Hermite–symmetric sequences $x(n)$, $y(n)$ are easily separated at the FFT processor output.

Taking into account the above we present the algorithm for calculating the AS sampling energy spectra from the M–points autocorrelation functions $B_1(m)$, $B_2(m)$, $B_3(m)$, $B_4(m)$, $m = 0, 1, \ldots, M-1$ calculated beforehand:

Step 1. To form two L–point Hermite–symmetric sequences

$$R_1(m) = \begin{cases} B_1(m) + (-1)^m B_2(m), & m = 0, 1, \ldots, M-1 \\ 0, & m = M, M+1, \ldots, L/2; \end{cases}$$

$$R_2(m) = \begin{cases} B_3(m) + (-1)^m B_4(m), & m = 0, 1, \ldots, M-1 \\ 0, & m = M, M+1, \ldots, L/2; \end{cases}$$

$$R_1(L-m) = R_1^*(m), \quad R_2(L-m) = R_2^*(m),$$

$$m = 1, 2, \ldots, L/2-1.$$

Step 2. To form a new L–point (but not Hermite–symmetric) complex sequence $B(m)$, $m = 0, 1, \ldots, L-1$:

$$\text{Re}\, B(m) = \text{Re}\, R_1(m) - \text{Im}\, R_2(m);$$

$$\text{Im}\, B(m) = \text{Im}\, R_1(m) + \text{Re}\, R_2(m).$$

Step 3. To calculate the L–point FFT with the coefficient Δt of the sequence $B(m)$ (i.e. to make use of Eq. (6)).

Step 4. To retrieve separate sampling energy spectra for $k = 0, 1, \ldots, L/2-1$:

$$G_1(k) = \text{Re}\, G(k), \quad G_2(k) = \text{Re}\, G(k + L/2);$$

$$G_3(k) = \text{Im}\, G(k), \quad G_4(k) = \text{Im}\, G(k + L/2).$$

This algorithm will allow simultaneous spectral processing of all four sodar channels (three slant and one vertical) to be performed with a single FFT processor. As for the periodogram technique, an additional volume of calculations according to steps 1, 2 which is needed for above noted possibility is rather small as compared with the volume of the calculational operations for the basic relation (6).

Let us consider the version when it is necessary to obtain the thinned out spectral readings for the same physical resolution ensured by the length of the correlation sequence $2M\Delta t$. In the acoustic sounding it corresponds to measurement of the wind velocity by means of the first rough determination of maximum of the sampling spectrum with its further refinement by different approximations.[4]

Let us turn to the initial relation (5). As compared with the classical version (6), we will increase a step of calculation of the frequency spectrum twice. (In so doing the obtained spectral resolution coincides with the periodogram resolution (1) in the particular case $M = N$, $L = 2N$). The sequence $B(m)$ is added by $L - 2M + 1$ zeros on the left and on the right. Then

$$G(2\, k/L\, \Delta t) = \Delta t \sum_{m=-L/2}^{L/2-1} B(m) \exp(-j4\pi\, k\, m/L) =$$

$$= \Delta t \left\{ \sum_{m=-L/2}^{-1} B(m) \exp(-j4\pi\, k\, m/L) + \right.$$

$$\left. + \sum_{m=0}^{L/2-1} B(m) \exp(-j4\pi\, k\, m/L) \right\}.$$

Having substituted $n = L/2 + m$ in the first sum and allowing for $B(m) = B^*(-m)$, and $\exp(j\,2\,\pi\,k) = 1$, we obtain

$$G(2\,k/L\,\Delta t) = \Delta t \sum_{m=0}^{L/2-1} [B(m) + B^*(L/2-m)] \exp(-j4\pi\, km/L).$$

Since $B(m) = 0$, $m \ge M$, and $B(L/2-m) = 0$, $m = 0, 1, \ldots, L/2-M$, then using the Hermite symmetry of the expression in the brackets about $m = L/4$ we finally obtain

$$G\left[\frac{k}{L/2\,\Delta t}\right] = \Delta t \sum_{m=0}^{L/2-1} P(m) \exp\{-j2\pi\, k\, m/(L/2)\}, \quad (7)$$

where $k = 0, 1, \ldots, L/2-1$,

$$P(m) = \begin{cases} B(m), & m = 0, 1, \ldots, L/2-M \\ B(m) + B^*(L/2-m), & m = L/2-M+1, \ldots, L/4, \end{cases} \qquad (8)$$

$$P(L/2-m) = P^*(m), \quad m = 1, 2, \ldots, L/4-1.$$

The spectral readouts sought are calculated by the $L/2$–point DFT with the use of only half of the correlation interval. Moreover, the $2M - L/2 - 1$ readouts of $B(m)$ from the unused half are superposed on this interval; $P(m)$

is the circular autocorrelation function[1,2] of the initial complex sequence corresponding to $G(2\,k/L\,\Delta\,t)$.

For the present case it is not difficult to modify the above described algorithm of simultaneous calculation of the four spectra sampled $B_1(m)$, $B_2(m)$, $B_3(m)$, $B_4(m)$, $m = 0, 1, \ldots, M - 1$. It is first necessary to form an $L/2$–point Hermite–symmetric circular autocorrelation sequences $P_1(m)$, $P_2(m)$, $P_3(m)$, $P_4(m)$ on the basis of Eq. (8), then to form the $L/2$–point sequences

$$R_1(m) = P_1(m) + (-1)^m P_2(m)\,,\ m = 0, 1, \ldots, L/4;$$

$$R_2(m) = P_3(m) + (-1)^m P_4(m)\,,\ m = 0, 1, \ldots, L/4;$$

$$R_1(L/2 - m) = R_1^*(m),\ \ R_2(L/2 - m) = R_2^*(m),$$

$$m = 1, 2, \ldots, L/4 - 1.$$

Then, the steps 2–4 are repeated with a replacement of L by $L/2$.

When processing long samples of real signals the technique of calculation of their autocorrelation functions with FFT[1,2] is used. As a matter of fact, in our case this technique is reduced to inversion of Eq. (6), where $G(k/L\,\Delta\,t)$ is determined by the periodogram method. The use of relation (7) is impermissible because it finally results in a circular correlation (8).

Taking into account the above material, we present the algorithm of simultaneous calculation of the autocorrelation functions (4) for two AS realizations $z_1(n\Delta t)$ and $z_2(n\Delta t)$, $n = 0, 1, \ldots, N - 1$ for the particular case $L = 2N$:

Step 1. Using Eq. (3), we form one complex (nonanalytical) sequence $z(n\Delta t)$ and complete it by N zeros.

Step 2. We calculate a $2N$–point FFT $Z(k)$, $k = 0, 1, \ldots, 2N - 1$ of the sequence $z(n\Delta t)$.

Step 3. We calculate $G(k) = |Z(k)|^2/N$, $k = 0, 1, \ldots, 2N - 1$. (If the spectral readouts are needed then $G(k)$ must be multiplied by Δt).

Step 4. We form the $2N$–point complex sequence $S(k)$

$$\mathrm{Re}\, S(k) = G(k),\ \ k = 0, 1, \ldots, N - 1,$$

$$\mathrm{Im}\, S(k) = G(k + N),\ \ k = 0, 1, \ldots, N - 1,$$

$$\mathrm{Re}\, S(k) = \mathrm{Im}\, S(k) = 0,\ \ k = N, N + 1, \ldots, 2N - 1.$$

Step 5. We calculate the $2N$–point inverse FFT

$$s(m) = \frac{1}{2N} \sum_{k=0}^{2N-1} S(k) \exp(j2\pi\, k\, m/2N)\,.$$

Step 6. We reconstruct the readouts of the autocorrelation functions $B_1(m)$, $B_2(m)$, $m = 1, 2, \ldots, N - 1$.

$$\mathrm{Re}\, B_1(m) = [\mathrm{Re}\, s(m) + \mathrm{Re}\, s(2N - m)]\,/\,2;$$

$$\mathrm{Im}\, B_1(m) = [\mathrm{Im}\, s(m) - \mathrm{Im}\, s(2N - m)]\,/\,2;$$

$$\mathrm{Re}\, B_2(m) = [\mathrm{Im}\, s(m) + \mathrm{Im}\, s(2N - m)]\,/\,2;$$

$$\mathrm{Im}\, B_2(m) = [\mathrm{Re}\, s(2N - m) - \mathrm{Re}\, s(m)]\,/\,2;$$

$$\mathrm{Re}\, B_1(0) = \mathrm{Re}\, s(0),\ \ \mathrm{Re}\, B_2(0) = \mathrm{Im}\, s(0).$$

This paper presents the algorithms allowing one to enhance the speed of the spectral–correlation processing of the stationary AS. Moreover, we did not touch the problems of the use of temporal, correlation, and spectral windows, which are well discussed in literature. Feasibility of all the proposed algorithms was confirmed by imitation simulation of samples of random AS with spectral–correlation and noise parameters characteristics of the acoustic sounding of the atmosphere.[4] The above considered algorithms are quite general. In our opinion, it is possible to use them in the other practical applications.

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