OPTIMIZATION OF THE SCHMIDT SYSTEM BY THE COMPENSATION METHOD

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An algorithm for optimization of the Schmidt high–transmission optical system according to the criterion of minimum wave aberration has been proposed in the paper, which ensures finding of a solution. The optimization results are discussed.

The Schmidt scheme is widely used in lidar optical systems. Its principal peculiarity is a refractive aspherical plate located in the center of curvature of a reflecting spherical surface to make this surface parabolic. Mounting of a thin afocal element in the system pupil does not affect astigmatism and wave curvature of the given system, and the spherical aberration of a basic system can be compensated by adjustment of the spherical aberration of the afocal element. Therefore, one of the main merits of this scheme is its wide field of view.

There are various techniques for calculation of a profile of the correction plate. For example, in Ref. 1 the ray paths for which the spherical aberration are compensated are assigned. The ray paths are defined by the ordinates of the points at which rays intersect the plane perpendicular to the system optical axis tangent to the corrector top and by the angles between the rays and the system axis. These paths determine the system spherical aberration in the angular form. In computational optics, the standard form of planoid surface is described by the equation[2]

$$z = c_1\, y^4 + c_2\, y^6 + \ldots \tag{1}$$

An expression is known[1] which connects a tangent of the angle of normal to the corrector surface G with the coefficients of the planoid surface equation

$$G = 4\, c_1\, y^3 + 6\, c_2\, y^5 + \ldots \tag{2}$$

Knowing the refracted ray angle B and assuming that the angle between the system axis and the incident ray equals zero, in accordance with Eq. (32) from Ref. 1, we have

$$G = -\, N \sin(B)/[\, N_1 - N \cos(B)]\ , \tag{3}$$

where N is the refractive index before the planoid surface, N_1 is the refractive index behind the planoid surface. Using Eqs. (2) and (3) and the ordinates of the points of intersection of the rays and the corrector surface y_i, we obtain the system of equations

$$\begin{cases} G_1 = 4\, c_1\, y_1^3 + 6\, c_2\, y_1^5 + \ldots \\ G_2 = 4\, c_1\, y_2^3 + 6\, c_2\, y_2^5 + \ldots \\ \cdots\cdots\cdots\cdots\cdots \\ G_m = 4\, c_1\, y_m^3 + 6\, c_2\, y_m^5 + \ldots, \end{cases} \tag{4}$$

where m is the number of rays.

If the spherical aberration is corrected for a nearly–parallel beam, the ordinates of the points of intersection of rays with the plane tangent to the corrector top and that of intersection of rays with the corrector surface can be considered equal. Then, having solved system (4), we obtain the coefficients of equation (1). However, in correcting large spherical aberration the ordinates of the points of intersection of rays with the corrector surface cannot be considered known. In this case the system of linear equations (4) is solved by the successive approximation method, and the ordinates of the points of intersection of rays with the plane tangent to the corrector top are considered as initial coordinates. As was noted in Ref. 1, the approximation process converged sufficiently fast.

Note the following shortcomings of this and similar algorithms. First, the form of the surface equation is assumed to be known, i.e., a certain number of the planoid equation coefficients are prescribed. And since it is not known *a priory* how many coefficients are required for correction, their number should be adjusted in the interactive regime. Second, as it is shown below, the values of the high–order coefficients of planoid equation may be as small as 10^{-99} and even smaller, what poses certain computational difficulties. Third, sufficiently large number of iterations is needed for inadequate initial approximation of the points of intersection of rays with the planoid surface.

The proposed method is based on minimization of the wave aberration calculated on the surface of the corrector by which the planoid surface is meant. In this case the wave front of a point monochromatic source can be represented as a surface of equal eikonal,[3] and the wave aberration is found as a difference between the wave fronts of an object and its aberrationless image on the corrector surface.

Eikonal of wave created by the point monochromatic source at a point is

$$F = \sum_i L_i\, N_i\ , \tag{5}$$

where N_i is the refractive index of medium i through which the wave passes a path of length L_i.

So, to satisfy the Fermat principle, the correction surface must convert the source eikonal into the image eikonal.

Using the standard program for calculation of the ray paths through an optical system, the object eikonal $F(x, y, z)$ and the image eikonal $F_m(x, y, z)$ are found,

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where in the general case x, y, and z are the curvilinear coordinates of the points on the corrector surface. The eikonal F_0 corresponding to the wave aberration being compensated can be written in the form

$$F_0 = F_m - F. \tag{6}$$

Phase modulation introduced by the corrector in case of normal light incidence has the form

$$F(x, y, z) = d\,(N - 1), \tag{7}$$

where d is the sag value. Therefore, to compensate for the wave aberration, the sag at the given point must be changed by the value

$$d = F_0/(N - 1). \tag{8}$$

In the general case the optical path length d with the sign is added.

For large values of F_0 the necessity of the iteration process stems from the error of translation of the surface points.

Calculation of the Schmidt high–transmission optical system was done in accordance with the proposed procedure. At the beginning of calculation, the system consisted of a correction plate 10 mm thick located in the curvature center of a spherical mirror with a curvature radius of 4000 mm. Input aperture of the system was 1000 mm. The optimization was carried out for an object located at infinity.

The calculated eikonals on the corrector surface were approximated by cubic splines. In accordance with the above–described procedure, the correction planoid profile on the assigned surface was calculated. After that the residual wave aberration was estimated by direct calculation of ray paths.

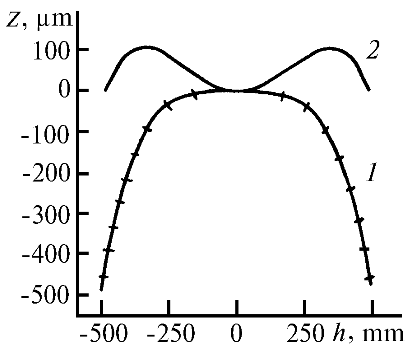

FIG. 1. Correction planoid profile of the Schmidt system: 1) profile of the corrector located in the center of a spherical mirror; 2) profile of the corrector displaced at a distance of 7.5 mm.

The correction required to calculate three ray path iterations. The residual wave aberration and the corrector profile are shown in Figs. 2*a* and 1 (curve *1*), respectively. The correction surface equation has the form

$$z = -\,7.65\ 10^{-21}\ y^4. \tag{9}$$

The wave aberration was 0.09 µm at the aperture edge and smaller than 0.002 µm at the aperture smaller than 400 mm. The calculated wavelength was $\lambda = 0.6328$. The error in determining the surface profile was 0.17 µm or $\approx \lambda/4$ at the aperture edge and smaller than 0.004 µm or $\approx \lambda/160$ at the aperture smaller than 400 mm.

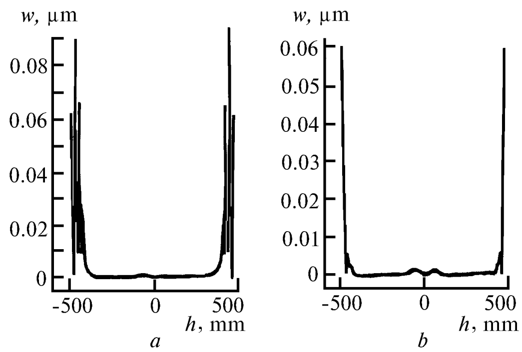

FIG. 2. The residual wave aberration of the Schmidt system for the correctors shown in Fig. 1: a) profile 1 and b) profile 2.

The corrector with such a surface profile is not optimal from the viewpoint of its production technology. In this sense, the corrector is of interest with the correction surface profile and the residual wave aberration shown in Figs. 1 (curve *2*) and 2*b*, respectively. It was obtained with mirror focal plane displacement (defocusing) of 7.5 mm.

The residual wave aberration in all cases did not exceed 0.09 µm at the aperture edge and 0.005 µm on the aperture smaller than 400 mm. The minimum residual aberration was observed for the corrector profile shown in Fig. 2*b*: at the aperture edge it did not exceed 0.06 µm, i.e., the error in determining the surface profile was no more than $\lambda/6$, and for aperture radius smaller than 480 mm the residual aberration did not exceed 0.0038 µm, i.e., the error in determining the surface profile was no more than $\lambda/167$.

Sharply oscillating increase in the error at the aperture edge can be explained by the edge effects of the computational process. These effects should most likely be attributed to errors in calculating the edge ray paths, and more reliable results can be obtained by implementation of the proposed algorithm on larger aperture of the order of 1.3 of the inner diameter to eliminate the edge points.

In lidar optics the diffraction–quality systems, i.e., the systems forming diffraction image of an object, are used. For systems of this type the total tolerable deformation of wave front W_{max} may not exceed $\lambda/4$ in accordance with the Rayleigh criterion. This criterion is successfully applied in the case in which the wave aberration is smooth.[4]

There are three optical surfaces in the Schmidt system: two refracting surfaces and one reflecting surface; therefore, the correction surface must satisfy $\lambda/12$ criterion. Obtained values of the sag for the planoid surface shown in Fig. 1 (curve *2*) were approximated by the least–squares technique. The coefficients of polynomial in the form of Eq. (1) and corresponding error in determining the corrector profile are presented in Table I.

TABLE I. Error in determining the correction surface profile Δd as a function of the degree of approximating polynomial (1).

Degree of polynomial	Planoid equation coefficients					
1	1.78 E – 02	1.52 E – 02	1.24 E – 02	5.17 E – 02	1.29 E – 02	4.14 E – 002
2	1.79 E – 09	1.79 E – 09	1.80 E – 09	1.78 E – 09	1.80 E – 09	1.78 E – 009
3	7.40 E – 21	– 7.53 E – 21	– 8.10 E – 21	– 6.09 E – 21	– 9.26 E – 21	– 5.72 E – 021
4	6.98 E – 34	1.28 E – 33	2.06 E – 32	– 4.88 E – 32	9.13 E – 32	– 1.03 E – 031
5		– 1.13 E – 44	– 2.92 E – 43	8.81 E – 43	– 2.21 E – 42	3.23 E – 042
6		2.17 E – 56	2.01 E – 54	– 8.71 E – 54	2.95 E – 53	– 5.74 E – 053
7			– 6.78 E – 66	4.73 E – 65	– 2.30 E – 64	6.14 E – 064
8			8.86 E – 78	– 1.33 E – 76	1.03 E – 75	– 4.03 E – 075
9				1.50 E – 88	– 2.50 E – 87	1.58 E – 086
10					2.49 E – 99	– 3.41 E – 098
11						3.10 E – 110
Δd, μm	3.98 E – 01	4.08 E – 01	2.96 E – 01	2.08 E – 01	1.16 E – 01	7.36 E – 002
W_{max}	λ/4.8	λ/4.8	λ/6	λ/9	λ/17	λ/26

As one can see from Table I, the Rayleigh criterion is satisfied when ten calculated coefficients are used for approximation of the corrector surface. In this case the error due to the corrector surface does not exceed λ/17, and the imposed requirements are wholly satisfied. The corresponding equation of the correction plate is written in the form

$$z=1.29\cdot 10^{-2}y^2 + 1.80\cdot 10^{-9}y^4 - 9.26\cdot 10^{-21}y^6 + 9.13\cdot 10^{-32}y^8 - 2.21\cdot 10^{-42}y^{10} + 2.95\cdot 10^{-53}y^{12} - 2.30\cdot 10^{-64}y^{14} + 1.03\cdot 10^{-75}y^{16} - 2.50\cdot 10^{-87}y^{18} + 2.49\cdot 10^{-99}y^{20}. \quad (10)$$

Choice of the wave aberration as a parameter to be optimized yields higher quality, simpler procedure of calculation of the correction optical system, essentially smaller computation time, and less stringent requirements for computer resources. The fact that application of the criterion of wave aberration minimum more efficiently optimizes an optical system having small wave aberration was noted already in Ref. 5. As calculation of the Schmidt high–transmission optical system showed, the method is also efficient for large wave aberration.

As distinct from the procedure of finding the equation of aspherical surface described in Ref. 5, the number of the coefficients was not assigned, and calculation was carried out for the corrector surface described by splines. Therefore, the procedure of adjustment of the number of coefficients was excluded, what allowed unattended system optimization process.

Our calculations testify the feasibility of the given method for optimization of the Schmidt high–transmission optical systems having large aberrations. Work is underway on calculation and experimental test of the method for a wider class of optical systems.

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