Multiple scattering noise in lidar measurements of backscattering matrices of crystalline clouds

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In this paper we discuss a feasibility to correct the distortions introduced by multiple scattering into lidar measurements of light backscattering matrices (LBSMs) of crystalline clouds. The correction algorithm is based on one of symmetry properties of LBSM and peculiarities of polarization structure of light scattered multiply by significantly non-spherical particles. These peculiarities are large degree of depolarization and its weak dependence both on the depth of light penetration into a cloud and on the feild-of-view angle of a lidar receiving antenna.

1. A consideration of higher degrees of multiplicity is a pressing problem when interpreting the results of laser sensing of such media as low-level clouds. However, it is not a challenge in case of crystalline clouds due to their low optical density. Nevertheless, because of large distance between these clouds and the Earth's surface, the cross size of a cloud layer falling within the field of view of a ground-based lidar may be comparable with mean free path of a photon. The multiple scattering in this case cannot be completely neglected, because an interpretation of lidar returns in terms of single-scattering approximation implies a distortion of results of the sensing. The above-said is especially true for sensing of crystalline clouds from space, because in this case the distance between the lidar and a cloud may achieve hundreds kilometers. These circumstances stimulate development of methods, which take into account the multiple scattering in polarization laser sensing of crystalline clouds. As will be seen below, this problem proves to be somewhat simpler than that for droplet clouds because of different mechanisms of formation of radiation polarization structure at scattering by spheres and significantly non-spherical particles.

2. It is convenient to consider the qualitative difference between mechanisms of formation of a depolarized component at scattering by spherical and non-spherical particles within the double-scattering approximation, since this approximation seems to be quite sufficient for ground-based lidar sensing. Analysis of trajectories and scattering volumes producing the double-scattered returns, which come to lidar receiver at the time $t = 2r/c$, where r is the distance between laser radiation wave train and the lidar, can be found in Ref. 1. Further analysis allows us to see that the double-scattered radiation flux is mostly formed due to forward–backward and backward–backward scattering at the moments following immediately after the laser radiation wave train enters the scattering medium. Only as the laser radiation wave train penetrates deeper into the medium, the scattering volumes corresponding to other directions of the scattering become comparable and begin to exceed the volume of frustum of cone

$$V = \frac{1}{3}\pi\theta_{\text{las}}(r^2 - R_0^2)\,(r - R_0), \tag{1}$$

where $2\theta_{\text{las}}$ is the laser radiation divergence angle, and R_0 is the distance between the lidar and the boundary of the scattering medium. Forward–backward and backward–backward types of scattering dominate within this volume, because other processes are hardly probable due to small cross size of the volume. As a consequence, the photons first scattered at the angles significantly differing from 0 or 180° leave the volume (1), so the second scattering event cannot occur in it.

Note here the first difference between the scattering by spherical and non-spherical particles. The scattering matrices $\mathbf{M}(0)$ and $\mathbf{M}(\pi)$ of spherical particles at the scattering angles of 0 and 180° have the form of unit matrices multiplied by scalars. The product of matrices $\mathbf{M}(0)\cdot\mathbf{M}(\pi)$ has the same form. As a result, the radiation double-scattered in the forward–backward and backward–backward directions does not undergo the depolarization. The situation is quite different for the scattering by non-spherical particles, where the depolarization does take place in the processes of this kind.

To illustrate our statement, let us consider the scattering by a pair of axisymmetric particles.

We refer to Fig. 1, which shows two particles on the z-axis, along which the laser radiation propagates. Both particles are axisymmetric with respect to the symmetry axes of an infinite order. The positions of particles' symmetry axes are characterized by the polar γ_1, γ_2 and azimuthal φ_1, φ_2 angles. The angles γ are measured from the z-axis, while the angles φ are measured from the plane XOZ (the plane P_0) to the planes P_1 and P_2, on which the z-axis and the particles' symmetry axes lie.

Elements of the scattering matrices $\mathbf{M}_1(\theta_1, \varphi_1, \gamma_1)$ and $\mathbf{M}_2(\theta_2, \varphi_2, \gamma_2)$ are functions of the scattering angles θ and the angles φ and γ defined above.

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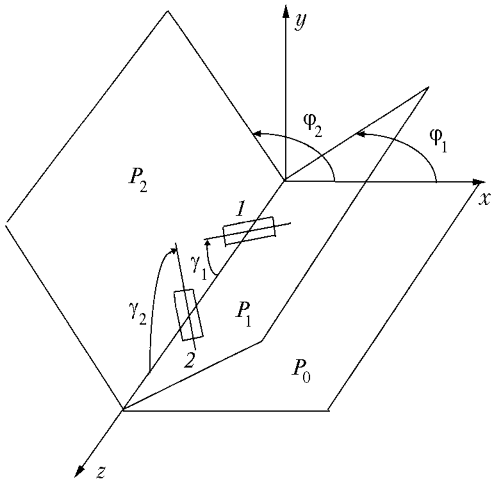

Fig. 1.

Consider the process, when forward scattering by particle *1*, $\theta_1 = 0$, and backward scattering by particle *2*, $\theta_2 = \pi$, occur. Scattering matrices of axisymmetric particles are characterized by their dependence on four parameters. They have a block-diagonal form if particle's symmetry axis lies in the scattering plane.[2] The scattering matrix of the first particle has the following form:

$$\mathbf{M}_1(0,\ 0,\ \gamma_1) = \begin{pmatrix} A & B & 0 & 0 \\ B & A & 0 & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -D & C \end{pmatrix},$$

if the plane P_1, in which the particle symmetry axis lies, is taken as a scattering plane.

The scattering matrix $\mathbf{M}_2(\pi,\ 0,\ \gamma_2)$ has the same form, if the plane P_2 is taken as the scattering plane. Its parameters are denoted respectively as a, b, c, and d. To describe the process of double scattering in the system of coordinates related to the reference plane P_0, one should determine the matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ with respect to this plane through transformations with the operators of the reference plane rotation

$$\Re = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\varphi & \sin 2\varphi & 0 \\ 0 & -\sin 2\varphi & \cos 2\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M}_1(0,\ \varphi_1,\ \gamma_1) = \Re(-\varphi_1)\ \mathbf{M}_1(0,\ 0,\ \gamma_1)\ \Re(\varphi_1)\ ;$$

$$\mathbf{M}_2(\pi,\ \varphi_2,\ \gamma_2) = \Re(\varphi_2)\ \mathbf{M}_2(0,\ 0,\ \gamma_2)\ \Re(\varphi_2)\ .$$

The Stokes vector of double-scattered radiation is proportional to the product of these matrices

$$\mathbf{S}_2 \sim \mathbf{M}_2(\pi,\ \varphi_2,\ \gamma_2)\ \mathbf{M}_1(0,\ \varphi_1,\ \gamma_1)\ \mathbf{S}_0,$$

where $\mathbf{S}_0$ is the Stokes vector of the incident radiation. In this expression, we omit spatial factors insignificant for our consideration. The matrix elements $\mathbf{M}_1(0,\ \varphi_1,\ \gamma_1)$ and $\mathbf{M}_2(0,\ \varphi_2,\ \gamma_2)$ include, as co-factors, the trigonometric functions sin and cos with the arguments $2\varphi_1$ and $4\varphi_1$ and, correspondingly, $2\varphi_2$ and $4\varphi_2$, except for rotation-invariant corner matrix elements. The matrix elements

$$\mathbf{N} = \mathbf{M}_2(0,\ \varphi_2,\ \gamma_2) \cdot \mathbf{M}_1(0,\ \varphi_1,\ \gamma_1)$$

are the polynomials, every term in which involves such products as $\sin n\varphi_1 \times \cos m\varphi_2$, $\sin n\varphi_1 \times \sin m\varphi_2$, etc., with n and m taking the values of 2 or 4. Only diagonal elements of the matrix **N** contain terms free of such factors. Therefore, when summing up the matrices **N** of a set of pairs of scatterers, whose symmetry axes are randomly oriented, the off-diagonal elements of the matrix of an ensemble vanish, and the matrix can be presented as:

$$\overline{\mathbf{N}} = \overline{A}\ \overline{a}\,\mathbf{n}\ ,$$

where elements of the normalized matrix **n** can be presented as:

$$n_{11} = 1,\ n_{22} = \frac{1}{2}\,(1 - \overline{c}\ /\ \overline{a}\,) = -\ n_{33}, \qquad n_{44} = \overline{c}\ /\ \overline{a}\ ,$$

$$n_{ij} = 0,\ \ \text{if } i \neq j.$$

The parameters $\overline{A}$ and $\overline{a}$ have the meaning of coefficients of directed light scattering in the forward and backward directions, respectively; n_{44} is the normalized element of the backscattering matrix. Here we have come to almost obvious result: at forward–backward and backward–backward scattering, the polarization changes only in those scattering events, in which the wave vector of the scattered radiation is opposite to the wave vector of the incident one. At strictly forward scattering by axisymmetric particles without birefringence, the polarization does not change. As applied to real crystalline clouds, our experiments on measurement of backscattering matrices[3] indicate that in the processes of forward–backward and backward–backward scattering the depolarization must be 0.3–0.4 for linearly polarized radiation and 0.6–0.7 for circularly polarized one.

As to our paper, it is important to emphasize that the depolarization of multiple-scattered radiation, when scattering by non-spherical particles, is non-zero near the medium boundary. Just this is the first difference from the scattering by spherical particles.

The second difference, which likely explains high depolarization noticed in Refs. 4 and 5, is different behavior of scattering phase functions at the angles close to 90°. The point is that the values of normalized scattering phase functions for crystal particles at such scattering angles are about an order of magnitude larger than those for spherical particles of similar size. This is indicated by both calculated (see, for example, Ref. 6) and experimental (Ref. 7) data. In the vicinity of the medium boundary, as the laser train comes deep into the depth, the scattering volume for scattering trajectories of the type $\theta_1 = 90°$ and $\theta_2 = 90°$ (towards the lidar receiver) becomes comparable or larger than the volume of frustum of cone (1). As a consequence, these trajectories begin to play a significant part. This does not take place at scattering by spheres, since the probability of the first scattering event at the angle $\theta_1 = 90°$ is far less than at scattering by crystal particles. Further analysis could show that depolarization of double-scattered radiation in the processes, when both scattering events occur at the angles of 90°, exceeds that for scattering at 0 and 180°. We also could explain the slight minimum of depolarization at some distance from the medium boundary at large (15 mrad) field-of-view angle of the receiving antenna (Ref. 5). However, the limited volume of this paper induces us to consider few results calculated by the Monte Carlo method.[4,5]

These results are significant for the algorithm taking into account multiple scattering in measurements of backscattering matrices. (This algorithm is considered below.) In Ref. 4, the calculations made for the crystalline cloud model have shown a fast growth of depolarization from ≈ 0.5 to 0.7–0.9 depending on the field-of-view angle and its weak dependence on the scattering coefficient and cloud structure.

In Ref. 5, the calculations were made for ensembles of Chebyshev particles with different values of the deformation parameter ε. At $\varepsilon < 0.1$ a particle looks like an elongated ellipsoid; at $\varepsilon \approx 0.2$ it becomes a cylindric column with rounded ends and the diameter-to-length ratio 1:1.5. Further increase of ε results in dumb-bell shape of the particle.

The calculated results are descriptive of transformation of the depolarization profile. As the deformation parameter increases, the depolarization profile characteristic of an ensemble of spherical particles gradually transforms to that typical for scattering by non-spherical particles.

For ensembles of particles with the deformation parameter $\varepsilon < 0.05$, depolarization behaves much as in the case of scattering by spheres. A gradual increase of depolarization is observed, as laser radiation penetrates deep into the medium. However, in contrast to spheres, its initial level in this case is not zero. It depends on the deformation parameter. The rate of increase depends on the field-of-view angle. The larger the angle, the faster the rate. For particles with a radius of equivalent media ρ_e such that their diffraction parameter $2\pi\rho_e/\lambda \le 6$, this tendency keeps up to $\varepsilon = 0.15$. However, the depolarization increases very fast and achieves its almost stationary value of 0.5 to 0.6 already at the depth of laser wave train penetration into the medium of 25 m ($\tau = 0.5$). Then it increases very slowly. For larger particles with $2\pi\rho_e/\lambda \ge 12$, the depolarization takes the value ~ 0.8 already at $\varepsilon = 0.08$ immediately near the medium boundary, and it is almost independent on both the depth of laser pulse penetration into the medium and the field-of-view angle.

3. The proposed algorithm for taking into account the noise due to multiple scattering in polarization lidar measurements of LBSM is based on some idealization of the above-listed peculiarities of the polarization structure of lidar response to multiple scattering in ensembles of non-spherical particles.

Let Stokes vector of the radiation coming to the receiver at the instant of time $t = 2r/c$ be presented in the following form:

$$\mathbf{S}(r) = \mathbf{S}_1(r) + \sum_{i=2}^{n} \mathbf{S}_i(r),$$

where $\mathbf{S}_1$ and $\mathbf{S}_i$ are Stokes vectors for the corresponding values of the scattering multiplicity.

The contribution of multiple scattering is taken into account through the lidar equation[8]:

$$P(r)\ \mathbf{S}(r) = \frac{1}{2}\, c\ W_0\, A r^{-2} [\mathbf{M}_\pi(r) + \\ + \mathbf{D}(r)]\ \mathbf{S}_0 \exp\{-2\int_0^r \alpha(z)\ \mathrm{d}z\}\ , \tag{2}$$

where $P(r)$ is the power of the scattered radiation, incident onto the receiving antenna at the time $t = 2r/c$; c is the speed of light; $\mathbf{S}(r)$ and $\mathbf{S}_0$ are the Stokes vectors of scattered and incident radiation, respectively, normalized to the intensity; W_0 is the energy of laser pulse; A is the area of the receiving antenna; $\alpha(z)$ is the extinction coefficient; $\mathbf{M}_\pi$ is the backscattering matrix. The matrix $\mathbf{D}$ is defined as follows:

$$\mathbf{D}(r) = D(r) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \delta & 0 & 0 \\ 0 & 0 & \delta & 0 \\ 0 & 0 & 0 & \delta \end{pmatrix} . \tag{3}$$

Matrix (3) is the product of the scalar $D(r)$, proportional to the intensity of the multiple-scattered radiation, and the matrix $\boldsymbol{\delta}$ of the depolarizer, which partially depolarizes the radiation. Definition (3) is a mathematical expression of the postulated idealization, according to which the state of polarization of the radiation multiple-scattered by non-spherical particles does not depend on the receiver's field-of-view angle and the depth of laser radiation penetration into the medium. This is the first ground for the proposed algorithm. The second ground is the known symmetry property of backscattering matrices

$$M_{11} - M_{22} - M_{44} + M_{33} = 0\ , \tag{4}$$

which follows from quite general property of the amplitude backscattering matrices $A_{12} + A_{21} = 0$ keeping true for any LBSM.[9]

The procedure of LBSM measurements[3] is constructed so that the measured parameters are the Stokes parameters normalized to the intensity and, correspondingly, elements of the backscattering matrix $\mathbf{M}_\pi$ normalized to the element M_{11}. If the scattered radiation includes some portion of the multiple scattering, then parameters are normalized to the total intensity. That is, in spite of the matrix

$$\mathbf{M}_\pi(r) = M_{11}(r)\ \mathbf{m}(r)$$

(where $m_{ij} = M_{ij}/M_{11}$), which would take place with no multiple scattering according to Eq. (2), the matrix

$$\mathbf{M}'(r) = \mathbf{M}_\pi(r) + \mathbf{D}(r) = [M_{11}(r) + D(r)]\ \mathbf{m}'(r)\ ,$$

$$m'_{11} = 1,\ m'_{ii} = (M_{ii} + \delta D)/(M_{11} + D)\ ,\ i = 2, 3, 4\ , \quad (5)$$

$$m'_{ij} = M_{ij}/(M_{11} + D)\ ,\ i, j = 2, 3, 4\ ,\ i \neq j\ ,$$

is determined. Hereinafter we omit the dependence of **M** and **D** on r, because we always imply a pair of matrices corresponding the same distance r, and according to definition (3) δ does not depend on r. According to Eq. (4), for non-distorted backscattering matrix (4) the condition

$$1 - m_{22} - m_{44} + m_{33} = 0 \qquad (6)$$

must be true. For the matrix $\mathbf{m}'$ this condition breaks down:

$$1 - m'_{22} - m'_{44} - m'_{33} = \Delta\ , \qquad (7)$$

where

$$\Delta = D(1 - \delta)/(M_{11} + D)\ .$$

Since m'_{ii} are the elements of the measured matrix, Δ is the experimentally found parameter. With known Δ, from Eq. (7) we can find the ratio of the intensity of multiple-scattered radiation to that of single-scattered radiation

$$I_\mathrm{m}/I_1 = D/M_{11} = \Delta/(1 - \delta - \Delta). \qquad (8)$$

Then we can re-normalize the matrix $\mathbf{m}'$ in order to determine the backscattering matrix $\mathbf{m}$, which is not distorted by multiple scattering.

Having determined from the above equation

$$D = M_{11}\ \Delta/(1 - \delta - \Delta)$$

and substituting it into Eqs. (5), we derive

$$m_{11} = 1,$$

$$m_{ij} = m'_{ij}\ (1 - \delta)/(1 - \delta - \Delta)\ ,\ \ i \neq j\ , \qquad (9)$$

$$m_{ii} = m'_{ii}\ (1 - \delta) - \delta\ \Delta/(1 - \delta - \Delta)\ ,\ i = 2, 3, 4\ .$$

The parameters m_{ij} are elements of the normalized LBSM corrected for distortions introduced by multiple scattering. We illustrate the above-said by an example. One measured LBSM had the following form:

$$\mathbf{m}' = \begin{pmatrix} 1 & -0.12 & -0.01 & 0.01 \\ -0.12 & 0.40 & -0.02 & 0.10 \\ 0.01 & 0.02 & -0.39 & -0.20 \\ 0.01 & 0.10 & 0.20 & -0.11 \end{pmatrix}. \qquad (10)$$

The absolute measurement error is estimated as $\sigma = \pm\ 0.04$.

As seen, $\Delta = 0.32$ according to Eq. (7). This value is large, because, according to Eq. (8) and assuming $\delta = 0$, the portion contributed by multiple scattering is 47% of the single-scattering one.

Below we present the result of correction of matrix (10) by Eqs. (9). It is assumed, that $\delta = 0$, what means the complete depolarization of multiple-scattered radiation:

$$\mathbf{m} = \begin{pmatrix} 1 & -0.176 & -0.015 & 0.015 \\ -0.176 & 0.588 & -0.029 & 0.147 \\ 0.015 & 0.029 & -0.573 & -0.294 \\ 0.015 & 0.147 & 0.294 & -0.162 \end{pmatrix}. \qquad (11)$$

It is easy to verify that the symmetry condition (6) holds true for this matrix.

4. The proposed method of correction of experimental LBSMs is based on the fundamental symmetry property of these matrices. Symmetry (6) can be violated only by multiple scattering, certainly, if it is not caused by experimental errors. Consequently, the violation of symmetry (7) can serve as a reliable indicator of presence of multiple scattering. The second statement this method is based on is the independence of the degree of polarization of the multiple-scattered radiation on the field-of-view angle and the depth of laser pulse penetration into a cloud. This statement is speculative, because it is some approximation, which requires *a priori* parameter δ (degree of polarization of multiple-scattered radiation) to be introduced. From the data available we can assume that δ varies from 0 to 0.3. Particular values of δ for different situations will likely be refined in further studies. We can also assume that in some cases, for example, for a cloud consisting of small particles with size comparable with the radiation wavelength, the proposed method is incorrect for the cloud front, for which the condition $\delta(r) = \mathrm{const}$ may be significantly violated. However, a low value of the ratio I_m/I_1 (8) can be expected in this case, so the correction will not be needed.

Note that the error due to multiple scattering has different effect on values of parameters of the scattering ensemble to be determined. On the one hand, the backscattering coefficient without correction will be overestimated by the factor $(1 - \delta)/(1 - \delta - \Delta)$. In the above example, this gives overestimation by almost one and half times. Consequently, correction of LBSM for multiple scattering can give significant gain in accuracy of determination of the backscattering coefficient. On the other hand, the noise due to multiple scattering practically does not influence the accuracy in determination of dominant orientation of particle axes. The angle of dominant orientation is determined from ratios of off-diagonal elements.[2] At LBSM correction, the matrix elements are multiplied by the same factor, therefore their ratios do not change.

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