MATHEMATICAL SIMULATION OF THE ANTHROPOGENIC AEROSOL DISTRIBUTION

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Mathematical simulation of the atmospheric aerosol distribution is based on the stochastic approach to description of climatic characteristics of the regions under study as multidimensional probability density functions for the occurrence of a specific meteorological complex. These characteristics close the differential equation describing the transport and the turbulent diffusion of a pollutant. Some examples are presented on the estimation of the contributions from particular industrial sources to the pollution of the atmosphere and the underlying surface in the region of South Baikal.

Environmental pollution with industrial and domestic wastes (smokes, sewage, ash and slag waste piles) has not only the direct effect on the human health but also the indirect one (soil erosion, flora and fauna damages). Environment self-cleaning from the pollution significantly depends on the climatic characteristics of a given region. Therefore, the evaluation of the meteorological balance of the atmosphere that determines its potentialities to spread and accumulate pollutants both in the near-ground layer and on the underlying surface is of great interest. From this viewpoint, the probability space is the functional space most closely describing the phenomenon under study because it minimizes the errors in contrast to other spaces. Let us give some explanations to the above statement. Usually the pollutant concentration is calculated for some instantaneous or averaged meteorological parameters. In the former case, the results obtained correspond just to these given instantaneous characteristics that can hardly occur simultaneously. In the latter case, the results correspond to the characteristics averaged over some period that can never occur under natural conditions. That is why it is more reliable to treat the transport and the turbulent diffusion of pollutants in the field of random velocities considering all meteorological complexes as a group of events for the time interval under study.

Such an approach does not assume the initial conditions and evolution of the process to be in one-to-one correspondence, while allowing, in principle, the possibility of the process to run in different ways under similar initial conditions.[1]

As the initial equation describing the process of transport and turbulent diffusion of a pollutant, let us take, for example, the following one:

$$u\,\frac{\partial s}{\partial x} - w_g\,\frac{\partial s}{\partial z} = \frac{\partial}{\partial y}\,k_y\,\frac{\partial s}{\partial y} + \frac{\partial}{\partial z}\,k_z\,\frac{\partial s}{\partial z} \qquad (1)$$

where x, y, and z are the axes of the Cartesian rectangular coordinate system, with the x and y axes being in the horizontal plane and x axis directed along the wind, whereas the z axis is looking vertically upwards; the main axes of the tensor of turbulent diffusion coincide with the coordinate axes ($k_{ij} = 0$, if $i \neq j$; $k_{ij} = k_i$, if $i = j$, $i, j = x, y, z$); s is the concentration of a pollutant considered; u is the horizontal (along x axis) component of the wind velocity vector; w_g is the rate of particle sedimentation due to gravitation.

The boundary conditions have the form

$$us = M\,\delta(y)\,\delta(z - H) \qquad \text{at } x = 0,$$
$$|s| \neq \infty \qquad \text{at } z \to \infty,\ |y| \to \infty,$$
$$k_z\,\frac{\partial s}{\partial z} = 0 \qquad \text{at } z = 0.$$

Here M is the source intensity (an amount of a substance emitted per unit time); H is its altitude, δ is the Dirac delta–function.

Solution of the initial differential equation, derived in Ref. 2 but written down in designations accepted in Ref. 3 with some corrections, takes the form

$$s = \frac{M(zH)^{-\mu(1+n)/2}\, z_1^{1+\mu(1+n)}}{2(1+n)\,k_1\sqrt{\pi\,k_0\,x^3}} \times$$
$$\times \exp\left[-\frac{y^2}{4\,k_0\,x} - \frac{u_1\,(H^{1+n} + z^{1+n})\,z_1^{1-n}}{(1+n)^2\,k_1\,x}\right] \times$$
$$\times\, I_\mu\left[\frac{2u_1\,(Hz)^{(1+n)/2}\,z_1^{1-n}}{(1+n)^2\,k_1\,x}\right],$$

where $\mu = w_g\,z_1/(k_1\,(1+n))$; I is the Bessel function of the imaginary argument; $k_z = k_1\,(z/z_1)$; $u = u_1\,(z/z_1)^n$; $k_y = k_0\,(z/z_1)^n$, $k_0 = L/2\sqrt{u^2 + v^2}$; L

is the scale of a computation step; v is the wind velocity component along the y axis; n is the dimensionless parameter for interpolation of the vertical profile of wind velocity; k_1 and u_1 are the coefficient of the turbulent diffusion and wind velocity at the altitude $z_1 = 1$ m.

The flow (F) of a suspended matter onto the underlying surface with regard for boundary conditions is expressed as

$$F = w_g\, s.$$

The problem solution assumes the use of data of meteorological observations on a stationary network, which form the basis for construction of the climatic probability density functions for the occurrence of meteorological complexes. Such an approach excludes many drawbacks of analytical solution arising due to simplification of the initial differential equations.

Since the analytical solution presented is valid for the differential equation (1) in which the abscissa is directed along the wind, we have to use the rotating coordinate system when calculating the pollution at every point of the space in order to fully describe wind statistical structure for the time interval considered. As to the problem of estimation of heavy particles accumulation on the underlying surface, in this case we have to use, as a rule, empirical distribution laws. When the ecological situation should be evaluated from the viewpoint of the probability of maximum permissible concentrations to be exceeded, the problem of describing the statistical structure can be reduced to the statistical distribution laws chosen to fit the real distribution with a minimum error. In this case, computations take much less time.

It should be noted that the idea of this method can also be applied to other types of analytical solutions.

To illustrate this, let us present the results of some computer experiments.

Experiment No. 1. The aim of the experiment was to estimate the contribution from anthropogenic sources of Angarsk and Irkutsk into the pollution of South Baikal. As a pollutant we consider dust (the total amount of dust emissions in Angarsk is approximately four times as large as that in Irkutsk) with regard for the percentage of its fractions. Then for each fraction of the rate of sedimentation due to gravitation was computed.[4] The data on meteorological regime from the six stationary stations for a ten-year period of observations allowed us to construct the empirical probability density function for the wind velocity vector as the stable climatic characteristic. The computational results are shown in Fig. 1. It should be noted that the computations were made separately for Irkutsk and Angarsk and then summarized in one figure to demonstrate the contribution from each industrial complex to the underlying surface pollution. As is seen, in spite of the fact that Angarsk pollution sources are farther from Baikal, their contribution to the total pollution of South Baikal water is no less than that from Irkutsk.

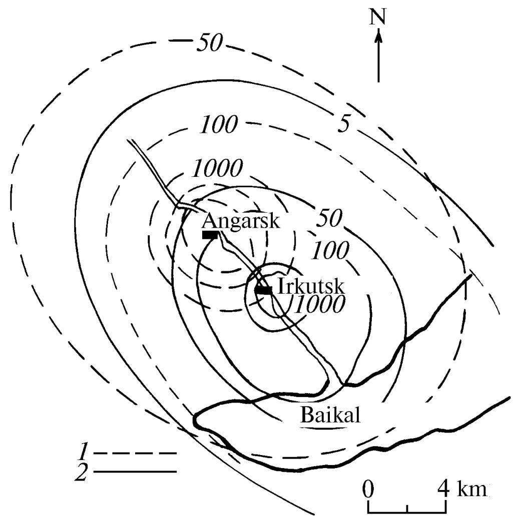

FIG. 1 Dust amount annually falling onto the underlying surface from the sources of Angarsk (1) and Irkutsk (2), in mg/m^2.

Experiment No. 2. The computations were made for the total amount of dust, emitted from the sources of Baikal'sk, felt out onto the underlying surface during the year (Fig. 2). The relief was mapped with the 1–km step up to 44 km to the east, 9 km to the west, and 15 km to the south from the Baikal'sk pulp and paper mill. The computational results were then compared with the data of snow surveys, from which the suspended matter accumulation was found for the period of steady–state snow cover over the region of South Baikal,[5] and have shown their good agreement.

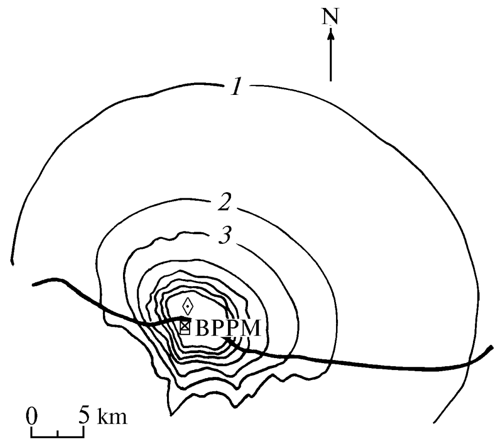

FIG. 2. Dust amount felt out annually onto the underlying surface from the Baikal'sk pulp and paper mill (BPPM). Isolines correspond to the following amounts: 100 *(1),* 500 *(2),* 1000 mg/m^2 *(3), and further with the step of* 1000 mg/m^2. *Solid line is for the Baikal shore line. The local maximum equal to* 37000 mg/m^2 *is marked by* ◇.

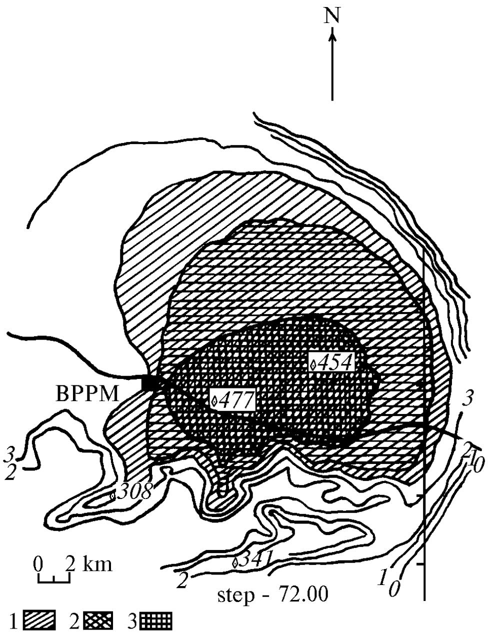

FIG. 3. Frequency of methyl mercaptan to be in excess over 20 *MPC for the region of the BPPM: probability no less than* 0.4 (288 h) *(1),* ≥ 0.5 (360 h) *(2), and* ≥ 0.6 (432 h) *(3).*

Experiment No. 3. For all months (averaged over 15 years) the calculations have been done for the probability of methyl mercaptan concentration to be in excess over the maximum permissible level (MPC = $9\cdot10^{-6}$ mg/m^3). Since the area of regions polluted with methyl mercaptan in concentrations above the MPC level is more than 100 km, the computation results are presented for the case when the excess in no less than 20 MPC. The relief was taken the same as in the experiment No.2. As an example we present the results obtained in July (Fig. 3). The shore line is given by the solid line as in Fig. 2. For convenience the repetition isolines for an excess over 20 MPC are shown with the step of 72 hours that corresponds to the probability of 0.1. The dangerous zones are shown by hatching. Additional computations have shown that in the zone of the Baikal'sk pulp and paper mill the concentrations of this pollutant may be even 90–100 times as large as the MPC level.

The models considered allow the predictions to be obtained (reconstruction of enterprises, fuel change, production regime change, improvement of waste cleaning, putting into operation of new objects, etc.). The results of such computations are presented, e. g., in Refs. 6–8.

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