NUMERICAL SIMULATION OF COHERENT ATMOSPHERIC OPTICAL SYSTEMS

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Methodical aspects of the computer simulation of optical systems operating through random-inhomogeneous media are examined. The Monte-Carlo method coupled with wave optics theory is used to estimate the image quality obtained in the turbulent atmosphere. The correct ion efficiency of a segmented adaptive mirror used to compensate for the wavefront phase distort ions is calculated.

The image performance of an optical imaging system is determined not only by a thorough system design, precise component workmanship, instrument adjustment, and stability, but also, to a considerable extent, by actual operating conditions. The purpose of the adaptive optical systems currently under design[1] and the segmented mirrors intended for future telescopes[2,3] is to compensate for the effect of the ambient medium (essentially, the atmosphere), thereby improving the optical image performance. For telescopes, for example, the most critical factor appears to be the correction for thermal and atmospheric turbulence distortions[4].

The design and construction of such sophisticated kinds of optical systems as adaptive (flexible and segmented) mirrors and telescopes requires a preliminary study of the system elements as well as an evaluation of their operating efficiency under atmospheric conditions. The most convenient and accurate means of exploring such problems has proved to be computer simulation[5]. The application of this approach to the design of conventional optical devices, viz. objectives, telescopic systems, etc. is well known[6]. Application software packages have been developed for computer calculations of optical components and systems using the geometrical optics approximation (ray tracing technique)[7].

However, the available optics design algorithms and programs . are found to be inapplicable to calculating adaptive and laser systems intended to operate through the atmosphere. There are a few reasons for that. First, the specific character of atmospheric paths due to large wave parameters of the beam dictates the use of waveoptics equations to provide a correct description of diffraction effects in finite beams. Second, since adaptive control optical systems rely on the phase information of the optical wave, the phenomenological representation of wave propagation, for instance, in terms of the radiative transfer equation appears inadequate. Third, an essential contribution to the formation of the image comes from the medium through which the optical wave propagates, which requires that not only the source but also the medium and the radiation detector be included in the simulation scheme (and, consequently, in the design). In the case of adaptive optics mirrors, reflectors and other elements of the system must also be accurately accounted for in the relevant mathematical model.

Thus, computer simulation of atmospheric optical systems calls for the development of new efficient computational procedures for wave optics design. In the earlier paper we proposed original algorithms for solving problems of nonlinear atmospheric optics[8], which we then used to develop mathematical software for optics quality control[9]. The present paper is concerned with the certain methodical aspects of application of the above-mentioned algorithms to numerical simulation of adaptive imaging systems operating through the turbulent atmosphere.

A state-of-the-art analysis of optical systems is based on a description of the image formation process in terms of the theory of linear systems and integral transformations[10]. Optical system components are treated as elements which transform the amplitude phase distribution and spatial frequency spectrum of the incident field. Let us consider mathematical models for some of the components and the most efficient means of their algorithmic implementation.

The pupil function can be regarded as a transparency with an amplitude phase distribution of the form

$$U(x,y)=A(x,y)\times\exp[i\varphi(x,y)],$$

whose effect on the incident field is the operation of complex multiplication. For a discrete computer approximation, the functions U, A and φ are replaced by their grid analogs, while radiative conditions are carried over to the grid boundaries. The latter are satisfied by the following profiles used in our simulation:

a) super-Gaussian shape

$$A(x,y)\sim\exp[-(x^2+y^2)^m],$$

which well approximates the circular pupil function for $m = 8$;

b) hyper-Gaussian shape

$$A(x,y)\sim\exp[-(x^m+y^m)],$$

which approximates the input hole in the form of a square or a rectangle.

The optical system includes a thin positive lens, which is described by a transmission function of the form

$$T(x,y)=\exp[-ik(x^2+y^2)/f], \tag{1}$$

where $k = 2\pi/\lambda$; λ is the wavelength.

The field in the focal plane of the lens agrees within the phase factor with a $2-D$ Fourier transform of the incident wave, whose discrete realization reads

$$\hat{U}(n_1,n_2)=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}U(k_1k_2)$$

$$\times\exp\left\{i\left[\frac{2\pi}{N_1}n_1k_1+\frac{2\pi}{N_2}n_2k_2\right]\right\}. \tag{2}$$

So far the most efficient and compact computational procedure for calculating the discrete Fourier transform (2) has been the Cooley-Tukey FFT algorithm, which, however, imposes limitations on the choice of N_1 and N_2 in the form

$$N_1\times N_2=2^{m_1}\times 2^{m_2}.$$

We have modified the Singleton mixed-radix algorithm for arbitrary values of N_1 and N_2 for use on BESM-6 computers, using a square grid with the maximum size of $N_1 \times N_2 = 100 \times 100$. For other types of computers, this algorithm enables one to select other values of N_1 and N_2 corresponding to the maximum size of the random access memory[8].

An optically homogeneous path travelled by the beam is modeled by means of a spatial Frequency filter. In the language of linear systems theory it is written in the form of a convolution integral:

$$U(x,y)\sim\iint_{-\infty}^{\infty}U(x',y')\times$$

$$\times\exp\left\{i\frac{k}{2z}[(x-x')^2+(y-y')^2]\right\}dx'dy'. \tag{3}$$

In its discrete variant the integral has the form of a double sum and is generally calculated by the moving summation technique[4]. The number of complex multiplications involved in this algorithm is proportional to $\sim N^4$ (N is the number of grid points in the transverse direction). We have developed an efficient algorithm for calculating Eq. (3) by the FT technique with separation of variables. This has resulted in a considerable reduction of the computer memory requirements of the algorithm through the use of a $1-D$ Fresnel function:

$$H(\kappa)=\exp(-i\kappa^2z/2).$$

Figure 1 shows an image correction scheme employed for numerical simulation of a coherent adaptive optical system operating through the atmosphere. The wave from the object, propagating through a turbulent atmospheric (random-inhomogeneous) layer, acquires an amplitude-phase distortion, some of which is compensated by the wavefront corrector.

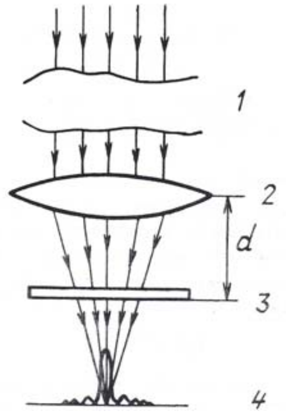

FIG. 1. Schematic of Image Correction: 1) distorting medium; 2) lens; 3) phase corrector; 4) image plane.

In the phase screen approximation the effect of the atmospheric turbulence on the imaging process is accounted for by introducing a random phase modulation of the complex pupil function

$$U(x,y)=\exp\{i[\varphi(x,y)+s(x,y)]\}, \tag{4}$$

where $s(x, y)$ is a random phase function with the power-law spectral density

$$F_s(\kappa_x,\kappa_y)=0,033C_T^2(\kappa_x^2+\kappa_y^2+\kappa_0^2)^{-11/6}. \tag{5}$$

Given an outer turbulence scale $L_0 = 2\pi/æ_0$, the fluctuation strength is characterized by the value of the phase structure function on the diameter 0 of the receiving aperture:

$$D_s(\varnothing)=\langle|s(r)-s(r+\varnothing)|^2\rangle. \tag{6}$$

The standard way to generate random phase function realizations $s(x,y)$ by the moving summation technique[4] for the power-law spectral density of Eq. (5) turns out to be unacceptable because it would entail lengthy computations. Therefore, we have developed a more efficient procedure for the synthesis of two-dimensional random fields, which is a generalization of the one-dimensional spectral algorithm[8].

In the program which numerically simulates the atmospheric turbulence an alternate way of assigning the distortions in the complex amplitude of the pupil is provided by direct numerical solution of the problem of propagation of a wave from the object through a random-inhomogeneous medium by the splitting method.

In the corrector plane, the calculated (by the Fourier method) wave field, distorted by the lens and the region of free space d, is used to calculate the phase function $\varphi(x, y)$ which is then joined together by a spiral-sweep algorithm and approximated by Zernike polynomials:

$$V_n^1(\rho\cos\theta, \rho\sin\theta) = R_n^1(\rho)\exp(il\theta). \quad (7)$$

The phase corrector operation was simulated by introducing an approximate form of $\varphi(x, y)$ with the opposite sign into the wavefront to be corrected.

To compensate for residual phase fluctuations which remain after approximating the wavefront by Zernike polynomials, the model included an adaptive segmented mirror with hexagonally packed elements possessing three degrees of freedom. The phase within a segment was approximated by the least-squares technique. Edge diffraction losses in the segmented mirror were ignored. The model of the corrector mirror included nineteen elements and the phase was approximated in the plane contiguous to the lens ($d = 0$).

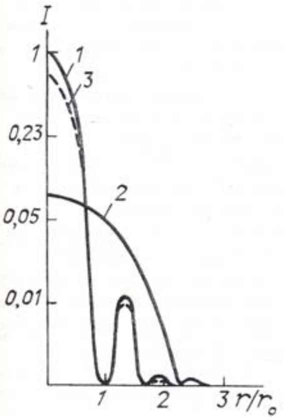

FIG. 2. Point spread function for phase screen model: $D_s(\varnothing) = 10.5$ rad^2; r_0 is the Airy radius; 1) ideal PSF; 2) turbulence-induced PSF; 3) PSF with phase correction.

The pulse and frequency response functions of the optical system were used to estimate its image performance. The pulse response - the point spread function (PSF) – is a characteristic of the image of a point object. It accounts for such effects as the image shift and the phase fluctuations of the incident wave caused by the atmospheric turbulence. Computational algorithms for estimating the PSF and the optical transfer function (OTF) were developed elsewhere[9] for mathematical software for optics quality control. Figure 2 shows the PSF of an adaptive optics correction of image distortions caused by a turbulent medium.

The mean PSF was estimated by multiple solution of the dynamic problem of image correction by means of a segmented mirror followed by ensemble averaging of the instantaneous PSR's using 100 realizations. For a moderate turbulence with the wave phase structure function $D_s(\varnothing) = 10$ rad^2 we achieved a practically complete restoration of the PSF for a diffraction-limited system.

It follows from a plot of the energy distribution over the blur circle (Fig. 3) that application of phase correction should result in a dramatic redistribution of the normalized energy (up to 80%) towards the central maximum of the PSF.

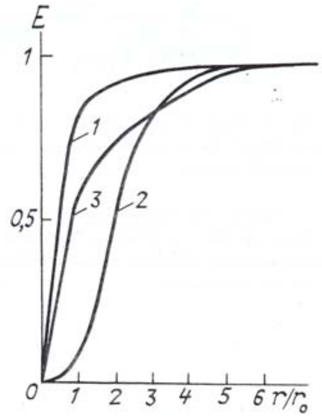

FIG. 3. Energy distribution over the blur circle: 1) no distortions; 2) phase fluctuations with $D_s(\varnothing) = 5.26$ rad^2; 3) phase correction by adaptive mirror; r_0 is the Airy radius.

The image performance of an optical system can also be estimated by means of the frequency response, i.e., the complex OTF, which characterizes extended objects of the half-tone picture type, and is used in vision theory. Shown for comparison in Fig. 4 are the OTF's of an optical system in the turbulent atmosphere with and without phase correction. The frequency axis is normalized to the critical frequency $\theta_0 = \varnothing/\lambda f$. The equivalent spatial frequency, which is the critical frequency for an optical system being capable of reconstructing undistorted images, is found by the Schade criterion

$$\theta_{eqv} = \int_0^{\theta_0} [W(\theta)]^2 \, d\theta, \quad (8)$$

where $W(\theta)$ is the normalized OTF of the system. The effect of the quality of the optics on the imaging fidelity is determined by the Linfoot criterion

$$S = \int_0^{\infty} \{1 - [1 - W(\theta)]^2\} \, d\theta. \quad (9)$$

As seen from Fig. 4, the phase correction by a segmented mirror leads to a 2.5-fold increase of the

Schade parameter while the value of S is increased as much as twofold.

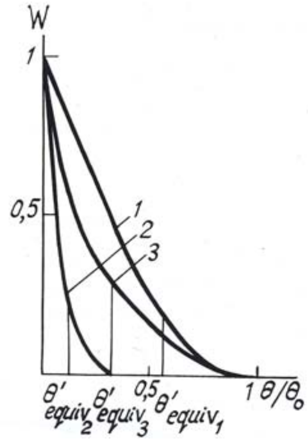

FIG. 4. Optical transfer function: 1) ideal OTF; 2) OTF in a turbulent medium; 3) OTF with correction applied.

The proposed numerical simulation algorithms for wave optics operating through a random-inhomogeneous medium as well as computational procedures for estimating the image performance on the basis of the PSF, the OTF, and the integrated parameters defined by Eqs. (8) and (9) have been embodied in an application program package written in FORTRAN for the BESM-6 and ES-1055 computers. The fairly high efficiency and compactness of the algorithms *has* enabled us to adapt the numerical simulation software for use in personal and minicomputers.

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