Bistatic and hybrid schemes for the formation of laser guide stars

V.P. Lukin

*Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk*

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Several approaches to the formation of a laser guide star for large-aperture telescopes are considered. Limiting capabilities of the optical arrangements considered are studied from the viewpoint of the wave front tilt correction.

Introduction

The up-to-date adaptive-optics telescopes necessarily have a laser system for the formation of a laser guide star (LGS). The laser guide is formed from the ground using backscattering of laser radiation on the atmospheric inhomogeneities. When an LGS signal is used for making corrections for phase fluctuations in an optical wave, the efficiency of one or another tilt correction scheme is of principal importance.

Historically, this research was first accomplished[1] by specialists in remote laser sensing of the atmosphere, who studied displacements of the image of an atmospheric scattering volume. In addition, the efficiency of correction for the wave-front tilts based on measured displacements of the image of the reference source was tentatively calculated.[2,3] In the 70s, the term "artificial reference source" was used in place of "laser guide star." It is well known that the vector $\boldsymbol{\varphi}_{\rm ins}$ characterizing the instantaneous angular location of the LGS image can be written as the following sum:

$$\boldsymbol{\varphi}_{\rm ins} = \boldsymbol{\varphi}_{\rm b.c} + \boldsymbol{\varphi}_{\rm s.s}, \qquad (1)$$

where $\boldsymbol{\varphi}_{\rm b.c}$ is the displacement of the centroid of a focused laser beam; $\boldsymbol{\varphi}_{\rm s.s}$ is the displacement of the image of the secondary source.

It is also known that the monostatic optical arrangement of the LGS formation is practically unusable[4] for the wave front tilt correction. That is why we consider here only bistatic optical arrangement, in which the axes of the main telescope and the auxiliary one (or an additional laser projector) are spaced at the distance Δ. The radius of the aperture of the main telescope is denoted here as $R_{\rm m}$, and the radius of the aperture of the auxiliary telescope (or additional laser projector) is denoted as $R_{\rm a}$.

Classification of the bistatic LGS formation techniques

It should be noted that there are bistatic optical arrangements of two types. A bistatic optical arrangement is classified as belonging to the type I if $\Delta \approx R_{\rm m}$. In this case, as calculations show,[5,6] the correlation between fluctuations of the forward and backward propagated waves almost vanishes.

A bistatic optical arrangement can be classified as belonging to the type II, if the separation Δ between the axes of the main telescope and the auxiliary one (or laser projector) is rather large. In this case not only the correlation between fluctuations of both the forward and the backward propagated waves is insignificant, but also the reference source cannot be considered as a point-like object – it is noticeably extended.[6–10]

In this paper, the algorithm of optimal correction[11] for fluctuations of the total wave front tilt is used. Then, as shown in Refs. 11 and 12, the relative efficiency of tilt correction made using a natural star $\boldsymbol{\varphi}_{\rm s}$ proves to be equal to

$$\beta^2 = \frac{\langle[\boldsymbol{\varphi}_{\rm s} - A\boldsymbol{\varphi}_{\rm ins}]^2\rangle}{\langle(\boldsymbol{\varphi}_{\rm s})^2\rangle} = 1 - \frac{\langle\boldsymbol{\varphi}_{\rm s}\ \boldsymbol{\varphi}_{\rm ins}\rangle^2}{\langle(\boldsymbol{\varphi}_{\rm s})^2\rangle\ \langle(\boldsymbol{\varphi}_{\rm ins})^2\rangle}, \qquad (2)$$

where

$$A = \langle\boldsymbol{\varphi}_{\rm s}\ \boldsymbol{\varphi}_{\rm ins}\rangle / \langle(\boldsymbol{\varphi}_{\rm ins})^2\rangle. \qquad (3)$$

The coefficient A is determined either from direct measurements or by calculation based on some atmospheric turbulence model.[6,12]

Bistatic optical arrangement of the type I

Assume that we use the bistatic optical arrangement, in which the separation Δ is such that the correlation between the forward and backward propagated waves vanishes. Then we have that

$$\langle\varphi^2_{\rm ins}\rangle = \langle\varphi^2_{\rm b.c}\rangle + \langle\varphi^2_{\rm s.s}\rangle, \qquad (4)$$

$$\langle\boldsymbol{\varphi}_{\rm s}\ \boldsymbol{\varphi}_{\rm ins}\rangle = \langle\boldsymbol{\varphi}_{\rm s}\ \boldsymbol{\varphi}_{\rm b.c}\rangle + \langle\boldsymbol{\varphi}_{\rm s}\ \boldsymbol{\varphi}_{\rm s.s}\rangle. \qquad (5)$$

For such bistatic optical arrangements, one of the terms in Eq. (5) is always identically equal to zero by definition, and the LGS jitter coincides with the angular displacement of the image of a point-like source:

$$\boldsymbol{\varphi}_{\rm s.s} = \boldsymbol{\varphi}_{\rm sp.w}\ .$$

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It is known[8–10] that two different on principle bistatic optical arrangements are possible (Fig. 1). The case that

$$\langle \varphi_s \, \varphi_{b.c} \rangle = 0$$

corresponds to the case *a* in Fig. 1 (with an additional laser projector[8]), and the case that

$$\langle \varphi_s \, \varphi_{s.s} \rangle = 0$$

which corresponds to the case *b* in Fig. 1 (an auxiliary telescope is used[10]).

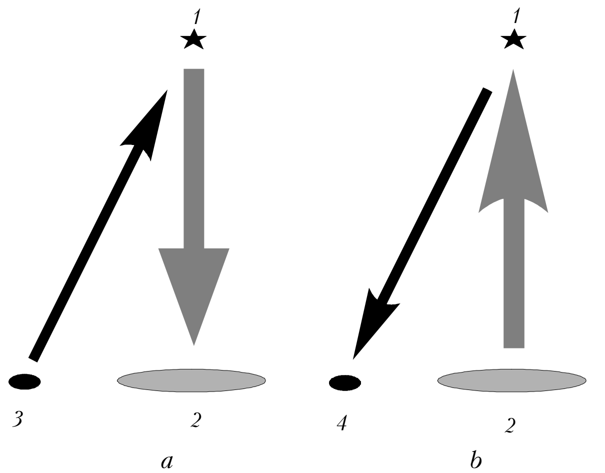

Fig. 1. Bistatic optical arrangement of the LGS formation: laser guide star *1*, aperture of the main telescope *2*, aperture of the auxiliary laser projector *3*, aperture of the auxiliary telescope *4*.

Assuming the laser guide star to be a point-like source, we have[13] from Eq. (2), with allowance for Eqs. (4) and (5), that in the case *a*

$$\beta^2 = 1 - \frac{\langle \varphi_s \, \varphi_{sp.w} \rangle^2}{\langle (\varphi_s)^2 \rangle \, [\langle (\varphi_{b.c})^2 \rangle + \langle (\varphi_{sp.w})^2 \rangle]} = 1 - \frac{K^2(X)}{(1 + \langle (\varphi_{b.c})^2 \rangle / \langle (\varphi_{sp.w})^2 \rangle)}, \quad (6)$$

where

$$K(X) = \frac{\langle \varphi_s \, \varphi_{sp.w} \rangle}{\sqrt{\langle \varphi_s^2 \rangle \, \langle \varphi_{sp.w}^2 \rangle}}. \quad (7)$$

It is just this function $K(X)$ that numerically characterizes the *cone anisoplanatism.* The values of the function $K(X)$ are given in the Table for the model of turbulence intensity $C_n^2(h)$ (Ref. 14) for different heights X of the guide star and the spectrum of turbulence with the infinite outer scale. Note that in this case

$$\langle \varphi_{b.c}^2 \rangle / \langle \varphi_{sp.w}^2 \rangle = (R_a / R_m)^{-1/3}. \quad (8)$$

The function $f(b)$ is defined below, see Eq. (18).

Summarizing Eqs. (6), (7), and (8), we can formulate the following recommendations: *under conditions of a particular observatory, the largest telescope available forms a bistatic guide star for the smaller telescopes in the observatory.*

Table

X, km	K	K^2	$1 - K^2$	$f(b)$
1	0.65	0.42	0.58	
2	0.72	0.52	0.48	
3	0.75	0.56	0.44	
5	0.79	0.62	0.38	
10	0.84	0.71	0.29	0.32 (for b = 100)
20	0.88	0.77	0.23	
40	0.89	0.79	0.21	
100	0.90	0.81	0.19	0.27 (for b = 1000)

Under the same conditions, for the case *b* the efficiency of the correction for the wave front tilt[13] can be written as

$$\beta^2 = 1 - \frac{\langle \varphi_s \, \varphi_{b.c} \rangle^2}{\langle (\varphi_s)^2 \rangle \, [\langle (\varphi_{b.c})^2 \rangle + \langle (\varphi_{sp.w})^2 \rangle]}. \quad (9)$$

Comparing Eqs. (6) and (9), one can easily see based on the results from Refs. 10 and 13 that only if the laser star is formed by the entire aperture of the main telescope, i.e.,

$$\varphi_{b.c}^2 = - \varphi_{sp.w}^2, \quad (10)$$

the optical arrangement provides for a sufficiently efficient correction, namely,

$$\beta^2 = 1 - \frac{K^2(X)}{1 + \langle \varphi_{sp.w}^2(R_a) \rangle / \langle \varphi_{sp.w}^2(R_m) \rangle}. \quad (11)$$

Here $\langle \varphi_{sp.w}^2(R_a) \rangle$ denotes the variance of jitter of the image of a point-like source (LGS) measured with the auxiliary telescope, i.e., on the aperture R_a. The comparison of Eqs. (11) and (6) shows[13] that the efficiency of correction for the tilts in the arrangements *a* and *b* is the same only if $R_a = R_m$, i.e., if the auxiliary telescope has the same radius as the main one.

So, we can summarize the main disadvantages of the optical arrangement *b*:

– the requirement that the focused laser beam should be formed by the entire aperture of the main telescope inevitably leads to phosphorescence of the entire optical channel of this telescope,

– in addition to the main telescope, one more large auxiliary telescope is needed for the efficient correction.

These disadvantages are critical therefore (in the bistatic optical arrangement of the type I, i.e., in the case that the separation between the main and auxiliary telescopes is not too large) the scheme with the auxiliary laser projector (case *a*) is preferable.

Bistatic optical arrangement of the type II

The arrangements of the type II are characterized by a sufficiently large separation between the main telescope and the projector (or auxiliary telescope). The separation is so large that the reference source cannot be considered as a point-like object.[8–10] The extension of the reference source is connected first with the fact that the optical system forming the LGS has a finite caustic. At the same time, for example, for the sodium star, the LGS size is determined primarily by the thickness of the Earth's sodium layer.

It is commonly accepted that the "length" of the Rayleigh guide star l_b is determined by the length of caustic of the beam-forming optical system and equals to 2–5 km, whereas for the sodium star it is about 10 km. This natural LGS extension, visible dimensions of the secondary source [the axes of the main and auxiliary telescopes are separated by the vector $\mathbf{\Delta} = (\Delta_y, \Delta_z)$] are as follows:

$$a_b^y = l_b \sin\theta_y, \ldots a_b^z = l_b \sin\theta_z, \quad (12)$$

where

$$\sin\theta_y = \frac{\Delta_y}{\sqrt{X^2 + \Delta_y^2}}; \quad \sin\theta_z = \frac{\Delta_z}{\sqrt{X^2 + \Delta_z^2}}; \quad (13)$$

X is the LGS altitude; l_b is the LGS length.

If

$$X \gg \Delta_y, \quad X \gg \Delta_z,$$

then

$$\sin\theta_y = \Delta_y/X, \ \sin\theta_z = \Delta_z/X$$

and

$$a_b^y = l_b\, \Delta_y/X, \ldots a_b^z = l_b\, \Delta_z/X.$$

In any case, the field of view of the measuring system (for measuring $\boldsymbol{\varphi}_{\text{ins}}$) should be larger than the angular size of the visible length of this secondary source ($a_b^y/X = l_b\, \Delta_y/X^2, \ldots a_b^z/X = l_b\, \Delta_z/X^2$).

So, assume that the separation $\mathbf{\Delta} = (\Delta_y, \Delta_z)$ is sufficiently large, then the reference star in the case *a* (Ref. 8) is seen already as an extended segment of a straight line (when using one auxiliary laser projector) or as crossed segments of two orthogonal straight lines (when using two projectors).

We can recommend the following differential scheme of signal processing, since it is most efficient.[15,16] A wave front sensor is placed in the main telescope; it measures random angular location of an LGS image. If the signal (or two signals) are integrated over the entire field of view, then

$$\begin{cases} \varphi_1^y = \varphi_{\text{b.c}}^y + \varphi_{\text{s.s}}^y, \\ \varphi_1^z = \varphi_{\text{b.c}}^z + \varphi_{\text{s.s}}^z, \end{cases} \quad (14)$$

and in the case of integration only over the central area of the LGS image

$$\begin{cases} \varphi_2^y = \varphi_{\text{b.c}}^y + \varphi_{\text{sp.w}}^y, \\ \varphi_2^z = \varphi_{\text{b.c}}^z + \varphi_{\text{sp.w}}^z, \end{cases} \quad (15)$$

Calculating the difference between the signals $\varphi_1^{y,z}$ and $\varphi_2^{y,z}$, we obtain the differential signal $\boldsymbol{\delta} = (\delta_y, \delta_z)$, where

$$\begin{cases} \delta_y = \varphi_{\text{s.s}}^y - \varphi_{\text{sp.w}}^y, \\ \delta_z = \varphi_{\text{s.s}}^z - \varphi_{\text{sp.w}}^z. \end{cases} \quad (16)$$

It is just this difference signal that serves as a correction one. Thus, for the algorithm of optimal correction, we have the following efficiency:

$$\beta^2 = 1 - \frac{\langle \boldsymbol{\varphi}_{\text{s}} (\boldsymbol{\varphi}_{\text{s.s}} - \boldsymbol{\varphi}_{\text{sp.w}})\rangle^2}{\langle \boldsymbol{\varphi}_{\text{s}}^2\rangle / \langle (\boldsymbol{\varphi}_{\text{s.s}} - \boldsymbol{\varphi}_{\text{sp.w}})^2\rangle} \approx$$

$$\approx 1 - \frac{K^2(X)}{1 + \langle (\boldsymbol{\varphi}_{\text{s.s}})^2\rangle / \langle (\boldsymbol{\varphi}_{\text{sp.w}})^2\rangle}. \quad (17)$$

It should be noted that the signal (14) is not directly suitable for making correction, because for the extended source the correlation $\langle \boldsymbol{\varphi}_{\text{s}}\, \boldsymbol{\varphi}_{\text{s.s}}\rangle$ is inefficient for tilt correction.

Here $\langle (\boldsymbol{\varphi}_{\text{s.s}})^2\rangle / \langle (\boldsymbol{\varphi}_{\text{sp.w}})^2\rangle$ is the ratio of the variance of the angular jitter of the image of an extended object with the length $a_b(a_b^y, a_b^z)$ to that of the jitter of the image of a point-like object. Denote this ratio as a function[6,15]

$$f(a, b, c) = \langle (\boldsymbol{\varphi}_{\text{s.s}})^2\rangle / \langle (\boldsymbol{\varphi}_{\text{sp.w}})^2\rangle, \quad (18)$$

in which the following parameters are used:

$$a = R_{\text{a}}/R_{\text{m}}; \quad b = a_b/R_{\text{m}}; \quad c = \sqrt{2}\, R_{\text{m}}^{-1}\, \iota_{\text{m}}^{-1}.$$

It should be noted that for the first time the jitter of an extended source was calculated in such a way in Ref. 17. Numerically calculated function $f(a, b, c)$ for the following model of the turbulence spectrum:

$$\Phi_n(\iota, h) = 0.033\, C_n^2(h)\, \iota^{-11/6} \{1 - \exp(-\iota^2/\iota_{\text{m}}^{-2})\}, \quad (19)$$

where $C_n^2(\xi)$ is the turbulence intensity profile and ι_{m}^{-1} is the outer scale, is shown in Figs. 2 and 3 as six fragments each for the altitude $X = 10$ and 100 km.

Thus, using our calculations (see Table and Figs. 2 and 3), we can find, from Eqs. (17) and (18), the relative efficiency of tilt correction by the use of the optical arrangement shown in Fig. 1, case *a*.

Using the designation (18), we can now write Eq. (17) in the following form:

$$\beta^2 = 1 - \frac{K^2(X)}{1 + f(1, b, c)}, \quad (20)$$

where the parameter $a = R_{\text{a}}/R_{\text{m}} = 1$, because the correcting signal (16) is independent of the characteristics of the laser projector, and the measurements are conducted on the aperture of the main telescope.

Note here the main disadvantages of this bistatic optical arrangement with an additional laser projector:

– if the separation between the main telescope and the projector is small, then the projector itself should be as large as possible,

– to use the effect of LGS extension, the separation between the projector and telescope axes should be about 40 km.

Let us analyze now the alternative approach[10] *b* that employs two auxiliary telescopes.

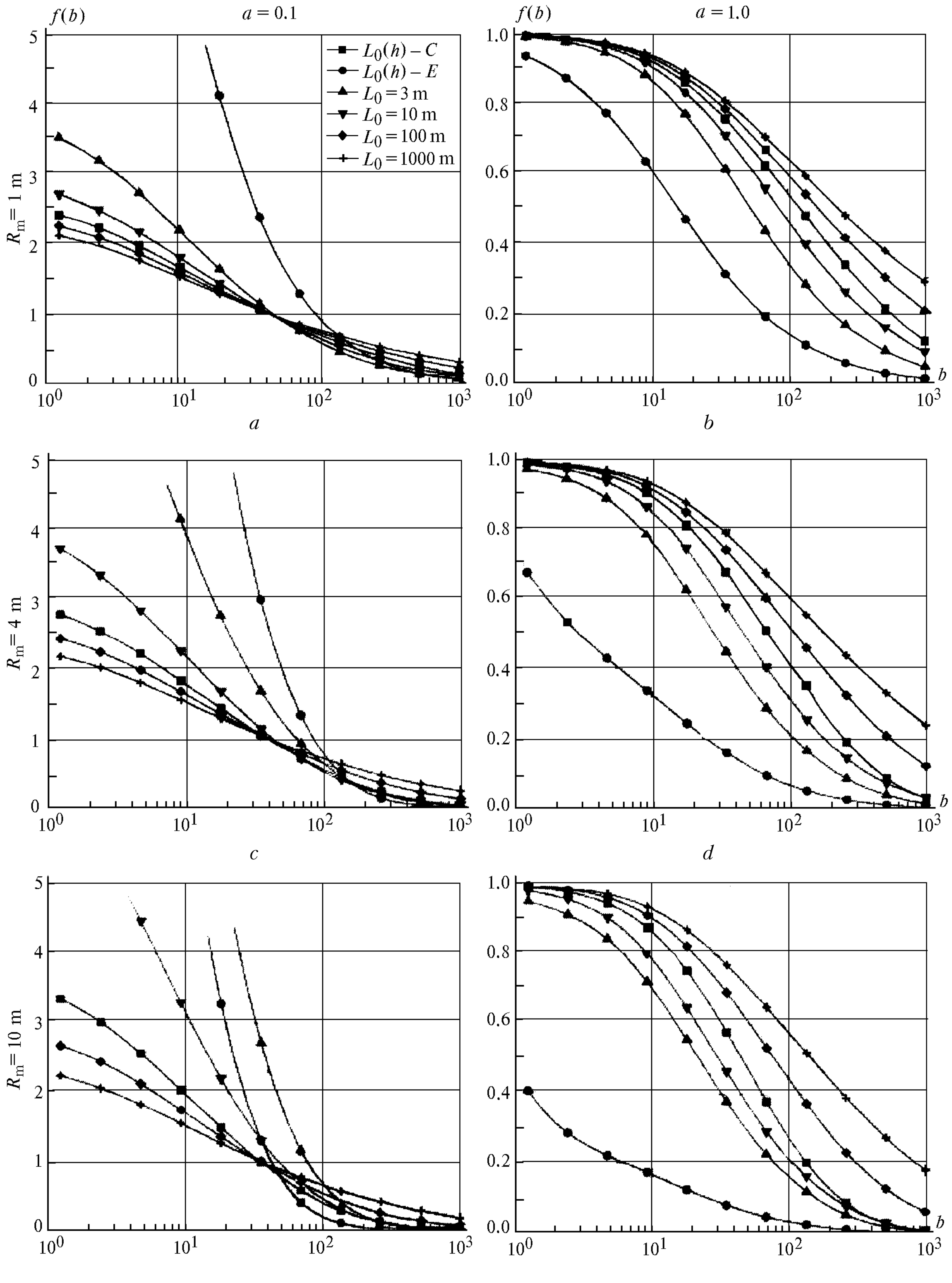

Fig. 2. Variance of the jitter of the image of an extended object normalized to the variance of the jitter of the image of a point-like source (spherical wave) $f(a, b, c) = \langle(\boldsymbol{\varphi}_{\rm s.s})^2\rangle / \langle(\boldsymbol{\varphi}_{\rm sp.w})^2\rangle$ as a function of the size of this extended source (parameter $b = a_b/R_{\rm m}$) for different apertures of the main telescope and different values of the parameters $a = R_{\rm a}/R_{\rm m}$ and $c = \sqrt{2}\, R_{\rm m}^{-1}\, \iota_{\rm m}^{-1}$ for the guide star located at the altitude $X = 10$ km. The curves correspond to different models of the outer scale of turbulence $\iota_{\rm m}^{-1} = L_0$. The left and right columns correspond to calculations made for the different aperture ratio of the auxiliary telescope to the main telescope: for the left column this ratio is equal to 0.1, and for the right column it is equal to 1.

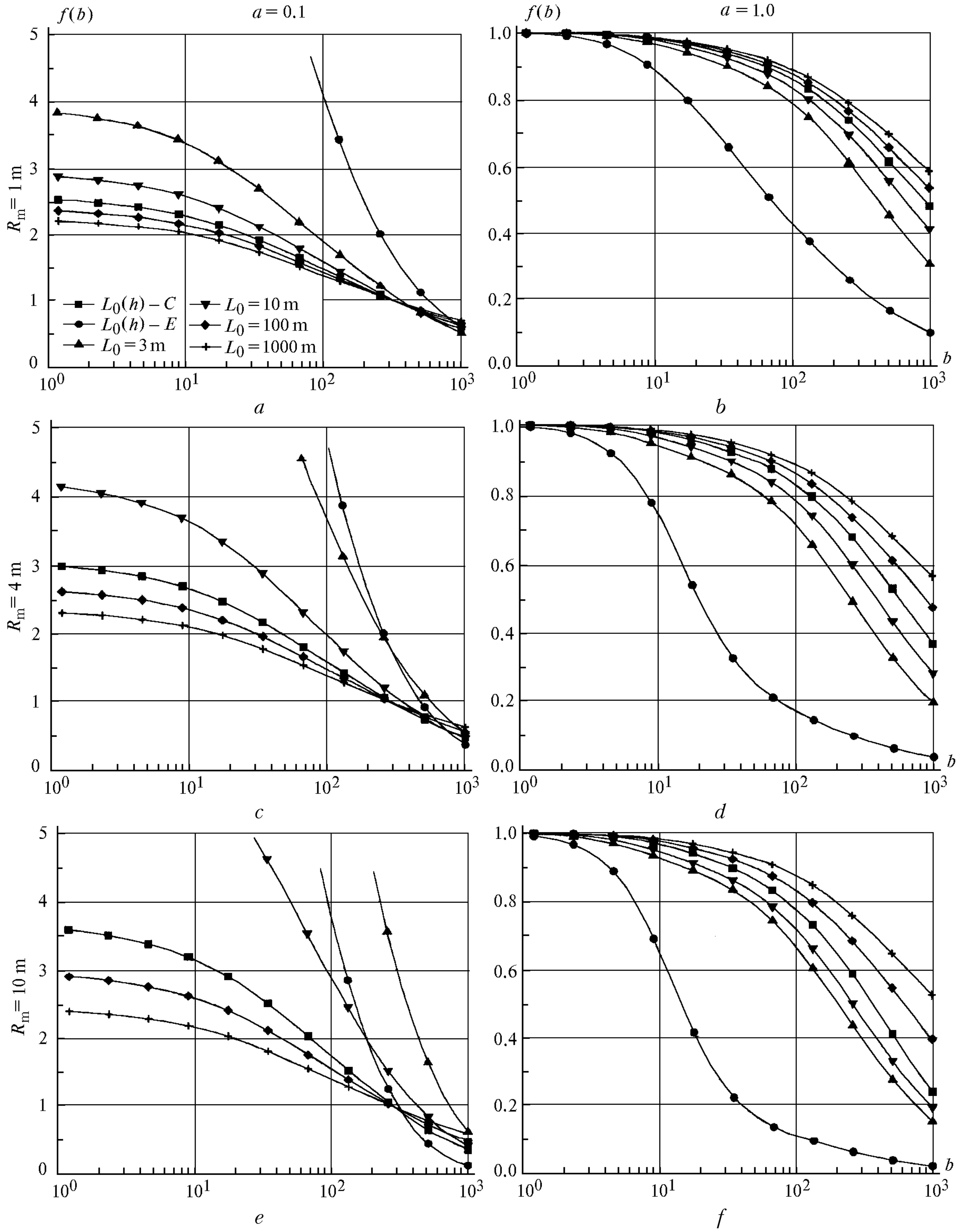

Fig. 3. The same as in Fig. 2, but for the altitude X = 100 km.

Here the measurements are conducted with two additional telescopes. The signals measured on the aperture R_a are written in the form

$$\begin{cases} \varphi^y_{\rm ins} = \varphi^y_{\rm b.c} + \varphi^y_{\rm s.s}\,, \\ \varphi^z_{\rm ins} = \varphi^z_{\rm b.c} + \varphi^z_{\rm s.s}\,. \end{cases} \quad (21)$$

As a result of optimal correction[16,18]

$$\beta^2 = 1 - \frac{\langle \boldsymbol{\varphi}_{\rm s}\ \boldsymbol{\varphi}_{\rm b.c}\rangle}{\langle(\boldsymbol{\varphi}_{\rm s})^2\rangle\,\langle(\boldsymbol{\varphi}_{\rm ins})^2\rangle}\,, \quad (22)$$

where

$$\langle(\boldsymbol{\varphi}_{\rm ins})^2\rangle = \langle(\boldsymbol{\varphi}_{\rm b.c})^2\rangle + \langle(\boldsymbol{\varphi}_{\rm s.s})^2\rangle;$$

$$\langle(\boldsymbol{\varphi}_{\rm s.s})^2\rangle = \langle(\boldsymbol{\varphi}^y_{\rm s.s})^2\rangle + \langle(\boldsymbol{\varphi}^z_{\rm s.s})^2\rangle.$$

From comparative analysis of Eqs. (22) and (17) it is seen that *only if the entire aperture* $R_{\rm m}$ *of the main telescope is illuminated*, i.e., if

$$\boldsymbol{\varphi}_{\rm b.c} = -\boldsymbol{\varphi}_{\rm sp.w},$$

we obtain sufficiently efficient correction using the optical arrangement b, and then

$$\beta^2 = 1 - \frac{K^2(X)}{1 + \langle(\boldsymbol{\varphi}_{\rm s.s})^2\rangle / \langle(\boldsymbol{\varphi}_{\rm sp.w})^2\rangle} = 1 - \frac{K^2(X)}{1 + f(a,\, b,\, c)}. \quad (23)$$

Comparison of Eqs. (23) and (17) shows that the efficiency of using the optical arrangement b can be equal to that in the case a if only $R_{\rm a} = R_{\rm m}$.

It follows from the above that

– a significant disadvantage of the case b is the requirement that the laser star is formed by the entire aperture of the main telescope, because of phosphorescence of the telescope optical channel;

– three identical (large) telescopes are needed for making an efficient correction;

– the separation between the axes of the main and auxiliary telescopes should be larger than 40 km (for the altitude $X = 100$ km and the telescope aperture $2R_{\rm m} = 8$ m).

One can see that the arrangement with two additional remote laser projectors has some advantages. At the same time, we should acknowledge that the optical arrangement b also has some disadvantages, namely:

– for efficient correction, the separation between the projectors should be larger than 40 km,

– the scanning over sky with the main telescope and two auxiliary projectors, whose radiation form the laser guide star shaped as a cross in the main telescope, should be matched with high accuracy (≈ 100 arcsec).

Hybrid optical arrangement

It should be noted that the hybrid optical arrangement using synchronous measurements with two (or three) telescopes is a rather efficient tool for making tilt corrections.[18] In this case, the LGS is formed by a narrow laser beam. In the main telescope, for which LGS is formed by the monostatic optical arrangement, the angular location of the guide star image is

$$\boldsymbol{\varphi}_{\rm ins} = \boldsymbol{\varphi}_{\rm b.c}(0) + \boldsymbol{\varphi}_{\rm sp.w} \quad (24)$$

and in the auxiliary telescope

$$\boldsymbol{\varphi}_{b} = \boldsymbol{\varphi}_{\rm b.c}(0) + \boldsymbol{\varphi}_{\rm s.s}. \quad (25)$$

Then the difference vector signal is calculated

$$\boldsymbol{\Delta} = \boldsymbol{\varphi}_{\rm ins} - \boldsymbol{\varphi}_{b} = \boldsymbol{\varphi}_{\rm sp.w} - \boldsymbol{\varphi}_{\rm s.s}, \quad (26)$$

which is used for making correction for the wave front tilt fluctuations of a natural star $\boldsymbol{\varphi}_{\rm s}$.

The limiting relative efficiency of correction with the help of the signal $\boldsymbol{\Delta}$ is

$$\beta^2 = 1 - \frac{\langle \boldsymbol{\varphi}_{\rm s}\, \boldsymbol{\Delta}\rangle^2}{\langle(\boldsymbol{\varphi}_{\rm s})^2\rangle\, \langle(\boldsymbol{\Delta})^2\rangle}, \quad (27)$$

where

$$\langle(\boldsymbol{\Delta})^2\rangle = \langle\boldsymbol{\varphi}^2_{\rm b.c}\rangle + \langle\boldsymbol{\varphi}^2_{\rm s.s}\rangle. \quad (28)$$

As a result,

$$\beta^2 = 1 - \frac{K^2(X)}{1 + \langle(\boldsymbol{\varphi}_{\rm s.s})^2\rangle / \langle(\boldsymbol{\varphi}_{\rm sp.w})^2\rangle}. \quad (29)$$

Note here the disadvantages of this hybrid arrangement:

– two additional (large) telescopes are needed;

– two additional wave front sensors are needed;

– to provide for small ratio $\langle(\boldsymbol{\varphi}_{\rm s.s})^2\rangle / \langle(\boldsymbol{\varphi}_{\rm sp.w})^2\rangle$, the separation between the axes of the main and auxiliary telescope should be large enough;

– the separation between the axes should be larger than 40 km.

Laser reference cross

Another technical realization of the optical arrangement, in which the laser guide star is formed as an extended source in the form of two crossing segments of straight lines, is possible as well.[19,20] It uses two narrow laser beams emitted from a point located near, but beyond the aperture of the main telescope. The beams experience angular modulation in two mutually normal directions. The frequency of this angular modulation is much higher than the characteristic frequency of the jitter of focused beams. Due to rather a rapid angular scanning of these two beams in their focal plane, a laser reference cross appears.

The advantage of this approach[19] is that it needs for only one main telescope equipped with the wave front sensor. Almost monostatic approach is realized: the LGS image is formed in the wave front sensor consisting of two identical CCD arrays. The optical arrangement of the wave front sensor is constructed in such a way that each of the CCD arrays records the image of only one of two crossing lines.

The signal from each of the CCD arrays is processed in the following way: two signals are measured simultaneously, namely, the signal recorded by the whole array φ^y (for the array Y or φ^z for the array Z) and the signal obtained from measurements only in the central part of the array $\varphi^y_{\rm c}$ (or $\varphi^z_{\rm c}$), and then the differences are calculated

$$\Delta^y = \varphi^y - \varphi^y_{\rm c}, \;\; \Delta^z = \varphi^z - \varphi^z_{\rm c}. \quad (30)$$

It can be readily shown[19] that the data of these current measurements obtained after averaging over the entire array and over time for the period T (or, what is the same, over the modulation angle θ), are expressed as follows:

$$\varphi^y = \frac{1}{\theta}\int_{-\theta/2}^{\theta/2} \varphi^y_{\rm b.c}(\theta)\, {\rm d}\theta + \frac{1}{\theta}\int_{-\theta/2}^{\theta/2} \varphi^y_{\rm sp.w}(\theta)\, {\rm d}\theta = \varphi^y_{\rm b.c}(0,\, t) + \varphi^y_{\rm s.s}(t), \quad (31)$$

$$\varphi^z = \varphi^z_{b.c}(0,\, t) + \varphi^z_{s.s}(t). \quad (32)$$

At the same time, the data obtained from measurements only in the central part of the CCD array are expressed as:

$$\begin{cases} \varphi^y_c = \varphi^y_{b.c}(0,\, t) + \varphi^y_{sp.w}(t)\ , \\ \varphi^z_c = \varphi^z_{b.c}(0,\, t) + \varphi^z_{s.s}(t)\ . \end{cases} \quad (33)$$

Measuring the difference between the values presented by Eqs. (33) and (30), we obtain the following difference signal:

$$\boldsymbol{\Delta} = (\Delta^y,\, \Delta^z) = \boldsymbol{\varphi}_{s.s}(t) - \boldsymbol{\varphi}_{sp.w}(t). \quad (34)$$

Now it is easy to calculate the level of residual fluctuations of the wave front tilt after having used the signal $\boldsymbol{\Delta} = (\Delta^y, \Delta^z)$ for making tilt corrections of the wave front of a natural star:

$$\langle[\boldsymbol{\varphi}_s(t) + \boldsymbol{\Delta}]^2\rangle \approx \langle[\boldsymbol{\varphi}_s(t) - \boldsymbol{\varphi}_{sp.w}(t)]^2\rangle + \langle(\boldsymbol{\varphi}_{s.s})^2\rangle, \quad (35)$$

and the relative variance is

$$\beta^2 = \frac{\langle[\boldsymbol{\varphi}_s(t) + \boldsymbol{\Delta}]^2\rangle}{\langle(\boldsymbol{\varphi}_s)^2\rangle} \approx \frac{\langle[\boldsymbol{\varphi}_s(t) - \boldsymbol{\varphi}_{sp.w}(t)]^2\rangle}{\langle(\boldsymbol{\varphi}_s)^2\rangle} + f(1,b,c). \quad (36)$$

The first term in Eq. (36) is connected with the effect of the so-called cone anisoplanatism, and the second term is connected with the finiteness of the LGS length.

Since the scanning angle θ of the laser beam is large enough, the linear dimension of the line may be much larger than the correlation length of the wave front tilt fluctuations, then

$$\langle\varphi^2_{s.s}\rangle / \langle\varphi^2_{sp.w}\rangle = f(1,\, b,\, c) \Rightarrow 0 \quad \text{at } b \Rightarrow \infty.$$

The result of suppression of the wave front tilt fluctuations in this case is better than that for any known bistatic optical arrangement. For the optimal correction algorithm, the level of residual fluctuations

$$\beta^2_{min} = \frac{\langle[\boldsymbol{\varphi}_s(t) - A\boldsymbol{\Delta}]^2\rangle}{\langle(\boldsymbol{\varphi}_s)^2\rangle} = 1 - \frac{\langle[\boldsymbol{\varphi}_s(t)\ \boldsymbol{\Delta}]\rangle^2}{\langle(\boldsymbol{\varphi}_s)^2\rangle\,\langle(\boldsymbol{\Delta})^2\rangle} = $$
$$= 1 - \frac{K^2(X)}{1 + f(1,\, b,\, c)}\ , \quad (37)$$

i.e., finally we can obtain additional decrease of the tilt fluctuations.

Let us note here the obvious advantages of the proposed approach:

– there is no need in additional telescopes;

– two laser projectors are mounted together with the telescope thus removing the problem of the joint scanning over sky;

– laser beams are formed beyond the aperture of the main telescope, therefore phosphorescence of the optical channel is absent;

– the signal $\boldsymbol{\Delta} = (\Delta^y, \Delta^z)$ of tilt correction is independent of laser beam parameters;

– dimensions of a laser guide star (angle θ) can be changed by changing only the control voltage of optical deflectors.

The image of the reference cross can be separated into two reference bands in two channels using, for example, orthogonal polarizations of the initial laser beams.

Note also that the jitter of the image of the reference band can be suppressed down to a sufficiently low level,[15,19] if the visible size $a_b > 10^3\, R_m$. Therefore, the field of view of the wave front sensor should be large enough for the whole reference band to be within it, that is,

$$\text{FOV} = \frac{2R_m}{f} > \frac{a_b}{X}\ .$$

Thus, the focal length f in the wave front sensor should be (for $X = 100$ km, $R_m = 4$ m) less than 200 m.

Summarizing the above-said, it should be noted that a sufficiently efficient method for the LGS formation is proposed; this method provides for high suppression of the wave front tilt fluctuations. This method can be used for tilt corrections in actual telescopes operating against a laser guide star.

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