STATISTICAL METHOD OF DESCRIPTION OF AEROSOL TRANSPORT IN A THERMALLY STRATIFIED ATMOSPHERIC BOUNDARY LAYER

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The paper deals with an exact analytical solution of the Fokker–Planck–Kolmogorov equation for the aerosol concentration distribution function in the atmospheric boundary layer that takes into account the effect of concentration intermittence. Practical application of the results obtained is discussed. Some examples of practical application of the concentration distribution functions are considered in examples of aerosol pollution spreading over Novosibirsk.

Theoretical study of aerosol transport in the atmosphere by traditional methods permits one to obtain only the mathematical expectations of the pollutant concentration. Since the spread of aerosols occurs in a turbulent atmosphere, these data are insufficient to solve a wide class of practical problems. In general, it is necessary to know the distribution functions of concentration, i.e., the statistical description of the propagation process. At present, only empirical probability density functions based on the results of the field observations[1] are applied to solve this problem. The hypothesis that the aerosol pollutant concentration in a given point obeys the logarithmic normal distribution is most widely used. For instance, satisfactory verification of the hypothesis was obtained by G.P. Zhukov[2] from the results of his experimental field study of pollutant diffusion in the near-ground atmospheric layer.

However, practical application of these results for a wide variety of meteorological conditions is very difficult because of the uncertainty in some parameters of the empirical distribution function of the concentration. The problem of determining the probability density functions is most deeply developed in an associated field, namely, in the theory of turbulent combustion.[3] Unfortunately, these data cannot be effectively used for calculations of pollutant diffusion in the boundary layer of the atmosphere due to the complexity and cumbersomeness of theoretical methods.

In this connection, we undertook a detailed theoretical study of the statistical character of the spread of aerosol pollutant in the atmosphere based on the semiempirical approach to the description of pollutant diffusion and the assumption that variations of aerosol concentration in a given point can be considered as a Markovian random process.[4]

According to Ref. 4, the exact analytical solution of the Fokker–Planck–Kolmogorov equation for a single-point probability density function of aerosol pollutant concentration C (at the point (x, y, z) at a given moment t), which is denoted by $f(C, t)$, has the form

$$f(C, t) = (1 - \gamma)\, \delta(C) + f^{(1)}(C, t)\ , \tag{1}$$

$$f^{(1)}(C, t) = \frac{1}{\pi^{1/2}\beta} \times$$

$$\times \left\{ \exp\left[-\left(\frac{C - \bar{C}}{\beta}\right)^2 \right] - \exp\left[-\left(\frac{C + \bar{C}}{\beta}\right)^2 \right] \right\},$$

where γ is the concentration intermittence, that is, the probability of the event that the pollutant concentration is greater than zero at a given point of the space; $\delta(...)$ is the Dirac delta function; $\bar{C}$ is the mathematical expectation of aerosol concentration; $f^{(1)}$ is the continuous part of the probability density function; β is the second parameter of the distribution function.

We see that the probability density function depends only on two parameters, $\bar{C}$ and β. So it is necessary and sufficient to set any two moments of concentration for practical application of probability density (1) to simulate the diffusion process. The mathematical expectation of the concentration can be obtained from a solution of the semiempirical equation of turbulent diffusion, and the second parameter β can be easily connected with the variance σ^2 of the aerosol concentration obtained by a solution of a similar equation (see, for instance, Ref. 5). Calculations lead to the relation[4]

$$\frac{\sigma^2}{\bar{C}^2} = \frac{\gamma}{2\beta_0^2} - (1 - \gamma) + \frac{1}{\pi^{1/2}\beta_0} \exp(-\beta_0^2),\ (\beta_0 = \frac{\bar{C}}{\beta})\ . \tag{2}$$

Thus, the probability density function of concentration (1) can be applied if and only if there is a model of the spread of the aerosol pollutant that

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permits one to obtain the mathematical expectation of the concentration and its variance.

The relation (1) was verified in the experiments performed in a wind tunnel of the Scientific Research Institute of Aerobiology[4]; it was also compared with the data of the field study of pollutant diffusion in the boundary layer of the atmosphere.[2] An analysis of the obtained information considering the most general properties of the concentration distribution function demonstrated satisfactory correspondence of the theoretical results to experimental data.[4]

To solve the semiempirical equation of turbulent diffusion and the corresponding equation for the concentration variance within the boundary layer of the atmosphere, it is necessary to assign the fields of mean values of the wind velocity and the set of the turbulent diffusion coefficients. In the present paper, the diurnal variations of the former meteorological parameter in the boundary layer were specified for the numerical analytical model,[6] and the diffusion coefficients were specified on the basis of the hypothesis that they are proportional to the corresponding components of the Reynolds viscous stress tensor.[5] The coefficients and some accompanying parameters were assigned for the algebraic model of turbulent flows and stresses.[7] The hypothesis that the diffusion coefficients are proportional to the components of the Reynolds viscous stress tensor in the ground layer of the atmosphere was verified by experimental data.[4]

A run of calculations for point stationary sources simulating the mouth of smoke stacks of the Novosibirsk Heat-and-Power Station (NHPS) was performed in order to illustrate the salient features of the spread of gaseous emissions from the NHPS and to implement the calculation algorithms in practice. The calculations were performed for winter and summer, night and day time, and different values of wind speed and direction. The actual height of buildings in residential areas, different types of the underlying surface (city building, parks, and forest tracts, the River Ob', etc.), and the actual relief of the terrain were taken into account. Below, we consider only two heat-and-power stations: HPS-2 and HPS-3 situated practically at the same place and having three stacks each. The heights of the stacks were 60–120 m. The effective height of the sources was determined by the well-known formulas.[8] The spread of nitrogen dioxide and sulfur dioxide was simulated.

Some results describing the spread of nitrogen dioxide (MPC = 0.85·10−4 g/m3) in summer are presented in Table I and are shown in Fig. 1. All characteristics were calculated for an altitude of two meters.

The designations used in Table I are as follows: t is the moment at which the concentration was calculated; v is the wind velocity; C_1, C_2, D_1, and D_2 are the minimum and maximum values of the concentration and of the standard deviation, respectively; S_{v1} and S_{v2} are the areas, calculated by the probability method, with the enhanced concentration above a certain limiting value C_0 (e.g., MPC) for minimum values of C_0 and close to maximum values, respectively; S_{S1} and S_{S2} are the same areas but calculated by the traditional method; D_1 and D_2 are their standard deviations corresponding to minimum and maximum values of C_0.

One can see that the maximum values of concentration depend strongly on meteorological conditions. As a whole, the concentration is higher in the daytime than at night. The concentration decreases monotonically in the daytime with the increase of the wind velocity; at night, the extremum is observed at about 5 m/s. The critical velocity, at which the maximum concentration is observed, equals approximately 5 m/s at night and 2 m/s in the daytime.

The coefficient of concentration variation K = D/C decreases from 0.5 at small concentration values under condition of stable stratification at t = 4 h to 0.09 at t = 15 h and wind velocity being equal to 7 m/s. A similar tendency is also observed for the variation coefficient of the area with enhanced concentration above a given limiting value C0. This coefficient equals approximately 0.5 for stable stratification and decreases to 0.15 when the stratification becomes unstable. It should be noted that the employed calculation algorithm for the area variance yields its estimate from below. Really, it must be higher.

The ratio of the areas S_v and S_S is close to unity for small values of C_0 and increases with the increase of C_0. In some cases, it reaches 1.5–1.6.

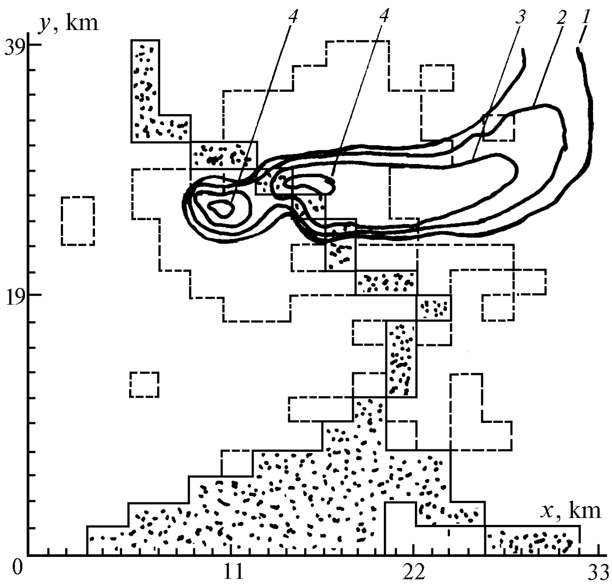

FIG. 1. Isolines of pollutant concentration (g/m^3): *1)* 6.9·10^{-6}, *2)* 1.7·10^{-5}; *3)* 3.5·10^{-5}; *4)* 2.8·10^{-4}. *The dotted areas is the River Ob' and the water storage reservoir. The boundary of the city is indicated by the dashed line.*

TABLE I. Results of calculation of the spread of nitrogen dioxide from HPS-2 and HPS-3.

t, h	v, m/s	C_1, C_2, g/m^3	D_1, D_2, g/m^3	S_{v1}, S_{v2}, m^2	S_{S1}, S_{S2}, m^2	S_v/S_S	D_{S1}, D_{S2}, m^2	D_S/S_v
4	2	$1.3\cdot10^{-7}$ $1.1\cdot10^{-5}$	$7.0\cdot10^{-8}$ $5.6\cdot10^{-6}$	$9.3\cdot10^{7}$ $2.9\cdot10^{7}$	$1.2\cdot10^{8}$ $2.2\cdot10^{7}$	0.80 1.32	$4.8\cdot10^{7}$ $1.4\cdot10^{7}$	0.50 0.48
4	5	$5.5\cdot10^{-7}$ $4.4\cdot10^{-5}$	$9.9\cdot10^{-8}$ $7.9\cdot10^{-6}$	$1.6\cdot10^{8}$ $4.6\cdot10^{7}$	$1.6\cdot10^{8}$ $4.0\cdot10^{7}$	1.00 1.15	$3.8\cdot10^{7}$ $8.7\cdot10^{6}$	0.24 0.19
4	7	$5.0\cdot10^{-7}$ $4.0\cdot10^{-5}$	$6.7\cdot10^{-8}$ $5.3\cdot10^{-6}$	$1.7\cdot10^{8}$ $4.9\cdot10^{7}$	$1.7\cdot10^{8}$ $3.9\cdot10^{7}$	1.01 1.26	$2.6\cdot10^{7}$ $7.9\cdot10^{6}$	0.15 0.16
15	2	$3.5\cdot10^{-6}$ $2.8\cdot10^{-4}$	$2.6\cdot10^{-7}$ $2.0\cdot10^{-5}$	$1.5\cdot10^{8}$ $1.1\cdot10^{7}$	$1.5\cdot10^{8}$ $6.0\cdot10^{6}$	1.03 1.48	$2.6\cdot10^{7}$ $5.0\cdot10^{6}$	0.17 0.45
15	5	$2.4\cdot10^{-6}$ $1.9\cdot10^{-4}$	$2.6\cdot10^{-7}$ $2.1\cdot10^{-5}$	$1.7\cdot10^{8}$ $5.0\cdot10^{6}$	$1.7\cdot10^{8}$ $4.0\cdot10^{6}$	1.00 1.25	$3.1\cdot10^{7}$ $1.8\cdot10^{6}$	0.18 0.36
15	7	$1.5\cdot10^{-6}$ $1.2\cdot10^{-4}$	$1.5\cdot10^{-7}$ $1.1\cdot10^{-5}$	$1.8\cdot10^{8}$ $6.5\cdot10^{6}$	$1.8\cdot10^{8}$ $4.0\cdot10^{6}$	1.00 1.60	$2.9\cdot10^{7}$ $2.9\cdot10^{6}$	0.15 0.43

TABLE II. Probability of MPC ($8.5\cdot10^{-5}$ g/m^3) *excess for different values of concentration and variation coefficient $K = D/C$.*

C	$K = 0.25$ $\gamma = 0.99$ $\beta_0 = 2.83$	$K = 0.50$ $\gamma = 0.95$ $\beta_0 = 1.41$	$K = 1.00$ $\gamma = 0.63$ $\beta_0 = 0.64$	$K = 1.25$ $\gamma = 0.49$ $\beta_0 = 0.47$	$K = 1.50$ $\gamma = 0.39$ $\beta_0 = 0.36$
$8.4\cdot10^{-4}$	0.99	0.95	0.63	0.49	0.39
$4.2\cdot10^{-4}$	0.99	0.94	0.62	0.49	0.30
$2.1\cdot10^{-4}$	0.99	0.88	0.60	0.48	0.38
$1.0\cdot10^{-4}$	0.73	0.62	0.51	0.43	0.36
$8.5\cdot10^{-5}$	0.50	0.50	0.46	0.41	0.35
$8.0\cdot10^{-5}$	0.40	0.45	0.45	0.40	0.34
$7.0\cdot10^{-5}$	0.19	0.33	0.40	0.37	0.33
$6.5\cdot10^{-5}$	0.10	0.27	0.37	0.36	0.32
$6.6\cdot10^{-5}$	0.05	0.20	0.34	0.34	0.31
$5.5\cdot10^{-5}$	0.01	0.13	0.30	0.31	0.29
$4.0\cdot10^{-5}$	0.00	0.01	0.15	0.21	0.23
$3.5\cdot10^{-5}$	0.00	0.00	0.10	0.16	0.19
$3.0\cdot10^{-5}$	0.00	0.00	0.05	0.10	0.15
$2.5\cdot10^{-5}$	0.00	0.00	0.01	0.05	0.10
$2.0\cdot10^{-5}$	0.00	0.00	0.00	0.01	0.04
$1.5\cdot10^{-5}$	0.00	0.00	0.00	0.00	0.01

Figure 1 shows the concentration isolines calculated for $t = 15$ h and a wind velocity of 2 m/s. A substantial displacement and deformation of the area with enhanced concentration are observed as a result of the influence of the difference of the underlying surface altitude varying from 13 to 230 m inside the city. The influence of the relief decreases at night and with the increase of wind velocity and the zone of spreading elongates along the nonperturbed wind velocity vector directed at an angle of 45° to the horizontal axis.

To obtain more detailed information about the behavior of the characteristics P, C_0, and γ, additional calculations of these values were performed for typical values of C and D close to the MPC. The results of these calculations are presented in Table II. The increase of the variation coefficient $K = D/C$ for C greater than MPC ($0.84\cdot10^{-4}$ g/m^3) leads to the decrease of the probability of MPC excess. For smaller C, the dependence is reverse: the probability increases with the increase of K. The values of the intermittence parameter γ and the parameter β_0 always decrease with the increase of the variance.

Thus, the following conclusions can be drawn from an analysis of calculation results. The statistical pollution characteristics calculated on the basis of the obtained probability density function are rather reasonable for typical values of nitrogen dioxide concentration observed in the atmosphere of Novosibirsk. Knowledge of the probability density function provides a basis for the statistical method of calculating the concentration variance, the zone with the enhanced concentration greater than C_0, and the probability of the MPC excess. This makes it possible to determine the above-considered characteristics of the spread of gaseous pollutant in the ground layer of the atmosphere more accurately.

Actual orograhical and thermal inhomogeneities of the underlying surface should be considered in simulation of the pollutant spread. Neglect of the above-mentioned facts may lead to gross errors in the estimation of the main characteristics of the atmospheric pollution.

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