LIDAR RETURN POWER IN THE CASE OF SOUNDING THE ATMOSPHERE ALONG A SLIGHTLY SLANT PATH OVER FOAM COVERED SEA SURFACE

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Received September 7, 1993

We investigate the power recorded with a lidar receiver when sounding the atmosphere along the path over sea surface partially covered with foam exhibiting strong shading. Presented are expressions for mean received power for cases of continuous and pulsed sensing of sea surface. The received power and the shape of return are shown to be strongly dependent on the degree of shadowing, sounding scheme, the atmosphere, and lidar parameters.

Analysis of energy of lidar returns from rough sea surface in various sensing schemes can be found elsewhere in the literature.[1–5] Such schemes are typical for airborne and spaceborne sensing, when mutual shading of surface elements is negligible. The shading is important at slant sensing of sea surface (e. g., from land or a vessel); in this case it can substantially distort the lidar signals.

Below we investigate the power of lidar returns from the partially foamed sea surface in the case of slant sensing.

We assume[4,5] that

– sensing is performed in IR where water absorption is strong, so that the return is mostly formed by the radiation reflected from the surface, and the diffuse radiation coming from under water is negligible;

– the wavelength of radiation is short compared to characteristic size of curvature and heights of water roughness;

– heights and slopes of the sea surface portions are normally distributed according to normal law;

– sensing beam is pencil–like;

– source, receiver, and their optical axes lie in the plane XOZ;

– foam is located at waves' slopes and reflects Lambertianly.

We assume that radiation from clear sea surface and radiation from surface with foam are added incoherently[4,5]

$$P = (1 - S_\mathrm{f})\, P_\mathrm{s} + S_\mathrm{f}\, P_\mathrm{f}, \qquad (1)$$

where S_f is the fraction of surface covered with foam and white–capped; P, P_s, and P_f are the mean powers of return from sea surface with a partial foam coverage, from clear portions, and from areas coated with foam, respectively.

Foamfree portions of the sea surface are modeled by randomly rough, locally mirror surface, while foamy portions are modeled by randomly rough, locally Lambertian surfaces whose slopes have the same distribution as the sea wave slopes.[4–7]

1. LIDAR RETURNS FROM A CONTINUOUSLY IRRADIATED SEA SURFACE

Consider first sensing of a sea surface under a continuous irradiation.

At slant sensing, with a source and a receiver being spaced, the average (over the ensemble of rough portions and the ensemble of atmospheric fluctuations) power P, recorded with the receiver, is[8]

$$P \simeq \frac{a_\mathrm{s}\, a_\mathrm{r}}{\tilde{L}_\mathrm{s}^2\, \tilde{L}_\mathrm{r}^2} (C_\mathrm{s} + C_\mathrm{r})^{-1/2} (C_\mathrm{s} \cos^2 \theta_\mathrm{s} + C_\mathrm{r} \cos^2 \theta_\mathrm{r})^{-1/2} \times$$

$$\times \frac{\exp\left(-\frac{1}{2}\Lambda\right)}{\Lambda} (\mathrm{e}^{b} - \mathrm{e}^{-b})\, M; \qquad (2)$$

$$M = (1 - S_\mathrm{f}) \frac{V^2 q^4}{8\, q_z^4} \frac{1}{(\overline{\gamma_x^2}\ \overline{\gamma_y^2})^{1/2}} \exp\left(-\frac{q_x^2}{2\, q_z^2\, \overline{\gamma_x^2}}\right) + S_\mathrm{f}\, A\, \omega;$$

$$\omega = \frac{1}{4\sqrt{\pi}\, (\overline{\gamma_x^2}\ \overline{\gamma_y^2})^{1/2}} \exp\left(\frac{1}{2\, \overline{\gamma_y^2}}\right) \sum_{k=0}^{\infty} (-1)^k \frac{\delta^k}{k!} \left(\frac{1}{4\, \overline{\gamma_y^2}}\right)^k \times$$

$$\times \left\{ \cos\theta_\mathrm{s} \cos\theta_\mathrm{r}\, \Gamma\left(k + \frac{1}{2}\right) G_{12}^{20}\left(\frac{1}{2\, \overline{\gamma_y^2}} \middle| \begin{matrix} 1/2 \\ -k - 1/2,\ 0 \end{matrix}\right) + \right.$$

$$+ \sin(\theta_\mathrm{s} + \theta_\mathrm{r})\, \Gamma(k + 1)\, G_{23}^{30}\left(\frac{1}{2\, \overline{\gamma_y^2}} \middle| \begin{matrix} 0,\ 1/2 \\ -k - 1,\ 0,\ 0 \end{matrix}\right) +$$

$$\left. + \sin\theta_\mathrm{s} \sin\theta_\mathrm{r}\, \Gamma\left(k + \frac{3}{2}\right) G_{12}^{20}\left(\frac{1}{2\, \overline{\gamma_y^2}} \middle| \begin{matrix} 1/2 \\ -k - 3/2,\ 0 \end{matrix}\right) \right\};$$

0235-6880/95/05 376-04 $02.00

$$\delta = 2\left(\frac{\overline{\gamma_y^2}}{\overline{\gamma_x^2}} - 1\right); \quad b = \frac{1}{2}\Lambda \text{ erf } X; \quad \text{erf } X = \frac{2}{\sqrt{\pi}}\int_0^X e^{-t^2}\, d\, t\,;$$

$$\Lambda = \tan\theta \int_{\text{ctanh}}^{\infty} (\gamma_x - \text{ctan}\theta)\, W(\gamma_x)\, \text{d}\, \gamma_x; \quad \theta = \max(\theta_s, \theta_r);$$

$$X = \left[\frac{C_s\, C_r \sin^2(\theta_s - \theta_r)}{C_s \cos^2\theta_s + C_r \cos^2\theta_r}\right]^{-1/2} \frac{1}{\sqrt{2}\,\sigma}\,;$$

$$\tilde{L}_{s,\, r} = L_{s,\, r} - \mu \sin\theta_{s,r}\,;$$

$$\mu = \zeta_m \frac{C_s \sin\theta_s \cos q_s + C_r \sin\theta_r \cos\theta_r}{C_s \cos^2\theta_s + C_r \cos^2\theta_r}\,; \quad \zeta_m = \frac{\Lambda\, \sigma\, F(\alpha)}{\sqrt{2\,\pi}\,(1 + X^{-2})}\,;$$

$$F(\alpha) \approx \left\{\frac{1}{2\,\alpha}\left[\ln 2\alpha - \ln\ln 2\alpha - \ln\left(1 - \frac{\ln\ln 2\alpha}{\ln 2\alpha}\right)\right]\right\}^{1/2};$$

$$\alpha = \frac{\Lambda^2}{4\,\pi\,(1 + X^{-2})^2}\,; \quad q_x = -(\sin\theta_s + \sin\theta_r);$$

$$q_z = \cos\theta_s + \cos\theta_r; \quad q^2 = q_x^2 + q_z^2;$$

Coefficients $a_{s,r}$, $C_{s,r}$ depend on parameters of a source and a receiver, as well as on the optical conditions in the atmosphere[9,10]:

For a clear atmosphere:

$$a_s = \frac{P_0}{\pi\alpha_s^2} \exp\left(-\int_0^{\tilde{L}_s} \alpha_s(z)\, d\, z\right);$$

$$a_r = \pi\, r_r^2 \exp\left(-\int_0^{\tilde{L}_r} \alpha_s(z)\, d\, z\right);$$

$$C_{s,\, r} = (\alpha_{s,\, r}\, \tilde{L}_{s,\, r})^{-2};$$

for a homogeneous turbulent atmosphere:

$$a_s = \frac{P_0}{\pi\,(\alpha_s^2 + \nu_s)}\,; \quad a_r = \pi\, r_r^2\, (1 + \nu_r / \alpha_r^2)^{-1};$$

$$C_{s,\, r} = (\alpha_{s,\, r}^2 + \nu_{s,\, r})^{-1}\, \tilde{L}_{s,\, r}^{-2}; \quad \nu_{s,\, r} = 0.4\,(k^{1/3}\, C_\varepsilon^2\, \tilde{L}_{s,\, r})^{6/5};$$

and for an inhomogeneous, optically dense atmosphere:

$$a_s = \frac{P_0}{\pi\,(\alpha_s^2 + \mu_s)} \exp\left(-\int_0^{\tilde{L}_s} (1 - \lambda)\, \alpha_t(z)\, d\, z\right);$$

$$a_r = \pi\, r_r^2 \exp\left(-\int_0^{\tilde{L}_s} (1 - \lambda)\, \alpha_t(z)\, d\, z\right);$$

$$\mu_{s,\, r} = \tilde{L}_{s,\, r}^{-2} \int_0^{\tilde{L}_{s,r}} \tilde{\alpha}_s(z)\, \langle s^2(z)\rangle\, (\tilde{L}_{s,\, r} - z)^2\, \text{d}\, z;$$

$$C_{s,\, r} = (\alpha_{s,\, r}^2 + \mu_{s,\, r})^{-1}\, \tilde{L}_{s,\, r}^{-2}; \quad \lambda = \frac{\tilde{\alpha}_s}{\alpha_t}\,.$$

Here $2\alpha_s$ and $2\alpha_r$ are the sounding beam divergence and the receiver field–of–view angles, P_0 is the power emitted by the source, r_r the effective size of the receiving aperture, V^2 is the Fresnel coefficient for the sea surface without foam, A is the albedo of a surface element with foam, $\theta_{s,r}$ are the incidence the angle of sounding radiation and the observation angle (counted from the normal to $z = 0$ plane), $L_{s,r}$ are the distances from the source and the receiver (along their optical axes) to $z = 0$ plane, $W(\gamma)$ is the probability density of slopes distribution, σ^2 and $\overline{\gamma_{x,y}^2}$ are the variances of heights and slopes of the water roughness, α_t and α_s are the medium extinction and scattering coefficients, $\tilde{\alpha}_s$ is the effective scattering coefficient, $\tilde{\alpha}_s = \alpha_s(1 - x_0)$, x_0 is the isotropic portion of the scattering phase function of the atmosphere,[10] $\langle s^2\rangle$ is the variance of scattering angle in the atmosphere, k is the wave number, C_ε is the structure constant of the dielectric constant of medium, $G_{p,\, q}^{m,\, n}\left(z \middle| \begin{matrix} a_1 \ldots a_p \\ b_1 \ldots b_q \end{matrix}\right)$ is the Meyer function, $\Gamma(k)$ is the gamma function.

Formula (2) is valid for a considerable mutual shading of surface elements ($\text{ctan}_{s,r} / (\overline{\gamma_{x,\, y}^2})^{1/2} \ll 1$) when the source and the receiver are on the same side of the normal to $z = 0$ plane. Angular directional patterns of the source and the receiver were assumed Caussian.[9,10]

Even the account for the first term in Eq. (2) already makes a good approximation for a slightly anisotropic roughness.

Figure 1 is an illustration of computating N (the ratio of power recorded by the receiver under a considerable shadowing to the power under the same conditions but without regard for shadowing) plotted versus the speed of nearwater wind V for different sensing angles θ. The computations of N were made for a monostatic sounding according to formula (2) based on results from Ref. 4 and using the following parameter values:

$L = 10$ μm, $C_\varepsilon = 0$, $\alpha_s = \alpha_r = 15'$,

$\theta = 88°$(curve *1*), 89°(curve *2*), and 89.5°(curve *3*).

Here and below the values of γ are calculated by Cox and Munk expressions,[11] while S_f and σ are those from the expressions[2,12]

$$S_f = 0.009\, U^3 - 0.3296\, U^2 + 4.549\, U - 21.33;$$

$$\sigma = 0.016\, U^2,$$

where U is the speed of nearwater wind (m/s).

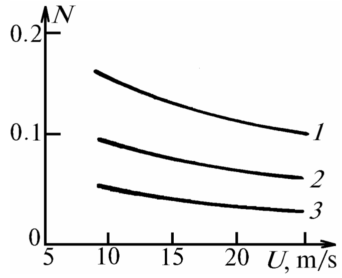

FIG. 1. The dependence of the received power on the nearwater wind velocity under the condition of a considerable shadowing.

From the figure we see that N strongly depends on the nearwater wind and viewing angle; the faster is the nearwater wind and the steeper are the roughness slopes, the more is the shading and the less is N. The same is valid when $\theta \to 90°$ (i. e., more oblique pathes). Note also that from the figure we can see that for the considered range of nearwater wind velocities, the signal recorded by the receiver is mostly due to scattering of a sounding beam by foam structures.

Figure 2 is a plot of N versus θ_s for different atmospheric conditions. Calculations used the following values of parameters:

$\theta_r = 0$, $C_s \gg C_r$, $L_r = 5$ km; $U = 18$ m/s, $\lambda = 1.06$ μm,

$C_\varepsilon = 0$ (curves *1*, *2*) and 10^{-6} m$^{-1/3}$ (curves *3*, *4*),

$2\,\alpha_r = 1'$(curves *1*, *3*) and $2'$ (curves *2*, *4*)

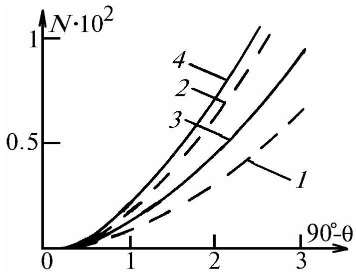

FIG. 2. The dependence of the received power on the sounding angle under the condition of considerable shadowing.

It is interesting from the figure that for narrow directional patterns of a source and a receiver and considerable shading atmospheric turbulence causes an increase in the received power. Physically, this is explained by the fact that in the considerably shading regimes the received power decreases for narrower directional patterns of a source and a receiver. On the other hand, the more turbulent is the atmosphere, the wider are the source's and receiver's patterns, hence the greater becomes the power recorded by the receiver. This effect is only observed when $\theta_s \neq \theta_r$ vanishing at $\theta_s = \theta_r$.

2. LIDAR RETURN FROM SEA SURFACE UNDER A PULSED IRRADIATION

Pulsed scheme of sensing of sea surface will be treated under the same conditions as above. In addition, we assume that the sea surface changes negligibly in the shape during its interaction with a light pulse, and that the pulse is short relative to the period of the return's carrier.

Then, for sensing of a sea surface with a foam using a δ –pulse, the mean power recorded by a receiver has the form[8]:

1. The size of a spot illuminated by a source and that of the receiver field of view are both much larger than the roughness height ($\sigma^{-2} \gg C_s\,(K\cos\theta_s + \sin\theta_s)^2$, $C_r\,(K\cos\theta_r + \sin\theta_r)^2$):

$$P(t) \simeq \frac{\alpha_s\,\alpha_r}{\sqrt{\pi}\,(C_s + C_r)^{1/2}\,\tilde{L}_s^2\,\tilde{L}_r^2}\,\frac{c}{q_x}\,\frac{1 - \exp(-\Lambda)}{\Lambda}\,G(t')\,M, \quad (3)$$

where

$$G(t') = \exp\left\{-C_s\left[\zeta_m\,(K\cos\theta_s + \sin\theta_s) + \frac{c\,t'\,\cos\theta_s}{q_x}\right]^2 - C_r\left[\zeta_m\,(K\cos\theta_r + \sin\theta_r) + \frac{c\,t'\,\cos\theta_r}{q_x}\right]^2\right\}; \quad (4)$$

$K = (\cos\theta_s + \cos\theta_r)\,/\,(\sin\theta_s + \sin\theta_r)$; $\alpha = \Lambda^2\,/\,(4\,\pi)$;

$\zeta_m = \Lambda\,\sigma\,F(\alpha)\,/\,\sqrt{2\,\pi}$; $t' = t - \dfrac{L_s + L_r}{c}$.

Figure 3 shows how the function $G(t')$ varies, being characteristic of the time behavior of received power, with increasing nearwater wind velocity. Calculations were performed for a monostatic sensing scheme ($\theta_s = \theta_r = \theta$; $L_s = L_r = L$) employing formula (4) for the turbulent atmosphere and for the following values of parameters:

$\theta = 89°$, $\lambda = 1.06$ μm, $\alpha_s = 0.1$ mrad, $\alpha_r = 1$ mrad, $L = 3$ km,

$U = 10$ (curves *1*, *2*), 16 (curves *3*, *4*), and 20 m/s (curves *5*, *6*),

$C_\varepsilon = 0$ (curves *2*, *4*, *6*) and 10^{-6} m$^{-1/3}$ (curves *1*, *3*, *5*) .

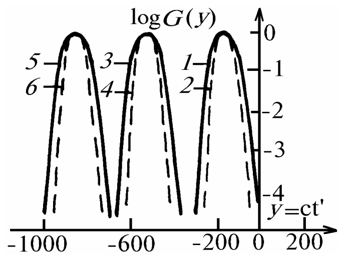

FIG. 3. The shape of a return for a weak distortion caused by a random relief of the sea surface.

As seen from this figure, when the sizes of an illuminated spot and the receiver field of view are both much larger than σ, the presence of shading leads to only a small distortion of the shape of return and to its time delay, the latter increasing with increasing nearwater wind velocity (enhanced shading). Physically, this delay is most likely associated with the effects occurring at the edges of illuminated spot of the surface sounded. Specifically, as nearwater wind increases (σ^2 and $\overline{\gamma_{x,y}^2}$ both decrease), roughness, located near spot edge closest to the source and being not previously illuminated, gets illuminated, whereas roughness near the farthest edge becomes increasingly shaded. Atmospheric turbulence tends to increase the return duration, due to spread of the laser beam and, as a consequence, smearing of the illuminated spot.

2. The size of a spot illuminated by the source is much smaller than that of the receiver field of view and the roughness height ($C_s(K\cos\theta_s + \sin\theta_s)^2 \gg C_r(K\cos\theta_r + \sin\theta_r)^2$, σ^{-2}):

$$P(t) \simeq \frac{a_s\, a_r}{\sqrt{\pi}\,(C_s + C_r)^{-1/2}\,\tilde{L}_s^2\,\tilde{L}_r^2}\;\frac{c}{q_x\sqrt{2}\,\sigma\,(K\cos\theta_s + \sin\theta_s)}\;\frac{C_s^{-1/2}\,F(t')\,M}{}, \quad (5)$$

where

$$F(t') = \exp\left\{-\frac{\zeta_n^2}{2\sigma^2} - \frac{1}{2}\Lambda\left[1 - \Phi\left(\frac{\zeta_n}{\sqrt{2}\,\sigma}\right)\right] - C_r\left[\zeta_n\,(K\cos\theta_r + \sin\theta_r) + \frac{c\,t'\cos\theta_r}{q_x}\right]^2\right\} \quad (6)$$

$\zeta_n = -\,c\,t'\cos\theta_s\,[K\cos\theta_s + \sin\theta_s]^{-1}\,q_x^{-1}$;
and $\Phi(x)$ is the probability integral.

Figure 4 depicts a plot of $F(t')$, showing the time behavior of received power, at various values of nearwater wind velocity. The calculations have been done for a monostatic sensing using formula (6) with the following values of parameters:

$\theta = 89°$, U = 10 (curve *1*), 16 (curve *2*), and 20 m/s (curve *3*).

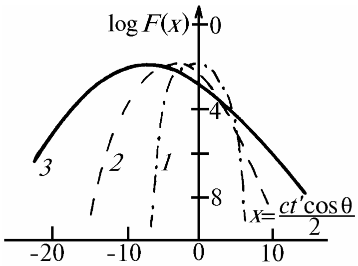

FIG. 4. The shape of return for a strong distortion due to randomly irregular sea surface.

From the figure we see that, as the nearwater wind increases, the returned pulse duration also increases, with the signal energy center arriving at the receiver sooner and sooner; physically, this is due to more rough sea surface (hence, the larger variance of roughness heights and slopes). In addition, because of the larger nearwater wind, the more important is shading, the closer the scattering centers to the lidar will be.

In the case of an illuminated spot size being much smaller than the roughness height, ($C_s\,(K\cos\theta_s + \sin\theta_s)^2 \gg \sigma^{-2}$), the atmosphere has little effect on the shape of returns when sensed monostatically.

When the source and the receiver are on the opposite sides of the normal to $z = 0$ plane, and the shading is important both on the way to the receiver and on the way from the source, formulas (2), (3), and (5) need for a replacement $\Lambda(\theta) \to (\Lambda(\theta = \theta_s) + \Lambda(\theta = \theta_r))$.

Summarizing the study of energy characteristics of return recorded by a receiver in sensing sea surface along slant paths, the following conclusions are in order.

a. The condition of a considerable mutual shading of surface elements is generally satisfied for strong nearwater wind causing foaming and white–capping.

b. In a considerably shading regime, the return recorded with a receiver in the infrared is mostly due to scattering of the laser beam by foam.

c. Power received by a lidar and time structure of the lidar signal significantly depend on the nearwater wind velocity, sensing scheme (monostatic or bistatic), as well as the relationship between the parameters of sounding radiation (sounding angle, the size of illuminated spot on the sea surface) and the statistical characteristics of waves (values of sea surface slopes and heights).

d. The atmosphere affects the return in a complicated way depending on sounding scheme and the parameters of a lidar.

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