Light scattering by rain droplets

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The review of results of investigations of the optical properties of non-spherical and vibrating rain droplets is presented. The effect of anomalously high modulation of light scattered by a vibrating droplet is observed under laboratory conditions and is confirmed by calculations of the scattering phase functions. Photodetection of the tracks of the anomalously high modulation formed by falling rain droplets made it possible to obtain the data on the size, shape, mode and amplitude of vibration of each observed droplet. Two analytical solutions are proposed based on the statistical processing of more than 100 tracks: for the dependence of the mean amplitude of vibration on the droplet size and for the mean shape of falling rain droplets.

It is well known that the falling rain droplets are not spherical and, besides, they can oscillate. The droplet shape and its periodical change affect the propagation and scattering of light in precipitation that should be taken into account in the problems of radiation transfer in the atmosphere, when interpreting the data of remote sensing, etc. However, majority of authors did not take into account the nonsphericity and oscillation of droplets, because the statistically provided data on the rain droplet shape, the modes of the excited oscillations and the optical properties of non-spherical and oscillating droplets practically were not available until recently.

This paper presents an overview of the results obtained by the author and devoted to the solution of the aforementioned problems.

The theory of natural oscillations of a droplet is developed quite fully, starting from classic papers by J. Rayleigh,[1] L.D. Landau and E.M. Lifshits[2] and finishing by the recent papers of American authors.[3,4] In the general case the mode of oscillation is determined by two numbers $n = 1, 2, \ldots$, and $l = 0, \pm 1, \pm 2, \ldots \pm n$. The first possible oscillation of non-compressible liquid corresponds to $n = 2$, and $l = 0$ for all axially symmetric modes.[2] The natural frequency of oscillation of spherical droplets is determined only by the number n and is degenerated on the parameter l:

$$f = \sqrt{\sigma/(3\pi\rho V)}\,\sqrt{(n-1)\,n(n+2)}, \qquad (1)$$

where $n = 1, 2, \ldots$; σ is the coefficient of surface tension of the liquid; ρ is the liquid density; $V = \pi D^3/6$ is the droplet volume, and D is the effective spherical diameter of the droplet. Typical values of the frequency for the principal oscillation mode ($n = 2$, $l = 0$) and for the droplet size $D = (1\ldots6$ mm) are from tens of Hz for big droplets to thousands of Hz for small ones.

Experimental measurements of oscillations were carried out in the majority of cases with artificial water droplets in aerodynamic tubes and shafts.[3,5,6] Such important factors as atmospheric turbulence, coagulation, etc. were excluded. Field investigations of rain droplets were very limited until Refs. 9–11 and 4 appeared and were too occasional and mainly took pictures by means of a flash lamp. The following questions remain open: what fraction of the total number of falling droplets oscillates? What are the amplitudes of oscillations and their dependence on the particle size? What is the mode composition of deformations excited in the droplets freely falling in the atmosphere? What are the optical properties of non-spherical and oscillating hydrometeors? The first step to answer these questions was investigation of the optical properties of non-spherical and oscillating droplets under laboratory conditions. Laboratory measurements were carried out with a droplet of distilled water of the mass 15–25 mg weighted on a thin hydrophyl ring, the periodic displacement of which led to the resonance excitation of the natural capillary oscillations of the droplet. Figure 1 presents the picture of the droplets at excitation of three first axially symmetric modes. This way made it possible not only to fix the droplet, but also to change its size and shape, to excite the needed axially symmetric modes of oscillation and to adjust the amplitude of oscillations in a wide range.[8,9] The droplet was illuminated along the upward looking symmetry axis by a collimated radiation in the form of short pulses of the constant intensity, the frequency of which differed a little bit from the droplet oscillation frequency. If the pulse repetition rate was the same as the frequency ν_0, the droplet would have been illuminated at the moment of the same phase, and the amplitude of the scattered signal would be constant. At some discrepancy between the frequencies, $\Delta\nu$, the droplet shape at the moment of illumination by the next pulse was bit different, then the amplitude of the scattered signal changed. The droplet oscillation phase, at the moments of illumination, smoothly changed by 2π during the sequence of pulses $N = \nu_0/\Delta\nu$. The technique used made it possible to simultaneously record constant and variable components of the scattered radiation excluding the adjacency effect.[10] Taking into account that primarily axially symmetric oscillations with $n = 2$ and $l = 0$ are excited in the rain droplets of up to 5 mm

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diameter, the main attention in laboratory experiments was paid to this mode. Let us note that at such an oscillation mode the droplet shape is either flattened or elongated rotation ellipsoid, the shape of which can be conveniently described by the coefficient $\gamma = c/a$, where c is the vertical and $a = b$ is the horizontal semi-axes.

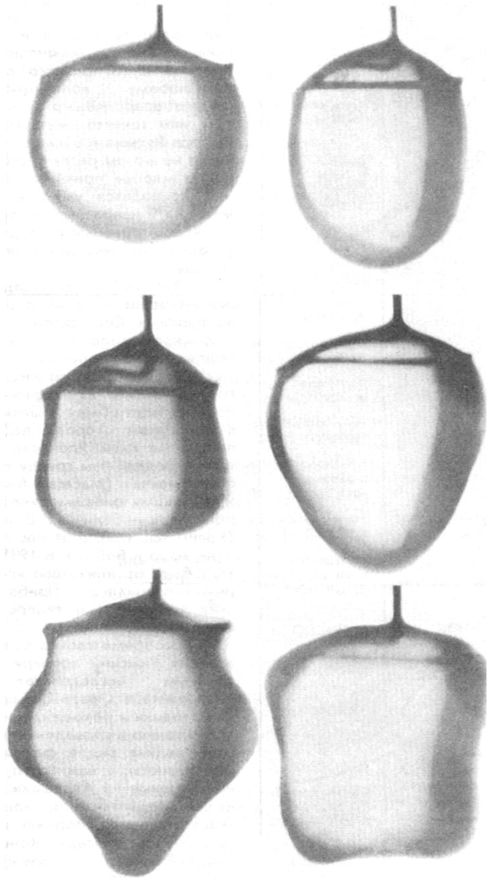


Fig. 1. Droplet shape at resonance excitation of three first axially symmetric modes (axis of symmetry is vertical). Upper pictures: $n = 2$, $l = 0$, middle row: $n = 3$, $l = 0$; lower pictures: $n = 4$, $l = 0$. Right pictures differ from the left ones by the phase shift of π.

Obviously, the radiation of the oscillating droplet at any scattering angle θ can be represented as a sum of constant and variable component caused by the periodic deformation of the droplet. Then, in addition to the traditional scattering phase function $I(V, \gamma, \theta)$ characterizing the properties of the motionless droplet of the volume V and the shape coefficient γ, it is convenient to consider the scattering phase function of the variable component of the scattered radiation $I_a(V, \gamma, \Delta\gamma, \theta)$ which additionally depends on the droplet deformation amplitude $\Delta\gamma$ and the mode of the excited oscillation (the angular dependence was determined by one angle θ counted from the vertical direction because of the axial symmetry).

The results of investigations have shown that the shape of the scattering phase function $I(V, \gamma, \theta)$ of the spherical droplet principally coincides with the classic curve,[13] while the scattering phase function of the variable component $I_a(V, \gamma, \Delta\gamma, \theta)$ significantly differs from $I(V, \gamma, \theta)$. It was expected that the depth of modulation at the deformation $\Delta\gamma = 0.01$ is approximately of the same order. We were very surprised when at some scattering angles we recorded the level of modulation some hundreds times greater than the expected one. In particular, at $\theta = 130–150°$ the oscillating droplet forms the pulses of the scattering radiation, the amplitude of which is some tens times greater than the intensity of light scattered by the motionless droplet. The revealed effect was called the effect of anomalously high modulation of the scattered light. The oscillograms of the pulses recorded at $\Delta\gamma = 0.04$ and different angles of observation are shown in Fig. 2. It is seen from the pictures that, as the scattering angle increases as $\theta = 132°$ (Fig. 2*a*), the pulses appear on the background of the small constant signal, the amplitude and duration of which increase as the scattering angle increases. At $\theta = 133°$ (Fig. 2*b*) the pulse amplitude more than ten times exceeds the constant signal (its ratio depends on the spectral range of measurements and the angular dimensions of the radiation source and receiver. As the observation angle subsequently increases up to $\theta = 136°$ (see Figs. 2*c* and *d*) the pulses formed by the droplet double, acquire a dumbbell shape with maxima at the edges and then close up. The decrease of the oscillation amplitude leads to the change of the flash shape and to the decrease of the angular range within which they are observed.

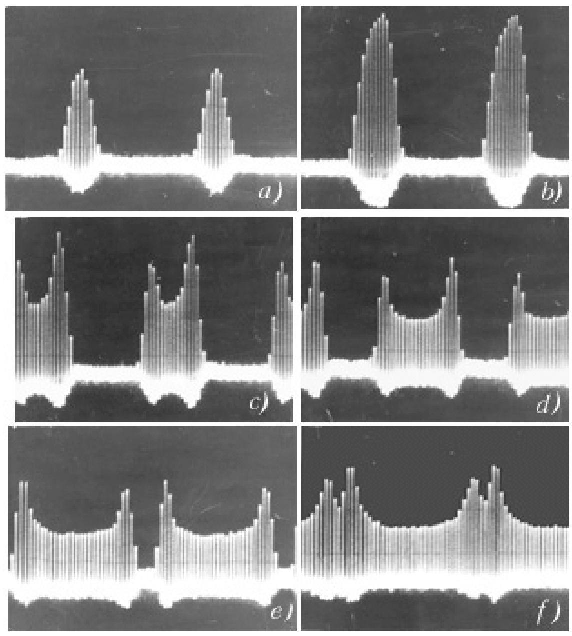


Fig. 2. Oscillograms of the pulses formed by an oscillating droplet at the deformation amplitude $\Delta\gamma = 0.04$. (*a*) flashes, the shape of which changes with the scattering angle, appear at $\theta = 132°$; (*b*) $\theta = 133°$; (*c*) $\theta = 136°$; (*d*) $\theta = 138°$; (*e*) $\theta = 140°$; (*f*) $\theta = 142°$.

Measurements of the scattering phase functions of non-spherical droplets have shown that the most significant effect of deformation is observed in the displacement of the angular position of the first order rainbow. As known, rainbow is the local increase of the intensity of the scattered light at certain angles. The first order rainbow is the most intense, and is observed for spherical particles at the angles of $\theta^{(1)}_{\rm rad}(\lambda = 0.8\ \mu m) = 137.2°$, $\theta^{(1)}_{\rm rad}(\lambda = 0.4\ \mu m) = 139.35°$. The value $\theta^{(1)}_{\rm rad}$ moves to smaller angles for the flatted droplet ($\gamma < 1$) and to lager ones for the elongated droplets. Thus, laboratory investigations have allowed us to reveal the effect of anomalously high modulation of light scattered by oscillating droplets and to suppose its relation with the rainbow properties.

The next step in investigating was calculation of the scattering phase function of non-spherical and oscillating droplets in the geometrical optics approximation.[11] The droplet shape was approximated by a rotation ellipsoid and the incident radiation was supposed to propagate along the axis of symmetry. The calculations have confirmed the conclusions of laboratory investigations, that the first and second order rainbows are very sensitive to the deformation of the ellipsoid. The first and second order rainbows are shown in Fig. 3 as functions of the shape coefficient γ for two values of the refractive index $n = 1.343$ and 1.328 that corresponds to the radiation at $\lambda = 0.4$ and 0.8 μm, respectively, for water at $t = 20°C$. The dependence of $\Delta\theta^{(1)}_{\rm rad}$ can be approximated by the formulas $\Delta\theta^{(1)}_{\rm rad}(n = 1.328) = 89° \cdot \Delta\gamma$ and $\Delta\theta^{(1)}_{\rm rad}(n = 1.343) = 95° \cdot \Delta\gamma$.

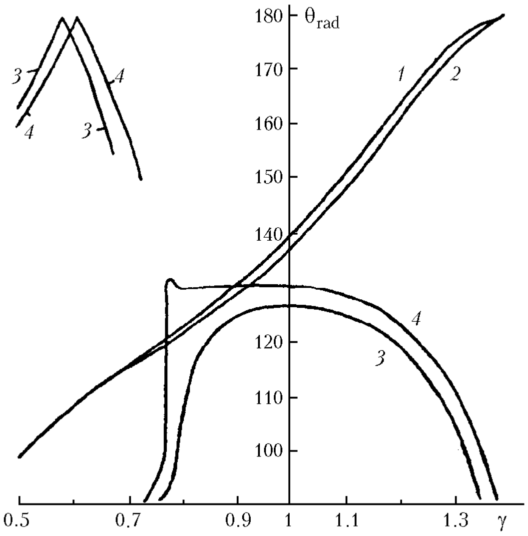
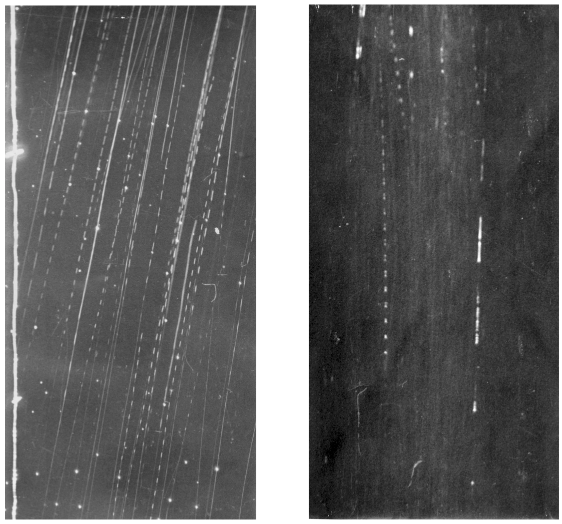


Fig. 3. Dependence of the angular position of first (curves *1* and *2*) and second (curves *3* and *4*) order rainbows on the droplet shape γ at different values of the refractive index n. Curves *1* and *3* correspond to $n = 1.343$, and curves *2* and *4* are for $n = 1.328$.

Explanation of the effect of anomalously high modulation of light scattered by an oscillating droplet is based on two factors. The first one is that the angular position of the first order rainbow is very sensitive to the droplet deformation. The second factor is caused by the big steep slope of the scattering phase function $\partial I(V, \gamma, \theta)/\partial\theta$ for the first order rainbow. Obviously, the amplitude of the variable component of the scattered radiation in linear approximation (here we ignore the change of the scattering cross section of the droplet, because its contribution is small) is determined by the product of the first and second factors: $I_{\rm a}(V, \gamma, \Delta\gamma, \theta) = dI = \{\partial I(V,\gamma,\theta)/\partial\theta\}\ \{\partial\theta/\partial\gamma\}\Delta\gamma$.

The important stage in investigating the optical properties of rains are field measurements.[9] It was supposed to record the oscillation of droplets in rain on the basis of the effect of anomalously high modulation of scattered light and to obtain the statistical data on this poorly studied process.



a *b*

Fig. 4. Pictures of rains illuminated from below by a stabilized light. The droplets in the left-hand picture are additionally illuminated by the stroboscope (bright points). The left-hand track in the right-hand picture corresponds to high amplitude of oscillations, at which the tracks have the dumbbell shape with maxima at the edges (compare with Fig. 2*c*). The right-hand track is caused by the overtone mode of oscillation.

The following technique for measurement was applied. Rain in the nighttime was illuminated upwards by a collimated light beam of a stable intensity of the visible wavelengths, and the radiation scattered by droplets was recorded onto a photographic film at open shutter of a camera. The droplets were additionally illuminated from one side by short pulses of the stroboscope at a fixed frequency $f_{\rm str}$. The spatially calibrated marks for geometrical reference of the pictures were situated side by side with the light beam on the vertically strained thread. The typical pictures obtained in the rain of the intensity about 3 mm/hour at recording the droplets at the scattering angles 120–145° are shown in Fig. 4. The larger scattering angle corresponds to the droplets at a larger height, and the scattering angle decreases as droplets fall down. It is seen from the picture that the track is solid at large scattering angles (upper part of the pictures), although its intensity is modulated. As the droplet falls down and the scattering angle decreases, the breaks of the track appear, the droplet "flares," then the flash duration decreases and they disappear at subsequent decrease of the scattering angle that is in agreement with the data obtained under laboratory conditions.

Let us consider, what information can be obtained from such pictures. Let us note that each track has its own "finger print": each oscillation mode forms the flashes of a certain shape and spatial length. The angular range, where the flashes of the anomalous scattering appear, is responsible for the mean shape of the droplet and amplitude of its oscillation. For example, at a high amplitude of oscillation and excitation of the main mode ($n = 2$, $l = 0$) at the scattering angles lower than the rainbow direction, the droplet forms the pulses of the dumbbell shape, the duration of which decreases as the droplet falls down and the scattering angle decreases (Fig. 4, left track). The trajectory of the droplet with higher harmonics of oscillation ($n > 2$) is shown in the right-hand part of the same picture. We have developed a technique for the track processing that makes it possible to determine all principal characteristics of the falling rain droplets: the size, the mean shape, the mode, and the amplitude of oscillation. The procedure of processing is the following. The bright points obtained due to the stroboscopic flashes were recorded on the trajectory of each droplet. Then the number of periods m_i of the oscillations of the ith droplet that are in the interval between two points of the stroboscope was determined. Frequency of the droplet oscillation f_i was calculated as $f_i = m_i f_{\rm str}$ where $f_{\rm str}$ is the stroboscope flash repetition rate. As the droplet oscillation frequency is strongly related to its volume V and the mode (the number n) by the Rayleigh formula (1), one can calculate both these parameters from the frequency. One should have in mind that, as a rule, the main mode of oscillations is excited in droplets. The higher modes of oscillation have approximately doubled or tripled frequency, and, besides, the flash shape is principally different from the flash shape of the main mode, so it is easy to determine the parameter n.

Another technique for determining the droplet size was also used. The spatial period L of the droplet oscillation, on which one full oscillation occurs can be easily determined at a fixed geometry. If one supposes that the vertical fluxes are small near the ground surface, this value is related with the velocity of gravitational fall of the droplets $v(D)$ and the frequency of their oscillation $f(D)$ by the relationship $L(D) = v(D)/f(D)$, where D is the equivalent spherical diameter defined as the diameter of spherical droplet of the same volume. Following the data of Gunn and Kinzer[12] for selection of the dependence $v(D)$ and using the Rayleigh formula (1) for $f(D)$, the dependence $L(D)$ was derived, which is monotonic and unambiguous. The use of this dependence allows one to determine the effective spherical diameter D from the known L value.

Analysis of the angular position of the flashes gives the next interesting possibility of interpreting the pictures. In particular, the mean flatness of the droplet leads to the displacement of the angular position of the flashes of anomalous scattering to the smaller angles, and the increase of amplitude of oscillations leads to an increase in the range of angles where the flashes can be observed. Then, we determined both the mean shape of the droplet $\gamma(D_i)$ and the amplitude of oscillations of each droplet $\Delta\gamma(D_i)$ from two parameters: the angular position of the first break of the droplet trajectory and the last flash. Processing of about 1000 tracks allowed us to obtain the statistical data, on the basis of which we have proposed analytical expressions for the mean shape of rain droplets $\gamma(D)$ and the amplitude of oscillations $\Delta\gamma(D)$:

$$\gamma(D) = 1 - 1.2\,D^2, \quad \Delta\gamma(D) = 0.7\,D^2,$$

where D is measured in centimeters. One should note that recording of a small-droplet fraction was limited by the sensitivity of the photography which was capable to record the droplets with the diameter $D > 0.12$ cm. The large-droplet fraction was limited by the size $D < 0.35$ cm because the probability of excitation of overtone modes of oscillation quickly increases in larger droplets, which were not interpreted at this stage, because such problem is too difficult.

Let us note that the sensitivity of the developed method is so high that it makes it possible to record the amplitude of deformation of some microns from the distance of 5 m. Recording of such deformations is difficult even at observing the droplet by microscope. As we know, no methods known provide such a possibility.

The effect of doubling the frequency of scattered radiation relative to the droplet oscillation frequency is of a certain interest. This nonlinear modulation effect was observed in the first order rainbow direction for a motionless droplet, and one can observe it on the oscillogram in Fig. 2*d* and in the picture of the left-hand track in Fig. 4*b*. The extreme, for which $\partial I(V, \gamma, \theta)/\partial\theta = 0$, is observed in the rainbow direction, and the square term of the expansion $(\partial^2 I(V, \gamma, \theta)/\partial\gamma^2)/\Delta\gamma^2$ makes the principal contribution to the variable component of the signal. The presence of this term leads to doubling the frequency.

Let us note for a conclusion that investigations have shown that nonsphericity and oscillation of rain droplets strongly affect scattering and propagation of optical radiation in rains. Besides, field observations have proved that oscillations of 93% of droplets leaving the track on the picture are recorded, i.e., oscillation has occurs on a mass scale. Obviously, for this reason one cannot ignore the effect of nonsphericity and oscillation of rain droplets on scattering and propagation of optical radiation in the atmosphere.

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