OPTIMIZATION OF THE OBSERVATION CONDITIONS IN THE PROBLEM OF ESTIMATING PARAMETERS OF THE GROUND ATMOSPHERIC LAYER

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We consider the problem of determining the parameters of the near–ground atmospheric layer from the data of gradient observations of wind velocity and air temperature. A numerical method of analysis of the measurement data and the experimental design is proposed. Numerical simulation of the optimum levels for location of the gradient observations is performed for thermally–stratified near–ground layer.

The uncertainty connected with measurement errors in the mean values of meteorological elements arise when determining the characteristics of turbulent regime in the near–ground atmospheric layer such as the friction rate u_*, vertical turbulent fluxes of heat $\overline{w'T'}$, and moisture $\overline{w'q'}$, by indirect methods, for instance, by data of gradient observations. Approximating the measured profiles of wind velocity and temperature by models corresponding to physical observation conditions one can formulate different criteria for estimating accuracy of the parameters of the near–ground layer using the least squares method (LMS). The parameters are: u_*, temperature scale T_*, and the Monin–Obukhov length scale L. Since one measures different physical values, the problem of multi–criterion estimation of the parameters arises. In this case, the technique of calculations can use only the measurements of wind velocity for determining u_*, and only gradient temperature measurements for T_*.[1, 2] In this paper we present a unified criterion which takes into account the variances of errors in measuring the wind velocity and temperature as weight factors for the estimation of u_*, T_*, and L.[3]

To reduce the influence of random measurement errors on the quality of the estimated parameters, we consider the problem of the choice of optimum location of the gradient observations. The problem of the construction of the experimental plan is formulated in correspondence with Ref. 4. for the case of simultaneous measurements of several values at the same levels.

1. FORMULATION OF THE PROBLEM OF ESTIMATING PARAMETERS IN THE NEAR–GROUND LAYER

Let us consider a stationary, horizontally homogeneous stratified near–ground layer of the atmosphere. Then, according to the similarity theory, the profiles of the mean values of wind velocity, potential temperature, specific humidity can be represented in the following form[5–7]:

$$u(z) = \frac{u_*}{\varkappa}\left[f_u\left(\frac{z}{L}\right) - f_u\left(\frac{z_0}{L}\right)\right],$$

$$T(z) = T_0 + T_*\left[f_T\left(\frac{z}{L}\right) - f_T\left(\frac{z_0}{L}\right)\right], \tag{1}$$

$$q(z) = q_0 + q_*\left[f_q\left(\frac{z}{L}\right) - f_q\left(\frac{z_0}{L}\right)\right].$$

Here u is the horizontal component of wind velocity; T is the potential temperature; q is the specific humidity; $\varkappa$ is the Karman constant; q_* is the measurement scale for the specific humidity; f_u, f_T, f_q are continuous universal functions; z_0 is the height of roughness of the underlying surface. The Monin–Obukhov length scale L is determined by formula:

$$L = u_*^2/(\varkappa^2 \lambda T_*), \tag{2}$$

where λ is the floatability parameter.

Furthermore, without any loss of generality, we consider the problem of determining dynamic and heat fluxes. Let the variations of wind velocity and temperature at N levels:

$$y_i = u(z_i, \theta) + \xi_1^{(i)}, \quad T_i = T(z_i, \theta) + \xi_2^{(i)},$$

$$E[\xi_j^{(i)}] = 0, \quad E[\xi_j^{(i)}\, \xi_j^{(i_1)}] = \delta_{ii_1}\, \sigma_j^2,$$

$$z_i \in [\hat{z}, h], \quad i, i_1 = \overline{1, N}, j = \overline{1, 2} \tag{3}$$

where E is the expectation; δ is the Kronecker delta; $\hat{z}$ is the lower level of measurements; h is the upper level of measurements be set to determine the unknown vector of parameters $\theta = (u_*, T_*, L, T_0, z_0)^T$.

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The estimation of the parameter vector θ is obtained by the minimum condition for the quadratic functional

$$I(\theta) = \sum_{i=1}^{N} \{\sigma_1^{-2} [u_i - u(\hat{z}) - u(z_i, \theta) + u(\hat{z}, \theta)]^2 + \\ + \sigma_2^{-2} [T_i - T(\hat{z}) - T(z_i, \theta) + T(\hat{z}, \theta)]^2 \}, \quad (4)$$

where the weighting factors σ_1, σ_2 are the rms errors of measuring the wind velocity and temperature, respectively.

To find the minimum of the function (4), let us write Eqs. (1) in the following form:

$$u(z) = \omega_1 f_u \left(\frac{z}{L}\right) + \omega_2, \; T(z) = \omega_3 f_T \left(\frac{z}{L}\right) + \omega_4;$$

$$\omega_1 = \frac{u_*}{\varkappa}, \quad \omega_2 = -\frac{u_*}{\varkappa} f_u \left(\frac{z_0}{L}\right), \quad \omega_3 = T_*,$$

$$\omega_4 = T_0 - T_* f_T \left(\frac{z_0}{L}\right). \quad (5)$$

Using linearity of the functions (5) with respect to ω_i, $i = \overline{1, 4}$ and taking into account the necessary conditions of the minimum for the functional (4) one can easily obtain an explicit representation for the coefficients ω_i for a fixed value of L. Substituting the obtained expressions for $u_*(L)$ and $T_*(L)$ into Eq. (2) we find L from the obtained equation by the dichotomy method. Then, the other parameters are calculated using given value L.

2. PLANNING GRADIENT OBSERVATIONS

Let us consider the problem of optimal location of gradient measurements of wind velocity and temperature, i.e., a plan of experiment corresponding to a certain optimum criterion. We use the criterion of D–optimality as the D–optimum solution minimizes the determinant of the corresponding covariance matrix of the parameter estimations and by virtue of the equivalence theorem[4] is simultaneously G–optimum, i.e., it minimizes the maximum variance of the response function.

Since the dependence of the regression functions (1) on θ is nonlinear, the *a priori* construction of the optimum solution is, generally speaking, impossible. The consequent procedure of analysis and observation planning is the most convenient for the case of simultaneous measurements of several values.[4] Such a formulation of the planning problem is expedient for stationary processes within a sufficiently large time interval.

The construction of a local D–optimum solution is realized by the iteration procedure.

1. Choose an arbitrary initial solution ε_0 satisfying the nondegeneracy condition for the information matrix

$$|M(\varepsilon_0, \theta)| = \left| \sum_{i=1}^{N_0} F(z_i, \theta) \, F^T(z_i, \theta) \right| \neq 0,$$

where

$$F(z_i, \theta) = \| \mathbf{f}_1(z_i, \theta), \mathbf{f}_2(z_i, \theta) \|,$$

$$\mathbf{f}_1^T = \left(\frac{1}{\sigma_1} \frac{\partial u}{\partial u_*}, \frac{1}{\sigma_1} \frac{\partial u}{\partial L}\right), \quad \mathbf{f}_2^T = \left(\frac{1}{\sigma_2} \frac{\partial T}{\partial u_*}, \frac{1}{\sigma_2} \frac{\partial T}{\partial L}\right).$$

2. Calculate the LSM estimation $\hat{\theta}_0$ and the matrix $M(\varepsilon_0, \hat{\theta}_0)$.

3. Find the point z^* corresponding to

$$\max_{z \in [\hat{z}, h]} = \mathrm{Sp}\, d(z, \varepsilon_0, \hat{\theta}_0),$$

where

$$d(z, \varepsilon_0, \theta) = F^T(z, \theta) \, M^{-1}(\varepsilon_0, \theta) \, F(z, \theta).$$

4. Construct the solution

$$\varepsilon = \varepsilon_{N_0+1} = \left(1 - \frac{1}{N_0 + 1}\right) \varepsilon_0 + \frac{1}{N_0 + 1} \varepsilon_1(z^*),$$

and repeat operations 2–3 in correspondence to it.

The cyclic performance of operations 2–4 is continued until the value

$$|M^{-1}(\varepsilon_N, \hat{\theta}_N)| / N$$

is less than a certain given value.

3. NUMERICAL EXPERIMENTS

Let us consider some examples of numerical construction of local D–optimum solutions for a variable upper boundary of the measurement and a given parameter vector θ which determines different conditions of the near–ground layer stratification. The Monin–Obukhov length scale L was chosen to be in the range from –50 to –10 and from 10 to 50 as they characterize two qualitatively different regimes of turbulence. The friction rate u_* was taken to be 0.5 m/s both for the stable and unstable stratification. So we emphasize the important part of temperature in the analysis of turbulence in the near–ground layer. Since the approximation of wind velocity and temperature profiles by formulas (1) and (2) assumes the presence of the initial measurement level $\hat{z}$ (the thickness of the displacement layer), the value $\hat{z}$ was taken to be 1 m for the unstable stratification (what corresponds, for instance, to the grass level of a wheat field[8]), and 0.5 m for the case of a stable stratified near–ground layer.

After the iteration procedure 1–4, we obtained the optimal plans of measurements. They are presented in the Table I. The analysis of the case of unstable stratification ($L > 0$) demonstrates that three-point optimum measurement plans with the heights $\hat{z} = 1$ m, $z_1 = 2$ m, $z_2 = 4$ m are preferable for the parameter estimation with the growth of instability of the turbulent regime. Besides, the weight of the middle point of the plan decreases with the decrease of the parameter L. If the upper boundary of gradient measurements increases to 10 m, the plan is still three-point with the mean level $z_1 \approx 3$ m. At a weak instability, the number of the plan points can be reduced to two depending on the height of the upper boundary of observations. The same is characteristic of weak stability. Thus, under the conditions close to neutral stratification, it is sufficient to estimate the parameters at two levels. For the conditions of strong and moderate stability, the obtained three-point optimum plans are as follows: $\hat{z} = 0.5$ m, $z_1 \approx 1$ m, $z_2 = 2$ m.

TABLE I. Local D-optimum plans of gradient observations of wind velocity and temperature.

The M-O length scale L, m	Dynamic rate u_*, m/s	Measure-ment level $\hat{z}$, m	Plan of points	
			z_1/P_1	z_2/P_2
– 10	0.5	1	1.96/0.5	4/0.5
			2.62/0.45	10/0.55
– 30	0.5	1	2.14/0.29	4/0.71
			2.98/0.42	10/0.58
– 50	0.5	1		4/1
			3.7/0.26	10/0.74
– 50	0.5	0.5		2/1
– 30	0.5	0.5	1.13/0.5	2/0.5
– 10	0.5	0.5	1.07/0.5	2/0.5

The numerical modeling of optimum plans performed shows that the number of levels required for estimation of the parameters of the temperature-inhomogeneous near–ground layer is rather redundant. This seems to be not accidental as the constructed optimum plans of observation level arrangement correspond also to recommendations presented in Refs. 8 and 9.

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