USE OF AN ADAPTIVE FILTER IN THE PROBLEM ON ESTIMATING COORDINATES OF AN OPTICAL RADIATION SOURCE

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Received March 6, 1994

In this paper we analyze a possibility of using Kalman linear adaptive filter in the problem on estimating coordinates of a pulsed source of optical radiation observed through the atmosphere.

The differential technique of ranging by retrieving coordinates of the end user of information from the NAVSTAR satellite network is well–known in radionavigation. It is based on detecting modulated signals from several space vehicles (SV).[1] If an optimal "constellation" of SVs is chosen, coordinates of the detector are calculated from time lags between signals arriving at the detectors and from the SVs coordinates. A slightly modified version of the same technique is used for estimating coordinates of a pulsed isotropic point source of optical radiation from observations available from several SVs.[2] According to this technique various SVs record different copies of one and the same signal, which has a complex, usually unknown profile. These copies are essentially altered in their scale and shifted in time. The initial data for calculating the coordinates of such a source are the time lags between such copies arrival at a SVs.

Determination of the time lags in the form of arrival times difference can be done accurate to the timing accuracy of SV recorders of the signal that depends on SNR (signal–to–noise ratio). A more reliable method of measuring time lags could be based on estimation of the time lad between copies as a whole, at which their matching becomes optimal in the sense of rms difference.

This task can be solved by a linear adaptive filter, capable of identifying unknown system connected to it in parallel.[3] The identification is implied of a linear system capable of transforming one of the compared copies of a signal into the other. Provided the profiles of the two copies coincide completely, the pulse response $h(t)$ of such a system would be the Dirac delta function $h(t) = \delta(t - \Delta t)$, where Δt is the time lag between the two copies. In practice the function $h(t)$ will naturally differ from $\delta(t - \Delta t)$, but one may expect that, for copies just slightly perturbed and scale normalized, the position of the average point (i.e., the first moment) and the maximum of $h(t)$ function are close to Δt. The same should hold for an adaptive digital filter with the pulse response $h_i(n)$ which approximates the system to be identified.

If the filter operates in a stationary mode ($n \gg N$), one may use its characteristic, that is, the position of its average point or maximum response $h_i(n)$ as a function of i, to estimate the sought time lags Δt between the compared copies of a signal. One may assume that the accuracy of such an estimation will worsen with Δt growth. Therefore, it is advisable, prior to comparing two copies, to bias one of them by an expected (nominal) lag Δt_0, thus reducing the problem to estimating the minor difference $\Delta t - \Delta t_0$. Below we understand by Δt just this difference between the actual and nominal values of a time lag.

In this study we just check up the above statements in a numerical experiment with a digital adaptive filter which has a finite pulse response $h_i(n)$, $i = 1, ..., N$.

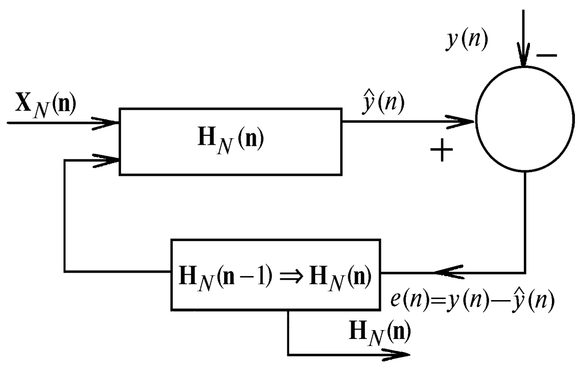

Fig. 1.

Figure 1 presents the operation of such a filter schematically. The transformation that an adaptive filter performs is described by a convolution equation, common to all the linear filters

$$\hat{y}(n) = \sum_{i=0}^{N-1} h_i(n)\, x(n-i) = \mathbf{H}_N^{\mathrm{T}}(n)\, \mathbf{X}_N(n) = \mathbf{X}_N^{\mathrm{T}}(n)\, \mathbf{H}_N(n)\ , \quad (1)$$

where $x(n)$ and $\hat{y}(n)$ are random signals, fed to the input and generated at the output of the filter, respectively; $\mathbf{H}_N(n)$ is the pulse transient characteristic of the filter at time n, presented by the column–vector of length N

$$\mathbf{H}_N(n) = [h_0(n)\ , \ ...,\ h_{N-1}(n)]^{\mathrm{T}}\ ; \quad (2)$$

$\mathbf{X}_N(n)$ is a portion of input signal which includes the values x for instants n, $n - 1$, ..., $n - N + 1$, presented by the column–vector of length N

$$\mathbf{X}_N(n) = [x(n),\ ...,\ x(n - N + 1)]^{\mathrm{T}}\ ; \quad (3)$$

subscript T denotes the symbol of matrix transposition. All elements of the vector $\mathbf{X}_N(n)$, for which $n - i < 1$ ($i = 0, 1, ..., N - 1$), are equal to zero. A peculiar feature of an adaptive filter is that its characteristic $\mathbf{H}_N(n)$ continuously changes for higher n, to make the output signal $\hat{y}(n)$ closer to the desired (reference) signal $y(n)$. In the ideal case the filter characteristic $\mathbf{H}_N(n)$ should be controlled by the statistical condition of minimum discrepancy

0235-6880/94/06 444-04 $02.00

$$\varepsilon(n) = E\Big[\,|y(n) - \hat{y}(n)|^2\Big] , \tag{4}$$

where $E[\bullet]$ is the operator of mathematical expectation of the random variable in brackets. In practice, however, a simpler condition is usually sufficient

$$\sum_{i=1}^{n} \lambda^{n-i}\, e^2(i / n) = \min , \tag{5}$$

where

$$e(i / n) = y(i) - X_N^{\mathrm{T}}(n)\, \mathbf{H}_N(n) \tag{6}$$

is the error in ith value of the sought signal $y(i)$ predicted by filter $\mathbf{H}_N(n)$, calculated for the time moment n, and λ is a weighting coefficient close to unit. Changes of $\mathbf{H}_N(n)$ at higher n, which it suffers while adapting to the task of matching $\hat{y}(n)$ to $y(n)$, are described by the recursive equation

$$\mathbf{H}_N(n) = \mathbf{H}_N(n-1) + \mathbf{g}_N(n)\,\big[y(n) - \mathbf{X}_N^{\mathrm{T}}(n)\,\mathbf{H}_N(n-1)\big] , \tag{7}$$

where $\mathbf{g}_N(n)$ is the vector of gain factors. In general, the task of designing an adaptive filter is reduced to calculation of these factors for every n. Just the latter equation, together with a procedure of $\mathbf{g}_N(n)$ renewal when one proceeds from $(n-1)$ to n, is what is called an adaptive filter. The filter functioning starts according to this equation at the time moment $n = 1$ and continues till the final value $n = p \geq N$, beyond which the value $\varepsilon(n)$ remains practically unchanged.

It is a recursive adaptive filter designed after the least–t square technique. Its operation may be presented by the following scheme. At $n = 0$ the initial conditions are:

$$\mathbf{H}_{NN}(0) = \mathbf{X}_{NN}(0) = 0 , \tag{8}$$

$$C_{NN}(0) = \delta\, I_{NN} , \qquad (\delta \gg 1) . \tag{9}$$

The input data, starting from $n = 1$ to the final value p are $y(n)$ and $\mathbf{X}_N(n)$. The current (running) nth computational cycle includes:

$$e(n / n-1) = y(n) - \mathbf{X}_N^{\mathrm{T}}(n)\,\mathbf{H}_N(n-1) \text{ – the forecast error,} \tag{10}$$

$$\mu(n) = \mathbf{X}_N^{\mathrm{T}}(n)\, C_{NN}(n-1)\, \mathbf{X}_N(n) , \tag{11}$$

$$\mathbf{g}_N(n) = \frac{C_{NN}(n-1)\,\mathbf{X}_N(n)}{1 + \mathrm{m}(n)} \text{ – gain vector ,} \tag{12}$$

$$\mathbf{H}_N(n) = \mathbf{H}_N(n-1) + \mathbf{g}_N(n)\, e(n / n-1) \text{ – renewed estimate,} \tag{13}$$

$$C_{NN}(n) = (1/\lambda)\,\big[C_{NN}(n-1) - \mathbf{g}_N(n)\,\mathbf{X}_N^{\mathrm{T}}(n)\, C_{NN}(n-1)\big]. \tag{14}$$

During the final pth cycle N values $h_0(p)$, $h_1(p)$, ..., $h_{N-1}(p)$, which form the column–vector $\mathbf{H}_N(p)$, appear at the filter output. We introduce the initial condition $C_{NN}(0) = \delta I_{NN}$ $(\delta \gg 1)$, where I_{NN} is the unit $N \times N$ matrix, to provide for the existence of the inverse matrix C_{NN}^{-1} during the first $n \leq N$ computations, since the matrix $C_{NN}(n)$ could be singular otherwise.

This algorithm should only be considered as basic one. Its purpose is to clarify the principle of operation of an adaptive filter in the mode of identification of a linear system. The cycle of such an algorithm needs $O(N^2)$ arithmetic operations. Their number may, however, be reduced to $O(N)$, if other more efficient procedures are used,[4,5] which are based on a scheme of quick Kalman filtration. Appendix to this paper contains a PASCAL program to compute such an adaptive filter. In addition to computing $\mathbf{H}_N(p)$ the program also enables one to estimate the time lag between the copies Δt according to formula

$$\Delta t = \sum_{i=0}^{N-1} i\, h_i(p) , \qquad p \geq N = 20\text{–}40 . \tag{15}$$

Convergence of the algorithm implementing this program is illustrated by Fig. 2 which shows coefficients $h_i(n)$ vs i for a signal–to–noise ratio of 30 at various n $(N = 20)$. The values of λ and δ were assumed to be 0.9 and 5, respectively. Figure 3 demonstrates the parameters of such an adaptive filter for the case of two signals of identical shape and no noise.

As seen from Fig. 3, the index of the maximum component of such an adaptive filter corresponds to a time lag $\Delta t = 4\tau$ between the two signals, where τ is the time step.

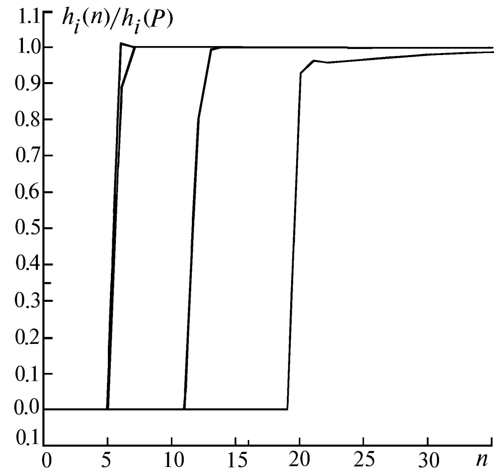

Fig.2. Parameters of the adaptive filter for two signals of identical shape in the case of "white" noise, $i = 0, 4, 10$, *and* 18.

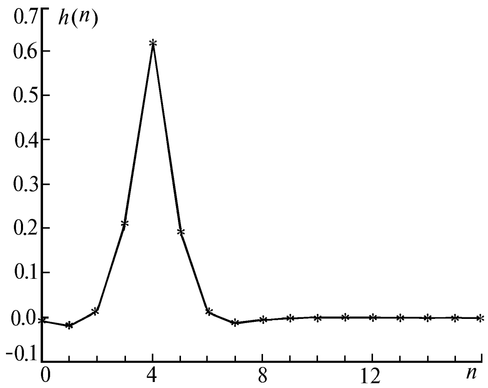

Fig.3. Parameters of the adaptive filter for two signals of identical shape.

Thus, one can find Δt from the maximum component of the adaptive filter accurate to τ value. Expression (15) then yields the same time lag between the two signals at a much higher accuracy. At the same time, when there is a noise in the signal, the value of p should not exceed the characteristic signal length too much. Otherwise the adaptive filter would start tuning to noise, instead of a signal. As demonstrated by computations for the case of two identical signals without noise the components of an adaptive filter remain practically independent of signal profile. Five to ten time steps after the start of the second pulse, the components of vector $\mathbf{H}_N(n)$ stop changing, while all the components with indices exceeding Δt by about 10 vanish. The index of the maximum component of the vector $\mathbf{H}_N(n)$ corresponds to the time moment shorter than Δt by less than $\tau/2$.

Assuming a Gaussian noise to perturb both the first and the second signal (the noises reaching the first and the second detector are thus mutually independent), and also accounting for inhomogeneous screening of signals which reach the first and the second SV detectors by the partially cloudy atmosphere, one might expect that the $\mathbf{H}_N(n)$ vector will noticeably be distorted. The components of vector $\mathbf{H}_N(n)$ will then not turn to zero at high indices. As a result, expression (15) will not yield a satisfactory accuracy. To determine Δt in this case one may use the time moment corresponding to the maximum (principal maximum) component of the $\mathbf{H}_N(n)$ vector. Then the accuracy of Δt retrieval will be equal to the time step τ. To reduce the effect of noise on the accuracy of retrieval of Δt, one may compare the signals after smoothing them. We choose an integrator (signal smoother) in the form of an RC–curcuit, described by the equation

$$V(t) = \frac{1}{t_I} \int\limits_0^t X(\tau) \exp\left(-\frac{t-\tau}{t_I}\right) \mathrm{d}\tau , \qquad (16)$$

where $X(\tau)$ is the initial, and $V(t)$ is the smoothed signal. Using formulas for numerical integration with the integration step τ, we obtain an expression for $V(t)$

$$V(t)=X(t)-t_I\, \mathrm{e}^{-t/t_I} \sum_{j=1}^{i} D_j \left(\mathrm{e}^{-t_j/t_I} - \mathrm{e}^{-t_{j-1}/t_I}\right) , \qquad (17)$$

$$D_j = \frac{X(t_j) - X(t_{j-1})}{t_j - t_{j-1}} . \qquad (18)$$

Signal smoothing significantly increases the signal–to–noise ratio. In this case the components of vector $\mathbf{H}_N(n)$ still do not turn to zero at high indices, and formula (15) does not yield any qualitatively different results on the time lag between the two signals. At the same time, integration for large t_I makes the principal maximum of $\mathbf{H}_N(n)$ more pronounced, so that one may then determine the time lag between the two signals using the principal maximum of $\mathbf{H}_N(n)$ at lower signal–to–noise ratios.

Computations show that the parameters of the Kalman adaptive filter, which transforms the first of the two signals into a signal only slightly different from the second, may be used to calculate the time lag between the moments the signals arrive at detectors. Even with some noise and if the broadening of signals is different (from SV to SV) due to partial cloudiness in the atmosphere, the error of time lag Δt determination from the parameters of the filter $\mathbf{H}_N(n)$ does not exceed the time step τ.

APPENDIX: DISCRETE ADAPTIVE KALMAN FILTER $\mathbf{H}_N(n)$ AND THE TIME LAG BETWEEN THE SIGNALS: COMPUTATIONAL PROGRAM

Temporal trends of the input and the reference signals are given by functions $Fx(j)$ and $Fy(j)$.

```
Program FK;
   Const N 1 = 50;                          {window width}
   Delta : Extended = 5.0; La : Extended = 0.9; P = 50;
   Ti : Extended = 10;                {integration constant}
   Var i, j, j1, j2, j3 : Integer;
      S, S1, S2, S3, S4, M, E, Y : Extended;
        X, X1, Y1, H, g, g1 : Array [0..N1–1] of
        Extended;
            C : Array [0..N1–1, 0..N1–1] of Extended;
   Begin    For j3 : = 0 to N1–1 do
            For i : = 0 to N1–1 do C[ j3, i] : = 0;
   For i : = 0 To N1–1 Do Begin
         X[i] : = 0; H[i] : =0; C[i, i] : = Delta End;
         For j2 : = 1 to P do Begin {j2}
            If j2 <= N1–1 Then Begin j : =j2; j1 : =j2–
            1 End
            Else Begin j : = N1–1; j1 : = j
                                              End;
   For i : = j Down To 1 do X[i] : = X[i–1];
        X1 [ j] : = Fx( j);              {input signal vector}
        Y1 [ j] : = Fy( j);             {output signal vector}
         {signal integration}
   S1 : = 0; S4 : = 0;
   For i : = 1 To j do Begin {i}
      S2 : = ( j – i)/Ti; If S2 > 60 Then S2 : = 0 Else
      S2 : = Exp(–S2);
        S3 : = ( j – i + 1)/Ti; If S3 > 60 Then S3 : = 0
        Else S3 : = Exp(–S3);
          S1 : = S1 + (X1[i] – X1[i – 1])*(S2 – S3);
            S4 : = S4 + (Y1[i] – Y1[i – 1])*(S2 – S3)
            End; {i}
            X[0] : = X1[ j ] – Ti*S1;  Y : = Y1[ j ] –
             Ti*S4;
                          {calculation of signal forecast
                          error}
   S : = 0; For i : = 0 to j1 do S : = S + X[i]*H[i];
   E : = Y – S;
                          {calculation of the value μ(n)}
   M : = 0; For j3 : = 0 to j1 do Begin S : = 0;
            For i : = 0 to j1 do S : = S + X[i]*C[ j3, i];
            M : = M + S*X[ j3]                 End;
                          {calculation of the vector of gain}
            For j3 : = 0 to j1 do Begin S : = 0;
   For i : = 0 to j1 do S : = S + C[ j3, i]*X[i];
                            g[ j3] : = S/(LA + M)   End;
                    {calculation of the updated estimate}
   For i : = 0 to j1 do H[i] : = H[i] + g[i]*E;
   For i : = 0 to j1 do Begin g1[i] : = 0;
   For j3 : = 0 to j1 do g1[i] : = g1[i] + X[ j3]*C[ j3, i]
   End;
     For j3 : = 0 to j1 do For i : = 0 to j1 do
     C[ j3, i] : = (C[ j3, i] – g[ j3]*g1[i])/La End; { j2}
        {calculation of time lag between the two signals}
     S : = 0; For i : = 0 to N1–1 do S : = S + i*H[i] End End.
```

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