Effective viscosity coefficient of rarefied gas nanosuspensions

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The effective viscosity coefficient of rarefied gas nanosuspensions is calculated by means of Boltzmann kinetic theory. The dependence of this coefficient on the radius of nanoparticles, their concentration, and temperature of a carrier gas is investigated. It is shown that the viscosity of gas nanosuspensions depending on the nanoparticle concentration and size can both increase and decrease compared with the viscosity of the carrier gas. The results obtained are compared with the data for binary mixtures of rarefied gases.

Introduction

The effective viscosity coefficient of rarefied suspensions (liquid + solid disperse particles) was calculated for the first time by Albert Einstein,[1] who took into account the influence of particles on the hydrodynamic velocity field of the carrier liquid and, as a consequence, on the stress tensor and the viscosity coefficient. It was shown that the effective viscosity coefficient of the suspended matter

$$\eta = \eta_0(1 + \frac{5}{2}\varphi) \qquad (1)$$

is always larger than the viscosity coefficient of the carrier liquid η_0 (φ is the volume concentration of the particulate matter). Later on in many papers the attempts were made to extend Einstein theory to the case of rather dense suspensions (see, for example, Refs. 2, 3, and references therein), and the equations similar to Eq. (1) were derived accurate to the terms on the order of φ^2.

In practice, the carrier medium often is a rarefied gas. Rarefied gases here are the gases, whose molecules interact with each other only through binary collisions. Thus, a gas (in particular, air) is rarefied under normal conditions. What is the effective viscosity of a rarefied gas suspension and how does it depend on the concentration of the dispersed matter? Both these questions are unanswered yet. Even more interesting is the situation with rarefied gas nanosuspensions (gas + nanoparticles), in which the disperse component is formed by nanoparticles. The rapid development of nanotechnologies determines the acute urgency of studying the effective viscosity of gas nanosuspensions.

The aim of this work was to calculate the viscosity coefficient of rarefied gas nanosuspensions. The data obtained are compared with the theoretical results calculated for various molecular mixtures.

Formulation of the problem and basic relationships

The problem formulated is very complicated because the nanoparticles have the size from one to tens of nanometers, which is on the order of or smaller than the hydrodynamic physically infinitely small scale for the carrier medium under normal conditions. Thus, even if the carrier gas can be described by fluid dynamics, the latter is inapplicable to description of the streamline of a nanoparticle, and the medium, on the nanoparticle scale, is not continuous. Therefore, kinetic theory should be used to describe the processes of nanoparticle transport in gases. As shown in Refs. 4 to 6 the dynamics of rarefied gas nanosuspensions is described by a system of Boltzmann kinetic equations. However, even in this case, the interaction of a nanoparticle with a gas molecule is collective, because it is necessary to take into account its simultaneous interactions with all atoms (molecules). This can be achieved by use of a specially constructed molecule–particle interaction potential[7,8]:

$$\Phi(r) = \Phi_9(r) - \Phi_3(r), \qquad (2)$$

$$\Phi_i(r) = C_i\Big\{\Big[(r-R)^{-i} - (r+R)^{-i}\Big] - $$
$$ - a_i\Big[(r-R)^{-(i-1)} - (r+R)^{-(i-1)}\Big]\Big\},$$

where

$$C_9 = 4\pi\varepsilon_{ij}\sigma_{ij}^{12}/(45V), \;\; C_3 = 2\pi\varepsilon_{ij}\sigma_{ij}^{6}/(3V);$$

$$a_9 = 9/(8r), \;\; a_3 = 3/(2r);$$

V is the effective volume per one molecule of a particle; ε_{ij}, σ_{ij} are the parameters of the Lennard–Jones 6–12 potential describing the interaction of gas molecules with every molecule (atom) of a nanoparticle.

This potential was used to study the diffusion of nanoparticles in rarefied gases within the framework of the Boltzmann kinetic theory.[8–10] In particular, it was shown that the properties of nanoparticle transport strongly differ from those of ordinary Brownian particles, and their diffusion cannot be described by Einstein theory and the Cunningham–Millikan–Davis experimental correlation based on it. At the same time, kinetic theory with the potential from Refs. 8 to 10 well agrees with the experimental data both the

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existing ones and those obtained especially.[9,10] No reliable interaction potential (in particular, some interaction models) between particles of the dispersion has been proposed so far. Therefore, in this paper the particle–particle interaction is modeled by the potential of solid spheres.

Thus, let us consider a rarefied gas nanosuspension as a binary mixture of particles of two sorts: carrier gas molecules and nanoparticles. As was already mentioned, the dynamics of such a binary disperse system is described by the ordinary system of Boltzmann equations. As a result, it can be shown that in the first approximation of the Sonin's polynomial expansion method that the viscosity coefficient of the considered rarefied gas nanosuspension is described by the equation[11]:

$$\eta = (1+Z)/(X+Y), \tag{3}$$

where

$$X = \frac{x_1^2}{\eta_1} + 2\frac{x_1 x_2}{\eta_{12}} + \frac{x_2^2}{\eta_2},$$

$$Y = 0.6A_{12}^* \left\{\mu\frac{x_1^2}{\eta_1} + 0.5x_1x_2\frac{(1+\mu)^2}{\mu}\frac{\eta_{12}}{\eta_1\eta_2} + \frac{1}{\mu}\frac{x_2^2}{\eta_2}\right\};$$

$$Z = 0.6A_{12}^* \times$$

$$\times\left\{\mu x_1^2 + 2x_1x_2\left[\frac{(1+\mu)^2}{4\mu}\left(\frac{\eta_{12}}{\eta_1}+\frac{\eta_{12}}{\eta_2}\right)-1\right]+\frac{1}{\mu}x_2^2\right\};$$

$$\eta_i = \frac{5}{16}\frac{(\pi\, m_i\, kT)^{1/2}}{\pi\,\sigma_{ii}^2\Omega_i^{(2,2)*}},\quad \eta_{12} = \frac{5}{16}\frac{(\pi\, m_{12}\, kT)^{1/2}}{\pi\,\sigma_{12}^2\,\Omega_{12}^{(2,2)*}};$$

$$\Omega_{12}^{(l,r)*}(T_{12}^*, \sigma_{12}^*) = \frac{2}{(r+1)!}\times$$

$$\times\int_0^\infty Q_{12}^{(l)*}(G^2T_{12}^*, \sigma_{12}^*)G^{2r+3}\exp(-G^2)\mathrm{d}G;$$

$$Q_{12}^{(l)*}(g_{12}^{*2}, \sigma_{12}^*) = 2\left(1-\frac{1+(-1)^l}{2(l+1)}\right)^{(-1)}\times$$

$$\times\int_0^\infty b^*\mathrm{d}b^*\left(1-\cos^l\chi(b^*,\, g_{12}^{*2},\, \sigma_{12}^*)\right);$$

$$\chi(b^{*2},\, g_{12}^{*2},\, \sigma_{12}^*) = \pi - 2b^*\times$$

$$\times\int_0^{y_m}\frac{\mathrm{d}y}{\sqrt{1-\left(\Phi^*(y,\sigma_{12}^*)+y^2b^{*2}g_{12}^{*2}\right)/g_{12}^{*2}}}.$$

Here $y_m(b^{*2}, g^{*2})$ is the minimum positive value of y corresponding to vanishing of the integrand; $y \equiv R/r$ is the reduced reciprocal distance between the centers of the carrier gas molecule and the nanoparticle normalized to the nanoparticle radius; $\Phi^* \equiv \Phi/\varepsilon_l$ is the reduced potential energy normalized to the value proportional to the depth of the potential well; x_1 and x_2 are the mole fractions of the components 1 and 2; $\mu = m_1/m_2$, m_1, m_2 are the molecular masses of the carrier gas and the nanoparticle, respectively (the subscript 2 corresponds to nanoparticles, unless otherwise specified); $m_{12} = m_1m_2/(m_1+m_2)$ is the reduced mass; $A_{12}^* = \Omega_{12}^{(2,2)*}/\Omega_{12}^{(1,1)*}$, $\Omega_{12}^{(l,r)*}$ are the reduced Ω-integrals; $\sigma_{22} = 2R$, R is the radius of nanoparticle; $T_1^* \equiv kT/\varepsilon_{11}$ is the reduced carrier gas temperature equal to the ratio of the gas temperature T to the depth of the potential well of the Lennard–Jones 6–12 potential, $T_{12}^* \equiv kT/\varepsilon_l$ is the reduced temperature of nanoparticle–gas molecule interaction normalized to the energy parameter $\varepsilon_l = 2\sqrt{10}\pi\varepsilon_{12}\sigma_{12}^3/(9V)$ proportional to the depth of the potential well of the potential (2); $\sigma_{12}^* \equiv \sigma_{12}/R$ is the spatial parameter proportional to the ratio of the size of the repulsion zone to the nanoparticle radius; $b^* \equiv b/R$ is the reduced impact parameter for the particle coming from the infinity; $g_{12}^{*2} \equiv m_{12}g_{12}^2/(2\varepsilon_l)$ is the reduced kinetic energy of the relative motion; $G_{ij}^2 \equiv g_{ij}^{*2}/T^* = m_{ij}g_{ij}^2/(2kT)$ is the kinetic energy of the relative motion normalized to the carrier gas temperature.

Boltzmann kinetic theory is applicable to description of gas nanosuspensions only in the case, when the volume concentrations of particles are low enough or, more precisely, when the corresponding Van der Waals parameter is small: $n_2R^3 \ll 1$ (only in this case the approximation of binary collisions of nanoparticles with the carrier gas molecules is valid). Therefore, it is useful to study the behavior of the viscosity coefficient (3) at small values of the mole fraction of the disperse phase: $x_2 \ll 1$. In the first approximation with respect to x_2, the viscosity coefficient (3) takes the form

$$\eta = \eta_1\left\{1+\frac{x_2}{(1+0.6A_{12}^*\mu)}\times\right.$$

$$\left.\times\left[\frac{1.2\sqrt{2}\alpha\Omega_{11}^{(2,2)*}}{s\,\Omega_{12}^{(1,1)*}} + 2 - 1.2A_{12}^* - 2\frac{\sqrt{2}\,\Omega_{12}^{(2,2)*}}{4s^2\,\Omega_{11}^{(2,2)*}}\right]\right\}, \tag{4}$$

where

$$s = \frac{\sigma_{11}}{\sigma_{22}} = \frac{\sigma_{11}}{2R} \ll 1;\quad \alpha = \frac{s^3}{\mu} = \frac{m_2\sigma_{11}^3}{m_1\sigma_{22}^3} = \frac{\pi}{6}\frac{\rho_2\sigma_{11}^3}{m_1} \tag{5}$$

(ρ_2 is the density of the particulate matter). In that case the parameter α has the physical meaning of the ratio of the particulate matter density to the density of the carrier gas.

The parameter x_2 can be related to the volume concentration of nanoparticles $\varphi = x_2v_pp/(kT)$, where v_p is the nanoparticle volume; p is the pressure of the gas nanosuspension. Then Eq. (4) takes the form similar to the Einstein equation (1). However, there is an important difference between them: the function (4) significantly depends on the mass, μ, and effective diameter, s, ratios between the molecules and nanoparticles, temperature, and parameters of the potential (2). Generally speaking, the term in the square brackets in Eq. (4) can alternate the sign at certain values of the above parameters. This means that when small volume concentrations of solid particles are added to the pure gas, the effective viscosity of the medium can both increase and decrease.

In particular, the derivative of the effective viscosity of the gas nanosuspension (4) with respect to the volume concentration of disperse particles is equal to

$$\frac{d\eta}{d\varphi} = \frac{6kTs}{\pi(1+0.6A_{12}^{*}\mu)p\sigma_{11}^{3}} \times$$
$$\times\left[1.2\sqrt{2}\alpha s\frac{\Omega_{11}^{(2,2)*}}{\Omega_{12}^{(1,1)*}} + 2s^2 - 1.2A_{12}^{*}s^2 - \frac{\sqrt{2}\,\Omega_{12}^{(2,2)*}}{4\Omega_{11}^{(2,2)*}}\right]. \quad (6)$$

The values of the Ω-integrals in this equation depend on the temperature and the parameters of the interaction potential for large particles on the order of unity increase with the decrease of the nanoparticle radius. Thus, for example, the function $\Omega_{12}^{(2,2)*}$ varies from unity to about 4.5 for different particles. As the radius of the disperse particles increases and, correspondingly, the parameter s decreases, the absolute values of all the terms in the square brackets decrease. Consequently, the addition to the effective coefficient of the gas nanosuspension caused by the presence of nanoparticles decreases as well. If the parameter s is small, then the density ratio α (5) of the particulate matter and the carrier gas molecules becomes significant. Thus, the decrease of the effective viscosity coefficient as compared with the coefficient η_1 should be expected at the strong difference between the sizes of the carrier gas molecules and the nanoparticles, when the last term in Eq. (6) turns out to be predominant. If the size of nanoparticles is small enough, then the decrease of the effective viscosity coefficient should be expected in the gas nanosuspensions with the relatively small parameter α. Certainly, here we should take into account the values of the Ω-integrals, which can change by 2 to 3 times.

Finally, note that for the binary mixture of gases with the strongly different molecular masses, when usually $\sigma_{11} \sim \sigma_{22}$ and $\mu \ll 1$, from Eq. (3) we obtain the following equation:

$$\eta = \eta_1\left[1 + 1.2\sqrt{2}\,x_2\frac{s^2\,\Omega_{11}^{(2,2)*}}{\mu\,\Omega_{12}^{(1,1)*}} + O(x_2)\right]. \quad (7)$$

The second term in the square brackets is on the order of $\mu^{-1}x_2$, while the third one, $O(x_2)$, is on the order of x_2, and at $\mu \ll 1$, $x_2 \ll 1$ the third term $O(x_2)$ is much smaller than the second one in the square brackets. From Eq. (7) it follows that small addition of a much heavier gas to the light one always leads to the increase in the effective viscosity of the mixture as compared to the viscosity of the light component alone, which agrees with Eq. (1).

Calculation of viscosity coefficients of rarefied gas nanosuspensions

The transport coefficients obtained using kinetic theory are governed by the values of the Ω-integrals. The integrals $\Omega_{12}^{(1,1)*}(C_3, C_9, R, T)$ have been studied in analyzing diffusion of nanoparticles in a rarefied gas.[8] The calculations performed for a wide range of binary gas nanosuspensions have shown that all the Ω-integrals for potential (2) (unlike the corresponding parameters for the rarefied gas) weakly depend on the use of various combination relations and widely different data on the parameters of intermolecular potentials. At the same time, their values are rather sensitive to variations of the nanoparticle radii, and for almost all pairs of molecules studied they are close to unity (different but no more than 5%) starting from the particle radii greater than 10 nm. On the other hand, for particles with the radii smaller than 10 nm, the Ω-integrals change monotonically in the range of 1.5–3.5, and the greater is the constant C_3, the larger is this change [see Eq. (2)].

The behavior of the function $\Omega_{12}^{(2,2)*}(C_3, C_9, R, T)$ that enters the Eqs. (3)–(6) for the viscosity coefficient was studied by calculating these Ω-integrals for rarefied gas nanosuspensions. The calculations have been carried out for particles of Zn, Cd, and Cu_2O in Ne, Zn, Cu_2O in Ar, and U in H_2. The particle radii varied from 0.5 to 100 nm. The parameters of the interaction potential (2) of these gas suspensions are presented in Table 1; the constants C_3 and C_9 were determined using the data from different sources summarized in Table 2. In all the cases, the results obtained coincide qualitatively with those described above for the integrals $\Omega_{12}^{(1,1)*}$.

Table 2. Parameters of nanoparticle–molecule interaction potential

Matter	C_3, K·Å^3	C_3, K·Å^9	σ_{ij}, Å	ε_{ij}/k, K
Cd–Ne	$7.760 \cdot 10^3$	$3.901 \cdot 10^5$	2.688	212.1
Zn–Ne	$8.664 \cdot 10^3$	$3.775 \cdot 10^5$	2.624	192.7
Cu_2O–Ne	$2.667 \cdot 10^4$	$5.473 \cdot 10^6$	3.398	322.3
Zn–Ar	$2.835 \cdot 10^4$	$2.501 \cdot 10^6$	2.952	311.5
U–H_2	$2.957 \cdot 10^4$	$3.852 \cdot 10^6$	3.150	300.1
Cu_2O–Ar	$9.041 \cdot 10^4$	$3.375 \cdot 10^7$	3.754	600.6

Table 2. Parameters of the Lennard–Jones 6–12 potential

Matter	ε_{ij}/k, K	σ_{ij}, Å
U	2703.6	3.343 (Ref. 12)
Zn	1040	2.460 (Ref. 12)
Cd	1260	2.580 (Ref. 12)
Cu_2O	2909.1	4.124 (Ref. 9)
Ne	35.7	2.789 (Ref. 11)
Ar	124.0	3.418 (Ref. 11)
Xe	229.0	4.055 (Ref. 11)
UF_6	236.8	5.967 (Ref. 13)
H_2	33.3	2.968 (Ref. 11)

As has already been mentioned above when nanoparticles are added to a gas at small volume concentrations the effective viscosity of the medium can both increase and decrease depending on the properties of the particulate matter, as well as on the nanoparticle radius and on the kind of the carrier gas. As an example, Fig. 1 depicts the derivative of the effective viscosity (6) with respect to the volume concentration of disperse particles normalized to the pure gas viscosity $\eta' = \eta^{-1}(d\eta/d\varphi)$ as a function of the nanoparticle radius for the U–H_2 gas nanosuspension (uranium particles of the radius R in hydrogen). The derivative η' actually alternates the sign nearby $R = 36$ nm. Thus, this gas nanosuspension with particles of a large radius will have the effective viscosity lower than the viscosity of the carrier gas. On the other hand, the viscosity of the gas nanosuspension U–H_2 with small particles turns out higher than the viscosity of the carrier gas.

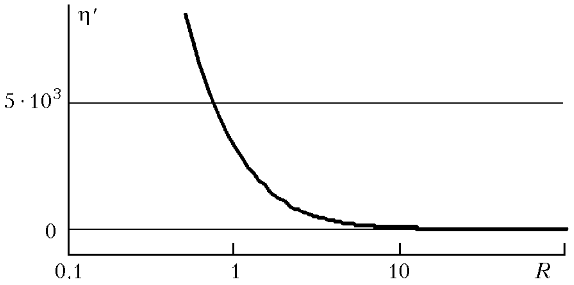

Fig. 1. Derivative η' of the viscosity coefficient of the gas suspension U–H_2 with respect to the volume concentration of particles as a function of the particle radius. T = 300 K.

The decrease of the effective viscosity coefficient relative to the viscosity coefficient of the pure gas can be expected, as was mentioned above, at the relatively small values of the parameter α (5). The values of this parameter for some gas suspensions studied are presented in Table 3. The gas suspension U–H_2 has the maximum value of this parameter. For the suspension Zn–Ne (zinc particles in neon), to the contrary, the parameter α (5) is much smaller than for U–H_2. As a result, when the zinc particles of small radius are added to the gas, the effective viscosity of the gas nanosuspension Zn–Ne decreases. This is illustrated by data presented in Fig. 2, which shows the dependence of η' on the radius of zinc particles. With the increase of the radius of the disperse particles, the absolute value of the function η' decreases. Thus, the addition of rather large nanoparticles (R > 30 nm) practically does not change the viscosity coefficient of the gas suspension as compared with the viscosity of the pure neon. Certainly, it should be kept in mind that the linear approximation (6) is applicable only at low concentrations.

Table 3. Values of the density ratio α (5)

Gas suspension	α
Cd–Ne	2.932
Zn–Ne	2.418
Cu_2O–Ne	2.068
Zn–Ar	2.248
U–H_2	77.864
Cu_2O–Ne	1.923

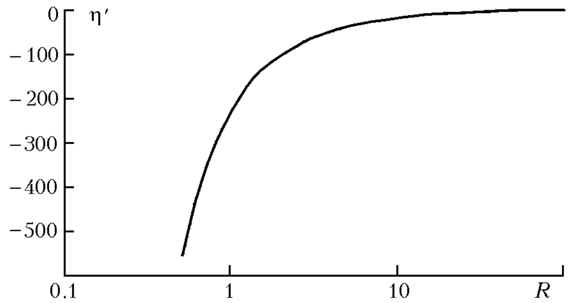

Fig. 2. Derivative η' of the viscosity coefficient of the gas suspension Zn–Ne with respect to the volume concentration of particles as a function of the particle radius. T = 300 K.

It should be emphasized that in all cases the derivative (6) has the highest values for particles of small radii (R < 2 nm), and just in this region it changed most strongly. This means that even low concentrations of ultrafine particles can significantly change the viscosity of the carrier gas. As an example, Fig. 3 shows the dependence of the effective viscosity coefficient (3) of the gas nanosuspension U–H_2 on the volume concentration of particles of the radius R = 0.5 nm at different temperatures. Here curve *1* corresponds to the temperature of 200 K; *2* – 300; *3* – 400; *4* – 500; *5* – 600; *6* – 800; and *7* – 1000 K. The dependence of the effective viscosity coefficient of this gas nanosuspension on the volume concentration of disperse particles has one peak. As the temperature increases, the height of this peak grows and it shifts toward lower concentrations. At all the temperatures, the relatively small addition of uranium particles causes the increase of the viscosity coefficient of the gas nanosuspension as compared to that of the pure gas. At the concentrations of about $2 \cdot 10^{-4}$ and at room temperature, the viscosity of the gas nanosuspension exceeds the viscosity of the carrier gas roughly by 90%. This effect significantly depends on the temperature and at T = 1000 K the ratio η/η_1 is approximately equal to 2.3, the concentrations being the same.

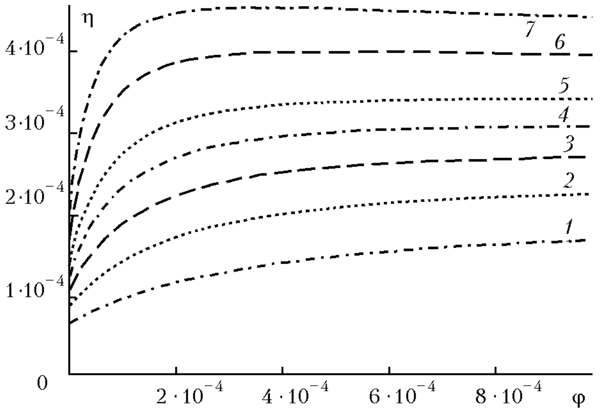

Fig. 3. Effective viscosity coefficient (in poise) of the gas nanosuspension U–H_2 (R = 0.5 nm) as a function of the volume concentration of particles.

As has already been mentioned above, the addition of large uranium particles should result in a decrease of the viscosity coefficient of the gas suspension as compared with the viscosity coefficient of pure hydrogen. This effect is really observed. The corresponding example can be found in Ref. 14 for the same gas suspension. For uranium particles of the radius $R = 70$ nm at $T = 300$ K the effective viscosity coefficient of the gas nanosuspension turns out to be 1% lower than the hydrogen viscosity coefficient at the uranium particle concentration of $2 \cdot 10^{-3}$. This effect increases with the temperature growth.

According to Fig. 2, the viscosity coefficient of the gas suspension Zn–Ne proves to be lower that the viscosity coefficient of pure neon. Actually, as was shown in Ref. 14, the viscosity of the gas nanosuspension at any temperature is lower than that of the carrier gas. At the room temperature and the volume particle concentration of $2 \cdot 10^{-4}$, the effective viscosity of this gas suspension is approximately 15% lower than the viscosity of pure neon, and this effect increases with the temperature growth.

Discussion

The main conclusion that can be drawn from the results obtained is that the viscosity of gas nanosuspensions, unlike the viscosity of coarse suspensions, can be both higher and lower than that of the carrier gas. The increase or decrease of the viscosity of the gas nanosuspension as compared with the viscosity of the pure gas depends mainly on the radius of nanoparticles and the parameter α (5), which is the density ratio of the particulate matter and the carrier gas. With the decrease of the nanoparticle radius, the effect of the change (increase or decrease) of the viscosity coefficient of the gas nanosuspension significantly increases in all cases. The growth of the suspension temperature also significantly intensifies this effect.

It is useful to compare the viscosity properties of gas nanosuspensions with the viscosity of binary mixtures of gases. However, it should be noted that the behavior of the effective viscosity coefficient of the binary mixture of gases in the general case is poorly studied even for the rarefied gases, though in this case kinetic theory allows the calculation to be, in principle, performed with quite a high accuracy. The viscosity of the binary mixture almost never is a linear function of the composition. Two typical situations are possible. In the first case the viscosity of the mixture increases monotonically with the increase of the mole fraction of one of the components, and in the second case it has a peak at some composition, and the amplitude of this peak is higher than the viscosity of any of the components.[13] The physical causes for such a behavior of the mixture viscosity are not fully clear. However, if the molecular masses of the components of the binary mixture differ widely, then the viscosity of the mixture in the linear approximation in terms of the molar concentration of the heavier component increases according to Eq. (7).

To describe the behavior of the viscosity of binary mixtures of rarefied gases with widely different molecular masses at arbitrary concentrations, the viscosities of the mixtures Xe–Ne and UF_6–Ne have been calculated. The intermolecular interaction was described by the Lennard–Jones potential with the constants presented in Table 2. The qualitative behavior of both mixtures is similar. Figure 4 shows the dependence of the viscosity of the mixture UF_6–Ne on the molecular concentration of the heavier component (here curves *1–7* correspond to the same temperatures as in Fig. 3). It is natural that at low concentrations of the heavier gas the viscosity of the mixture behaves in accordance with Eq. (7). However, at high concentrations the dependence of the viscosity on the composition becomes nonlinear and has a peak, whose position significantly depends on the temperature of the mixture. As the temperature increases, the mole fraction of the heavier gas x_2 corresponding to the maximum value of the effective viscosity coefficient of the mixture at this temperature increases monotonically, and the amplitude of the peak increases as well. The viscosity coefficients of pure Xe and UF_6 are lower than that of pure Ne, therefore the effective viscosity coefficient of the mixtures of the rarefied gases studied has single maximum.

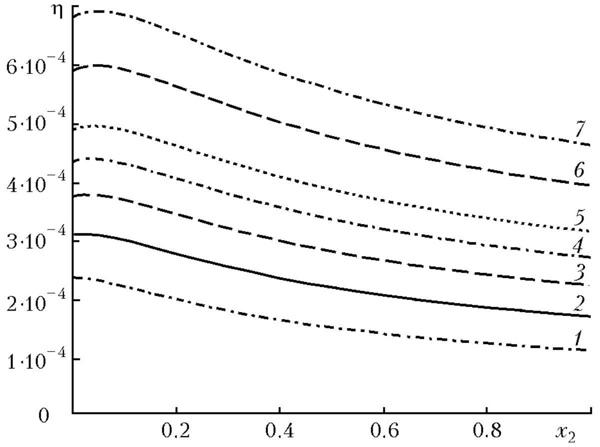

Fig. 4. Viscosity coefficient (in poise) of the mixture of Ne and UF_6 gases as a function of the mole fraction of UF_6.

It should be noted that at high concentrations of the heavier component the viscosity of the mixture becomes lower than that of the light gas, but, unlike the gas suspension, this is observed at high molar concentrations of the heavier component, for example, for the mixture UF_6–Ne at the UF_6 molar concentration of about 7–10%.

It should be emphasized once again that always the viscosity of the mixture of gases, in particular, with the widely different masses first increases upon addition of the heavier component. Thus, the mechanism of momentum transfer operates in gas nanosuspensions, while in gases it can be neglected. To understand the nature of the transfer mechanisms, it is useful to apply kinetic theory of viscosity. It can be shown (see, for example, Ref. 15), that within the framework of elementary kinetic theory the effective viscosity coefficient of the rarefied gas nanosuspension η_e is described by the following equation:

$$\eta_e = \frac{5}{16}\sqrt{\frac{kT}{\pi}}\left[\frac{x_2\sqrt{m_2}}{x_1\sigma_{12}^2 + x_2\sigma_{22}^2}k + \frac{x_1\sqrt{m_1}}{x_1\sigma_{11}^2 + x_2\sigma_{12}^2}\right], \quad (8)$$

where $k = l_{22}/l_{21}$ and l_{22} is the mean free path of the heavy particle (subscript 2) with respect to particles of the kind 2; l_{21} is the mean free path of the heavy particle with respect to molecules (particles with the subscript 1). Simple estimates show that

$$k \sim \frac{n_1\sigma_{11}^2 c_1}{n_2\sigma_{22}^2 c_2} \sim \frac{n_1\sigma_{11}^2}{n_2\sigma_{22}^2}\sqrt{\frac{m_2}{m_1}} \sim \frac{\sigma_{22}}{\sigma_{11}}\sqrt{\frac{m_2}{m_1}},$$

where c_1, c_2 are the thermal velocities of the molecules and particles, respectively.

For all the considered gas nanosuspensions, there is the value of k, at which Eq. (8) yields, in the entire temperature range considered, the dependences of the viscosity coefficients of gas suspensions on the molar and mass concentrations almost coinciding with the exact calculation by Eq. (3). The expansion of Eq. (8) into a series over small x_2 parameter

$$\eta_e = \frac{5}{16\sigma_{11}^2}\sqrt{\frac{m_1 kT}{\pi}}\left[1 + \frac{x_2\sigma_{11}^2}{x_1\sigma_{12}^2}\sqrt{\frac{m_2}{m_1}}k - \frac{x_2\sigma_{12}^2}{x_1\sigma_{11}^2}\right] \quad (9)$$

shows the physical meaning of two main terms of the expansion in the square brackets in Eqs. (4) and (6). The first, positive, term is associated with the increase of the viscosity due to momentum transfer by nanoparticles. The negative term is caused by the decrease in the frequency of molecule–molecule collisions owing to the collisions of molecules with nanoparticles, as well as owing to the decrease of the mole fraction of the gas molecules. For gases with strongly different molecular masses, $\sigma_{11} \sim \sigma_{12}$ and the second term in Eq. (9) can be neglected as compared with the positive term, and thus we obtain the equation of the form (7).

The influence of the parameter α (5) on the viscosity of the gas suspension is physically clear. To provide for the transfer of the same momentum, particles of the higher density should have smaller radii and, consequently, the longer mean free paths. That is why the effective viscosity coefficient of the gas nanosuspension increases with the growth of α.

In conclusion, it should be noted that in this paper the potential of solid spheres was used to calculate the interaction. For particles of the dispersion, this potential is far from realistic in the general case. However, this has almost no effect on the results and conclusions drawn in this paper, because at low volume concentrations of the disperse phase the contribution of the particle–particle interaction to the effective viscosity coefficient is negligibly small.

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