

POLARIZATION SOUNDING OF HIGH-ALTITUDE AEROSOL FORMATIONS

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The technique and the results of sounding of the crystal clouds using a lidar capable of measuring the Stokes parameters of the scattered radiation are presented. Transmitted laser radiation was linearly polarized. Five combinations of the Stokes parameters were identified against the criterion of their difference from the corresponding values characterizing the molecular scattering. Possible relations between the above-mentioned combinations of the Stokes parameters and either type of the symmetry of the ensemble of the scattering particles are discussed.

INTRODUCTION

Owing to the fact that high clouds have usually small optical thicknesses, the lidars can be successfully used for studying the cloud structure as a function of its thickness and for sounding multilayer cloudiness.^{1,2} An important specific feature of cirrus is a pronounced anisotropy of optical properties of their particles and often of the entire ensemble of the particles. The latter fact can cause the unexpected effects attendant to the propagation of laser beams through cirrus and it is worth of careful study. The optical anisotropy of the ensemble of the ice crystals depends on the modification of their crystallographic structure and spatial orientation. The determination of the above-indicated characteristics is necessary to predict the scattering matrix. Measurement of the Stokes parameters of the radiation being the lidar response function to a transmission of a laser pulse through a cloud provides the good possibilities for solving this problem. In addition to the intensity, measurements of the second Stokes parameter depolarization have been widely used. But these data are sufficient only to confirm the crystal structure of cirrus.

Measurements of all four components of the Stokes vector are much more informative. In the previous papers devoted to these measurements³⁻⁵ it was noted that the anisotropy of the optical properties of aerosol formations was manifested in deviations of the third and fourth Stokes parameters u and v from zero. In the present study the results of recent measurements carried out in 1988 and 1989 are analyzed using the lidar whose parameters were listed in the catalog given in Refs. 6 and 7.

1. THE VECTOR LIDAR EQUATION AND THE PROCEDURE OF MEASURING THE STOKES PARAMETERS

The lidar equation written for the radiation power is easy to generalize for the Stokes parameter vector⁸

$$I(h)\mathbf{s}(h) = 1/2c\Delta t\kappa P_0 T(h)\hat{\beta}(h)\mathbf{s}_0, \quad (1)$$

where $I(h)$ is the intensity of the scattered light as a function of the distance h , P_0 is the power of a laser transmitter; c is the velocity of light, Δt is the laser pulse duration, κ is the transmittance of the lidar optical parts, $T(h)$ is the square transmittance of the path from the lidar

to a scattering volume, $\hat{\beta}(h)$ is the backscattering matrix of this volume, and $S(h)$ and S_0 are the dimensionless column-vectors of the form

$$\mathbf{s}(h) = \begin{pmatrix} 1 \\ q(h) \\ u(h) \\ v(h) \end{pmatrix}, \quad \mathbf{s}_0(h) = \begin{pmatrix} 1 \\ q_0 \\ u_0 \\ v_0 \end{pmatrix},$$

which have the meaning of the Stokes vectors of the scattered and sounding radiation being scaled to the radiation.

The scattered light intensity $I(h)$ is numerically equal to the ratio of the light flux incident on the antenna area to the solid angle $\omega = A/h^2$

$$I(h) = P(h)h^2/A, \quad (2)$$

where A is the antenna area.

A photodetector response is proportional to the light energy received in time $\Delta\tau$ characterized by a recording system. Let us introduce the notation

$$F(h) = mP(h)\Delta\tau h^2, \quad (3)$$

where m is the proportionality factor between the light energy and the photodetector response. On account of Eqs. (2) and (3), we can write Eq. (1) in the following form:

$$F(h)\mathbf{s}(h) = m\kappa\Delta hAE_0T(h)\beta(h)\mathbf{s}_0, \quad (4)$$

where $E_0 = P_0\Delta t$ is the laser pulse energy and $\Delta h = c\Delta\tau/2$ is its spatial length corresponding to the period of integration $\Delta\tau$. Since the measured function $F(h)$ is proportional to the radiation intensity, the left side of Eq. (4) is proportional to the Stokes vector of the scattered radiation.

Let us describe the procedure for determination of the dimensionless Stokes parameters of the scattered radiation. As is well known (see, for example, Ref. 9), in order to determine all Stokes parameters, we must measure the radiation intensity for the six states of the polarization basis. These states are obtained with the help of certain combinations of the positions of the $\lambda/4$ plate and a linear polarizer. Before describing the

procedure for measurements we will recall the form of the operators of an ideal linear polarizer $L(\theta)$ and the $\lambda/4$ phase plate $M(\theta)$ oriented at an angle θ

$$L(0) = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad L(\pm\pi/4) = \frac{1}{2} \begin{pmatrix} 1 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$L(\pi/2) = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M(0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad M(\pm\pi/4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 \\ 0 & 0 & 1 & 0 \\ \pm 1 & 0 & 0 & 0 \end{pmatrix}.$$

The $\lambda/4$ plate and the Wollaston prism were placed one after another along the optical axis of the receiving antenna of our lidar. The units, in which these elements were positioned, provided their rotation of the optical axis at the $\pm\pi/4$ angles. At the same time the unit consisting of the two photodetectors rotated together with the prism. The first beam emanated from the Wollaston prism was the result of action of the operator $L(0)$ while the second – of the operator $L(\pi/2)$. The intensities of these beams were measured for three combinations of relative position of the phase plate and the prism.

The first combination was determined as follows: the fast axis of the $\lambda/4$ phase plate was aligned with the x axis. This direction, for definiteness, was chosen along the normal to the plane containing the two beams emanated from the Wollaston prism. In what follows that the oscillations of the electric vector were parallel to the given direction x selected in one beam and perpendicular to this direction in another beam. The detectors placed on the beam paths measured the quantities which were proportional to the intensities of x and y components of the field oscillations. We assume that a unitary light flux described by the column-vector s is incident on the phase plate positioned in such a way and on the prism positioned immediately behind. Then the Stokes vector of the first beam will be described by the equation

$$s_x = L(0) M(0)s,$$

and of the second

$$s_y = L(\pi/2)M(0)s. \tag{5}$$

In addition the first components of the vectors s_x and s_y will be the intensities I_x and I_y , and in accordance with the above assumption, $I_x + I_y = 1$. Since only the intensities can be measured, we are interested in the following scalar products of vector equations (5):

$$I_x = \frac{1}{2} (1, 1, 0, 0) \begin{pmatrix} 1 \\ q \\ u \\ v \end{pmatrix} = \frac{1}{2} (1 + q);$$

$$I_y = \frac{1}{2} (1, -1, 0, 0) \begin{pmatrix} 1 \\ q \\ u \\ v \end{pmatrix} = \frac{1}{2} (1 - q). \tag{6}$$

From this we find $q = I_x - I_y$. The first rows of the matrices $\hat{K}_x^{(q)} = L(0)M(0)$ and $\hat{K}_y^{(q)} = L(\pi/2)M(0)$ serve as the row-vectors in Eq. (6). Let us refer to these rows as instrument vectors and denote them by $\mathbf{K}_x^{(q)}$ and $\mathbf{K}_y^{(q)}$, where the upper subscript means that this vector is used for determination of the parameter q .

For determining the parameter u , the phase plate and prism are rotated at 45° counterclockwise if one looks counter to the incident beam. It is easy to see that the instrument vectors will have the form

$$\mathbf{K}_x^{(u)} = \frac{1}{2} (1, 0, 1, 0),$$

$$\mathbf{K}_y^{(u)} = \frac{1}{2} (1, 0, -1, 0), \tag{7}$$

and, respectively,

$$u = I_x^{(u)} - I_y^{(u)}.$$

For determining the parameter v , only the phase plate is rotated at 45° , while the position of the prism remain the same as in the case of the determination of the parameter q . The instrument vectors in this case are

$$\mathbf{K}_x^{(v)} = \frac{1}{2} (1, 0, 0, -1);$$

$$\mathbf{K}_y^{(v)} = \frac{1}{2} (1, 0, 0, 1); \tag{8}$$

$$-v = I_x^{(v)} - I_y^{(v)}.$$

Let us recall that $I_x + I_y = 1$ in all three cases, because the ideal measurement procedure is considered. We note that the parameters q and u can be measured without the phase plate. The above-described procedures were chosen in order to avoid the inconvenient procedure of placing and removing the plate as well as to fix the Fresnel reflection from the optical surfaces.

2. THE RELATION BETWEEN THE STOKES PARAMETERS AND THE MEASURED QUANTITIES AND THE COEFFICIENTS OF THE SCATTERING MATRIX

Let us come back to Eq. (4) in order to express the dimensionless Stokes parameters in terms of the quantities being measured in sounding and to determine the relation between the coefficients of the scattering matrix and these parameters. To simplify the formulas, let us introduce the notation

$$C = m\kappa\Delta hAE_0$$

and, for definiteness, fix the dimensionless Stokes vector of laser radiation

$$s_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

as it was in experiments described below.

The parameter q is assumed to be measured. Then two equations follow from Eq. (4) on account of Eq. (6), namely:

$$F_x^{(q)}(h) = \frac{1}{2} (1, 1, 0, 0) \hat{\beta}^{(q)}(h) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot [CT(h)]^{(q)};$$

$$F_y^{(q)}(h) = \frac{1}{2} (1, -1, 0, 0) \hat{\beta}^{(q)}(h) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot [CT(h)]^{(q)}$$

Multiplying the matrix, we have

$$\begin{aligned} F_x^{(q)}(h) &= \frac{1}{2} [CT(h)]^{(q)} \cdot \{[\beta_{11}(h) + \beta_{12}(h)] + \\ &+ [\beta_{21}(h) + \beta_{22}(h)]\}^{(q)}; \\ F_y^{(q)}(h) &= \frac{1}{2} [CT(h)]^{(q)} \cdot \{[\beta_{11}(h) + \beta_{12}(h)] - \\ &- [\beta_{21}(h) + \beta_{22}(h)]\}^{(q)}. \end{aligned} \quad (9)$$

After subtraction and summation of Eqs. (9), we obtain

$$\begin{aligned} [F_x(h) + F_y(h)]^{(q)} &= [CT(h)]^{(q)} \cdot [\beta_{11}(h) + \beta_{12}(h)]^{(q)}; \\ [F_x(h) - F_y(h)]^{(q)} &= [CT(h)]^{(q)} \cdot [\beta_{21}(h) + \beta_{22}(h)]^{(q)} \end{aligned}$$

and after the second equation is scaled to the sum $F_x(h) + F_y(h)$, which is proportional to the total radiation intensity, we have

$$\begin{aligned} [\beta_{11}(h) + \beta_{12}(h)]^{(q)} &= [F_x(h) + F_y(h)]^{(q)} / [CT(h)]^{(q)}; \\ \frac{[\beta_{21}(h) + \beta_{22}(h)]^{(q)}}{[\beta_{11}(h) + \beta_{12}(h)]^{(q)}} &= q. \end{aligned} \quad (10)$$

The first relation determines the backscattering coefficient of a linearly polarized radiation in terms of the measured signal, the instrument constant, and the transmittance of the path. The second formula provides the relation between the dimensionless parameter q and a certain combination of the coefficients of the scattering matrix. In the same way the following two pairs of relations are determined:

$$\begin{aligned} [\beta_{11}(h) + \beta_{12}(h)]^{(u)} &= \frac{F_x(h) + F_y(h)]^{(u)}}{[CT(h)]^{(u)}}, \\ \frac{[\beta_{31}(h) + \beta_{32}(h)]^{(u)}}{[\beta_{11}(h) + \beta_{12}(h)]^{(u)}} &= u; \end{aligned} \quad (11)$$

$$\begin{aligned} [\beta_{11}(h) + \beta_{12}(h)]^{(v)} &= \frac{[F_x(h) + F_y(h)]^{(v)}}{[CT(h)]^{(v)}}, \\ \frac{[\beta_{41}(h) + \beta_{42}(h)]^{(v)}}{[\beta_{11}(h) + \beta_{12}(h)]^{(v)}} &= v. \end{aligned} \quad (12)$$

The superscripts (q) , (u) , and (v) in Eqs. (10–12) enumerate the measuring procedures and imply the fact that the conditions of light passing and scattering as well as the instrument constant C can be changed owing to the instability of laser radiation and the sensitivity of the recording system when the measurements of the Stokes parameters are not coincident in time.

The variability of the optical properties of the scattering volume is the most principal point. Successive measurements of the parameters q , u , and v are based on the use of the following assumption:

$$\frac{\beta_{i1} + \beta_{i2}}{\beta_{11} + \beta_{12}} = \text{const}, \quad i = 1, 2, 3, 4. \quad (13)$$

In other words, a hypothesis is accepted that the micro-physical characteristics are independent of the transformation of the polarization characteristics of radiation. In addition, the total content of the particles is assumed to vary, in such a way that the relative contributions of particles of different shape, and orientation remain unchanged. When Eq. (13) is satisfied, the measured parameters q , u , and v can be considered to belong to a certain averaged ensemble of the particles.

To estimate the absolute values of the combinations of the coefficients of the scattering matrix of the form $\beta_{i1}(h) + \beta_{i2}(h)$, where $i = 1, 2, 3$, and 4, we must know the instrument constant C and the transmittance $T(h)$. The well-known procedure for calibrating according to molecular scattering appears to be appropriate in sounding the optically thin cirrus. The main idea of this procedure consists in following. If the magnitude of the backscattering coefficient $\beta(h_k)$ can be assigned at an altitude h_k from the results of modeling or aerological sounding then the first relation of Eqs. (10) can be written as follows:

$$[\beta_{11}(h) + \beta_{12}(h)]^{(q)} = \beta(h_{kc}) \frac{T^{(q)}(h_k) [F_x(h) + F_y(h)]^{(q)}}{T^{(q)}(h) [F_x(h_k) + F_y(h_k)]^{(q)}}. \quad (14)$$

The algorithm of retrieval of an extinction coefficient profile described in Ref. 10 is used here to find the ratio $T(h_k)/T(h)$. The same relations can be written for Eqs. (11) and (12). We can find the average backscattering coefficient from the results of three measurements

$$\bar{\beta}(h) = \frac{1}{3} \sum_{j=1}^3 [\beta_{11}(h) + \beta_{12}(h)]^{(j)}, \quad j = (q), (u), (v).$$

Taking Eq. (13) into account, we obtain

$$\overline{[\beta_{21}(h) + \beta_{22}(h)]} = q(h) \bar{\beta}(h);$$

$$\overline{[\beta_{31}(h) + \beta_{32}(h)]} = u(h) \bar{\beta}(h);$$

$$\overline{[\beta_{41}(h) + \beta_{42}(h)]} = v(h) \bar{\beta}(h);$$

Thus, the measured dimensionless Stokes parameters are associated with a certain averaged scattering ensemble of the particles.

3. EXPERIMENTAL RESULTS

More than 100 vertical profiles of the Stokes parameters were obtained within the 5–25 km altitude ranges in 1988–1990. In all cases the linear polarized laser radiation was characterized by the column–vector

$$\mathbf{s}_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

The procedure of successive measurements of the parameters q , u , and v was employed. The recording system operated in the photon counting mode. The photocounts were integrated over the series of laser shots. The number of shots in one series varied from 1000 to 2000 depending on a signal level. The time required to measure one Stokes parameter varied from 40 to 80 s at a pulse repetition frequency of 25 Hz. The spatial resolution was 96 m at those altitudes where cirrus were observed. The strobe increased up to 960 m at the altitudes higher than 20 km. A minimum level of the recorded signal was taken to be equal to $1.5 \cdot 10^9$ photocounts in one spatial strobe that corresponds to the relative error less than 3% for the mean rate of the photocounts of the background noise $2-5 \cdot 10^3 \text{ s}^{-1}$. The vertical profiles of the Stokes parameters $q(h)$, $u(h)$, and $v(h)$ and the scattering ratio

$$R(h) = [\beta_a(h) + \beta_m(h)]/\beta_m(h),$$

were calculated from remote sounding data, where $\beta_a(h)$ and $\beta_m(h)$ are the backscattering coefficients of aerosol and molecular atmospheric constituents.

The profiles drawn with a plotter were qualitatively analyzed, and five combinations of the Stokes parameters were identified in the process of light scattering by the aerosol layers. In analysis the Stokes parameter was tentatively assumed to be anomalous if its absolute value differed by 0.1 and more from the absolute value of the corresponding parameter for molecular scattering.

The classification of the scattering patterns made in such a manner is given in Table I, while the examples of the individual realizations are presented in Figs. 1–3.

TABLE I. The obtained combinations of the Stokes parameters (scattering pattern) in sounding by the linear polarized light.

Scattering pattern	The value of the Stokes parameter		
	$ q $	$ u $	$ v $
I	≈ 1	0	0
II	< 1	0	0
III	< 1	> 0	0
IV	< 1	0	> 0
V	< 1	> 0	> 0

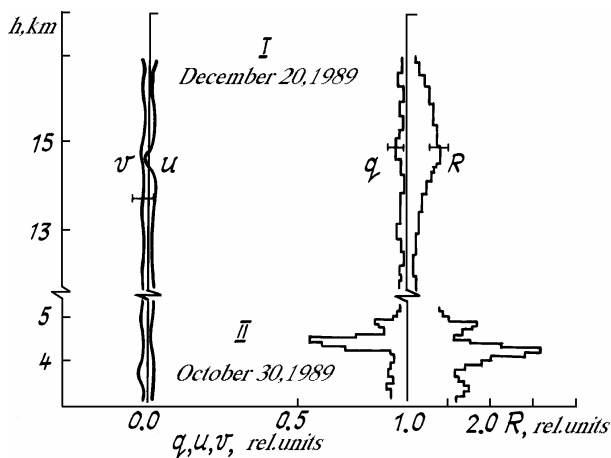


FIG. 1. The vertical profiles of the Stokes parameters q , u , and v and the scattering ratio R for the first and the second scattering patterns.

Figure 1 illustrates first and second scattering pattern. The profiles were obtained on different days. The first case corresponded to the weakly pronounced stratospheric layer ($R = 1.35$), but the similar behavior of the Stokes parameters was also noted at the altitudes 6–7 km in the well-pronounced thin aerosol layers ($R = 8-10$). The second scattering pattern shown in Fig. 1 was typical. Figure 2 shows the case when the parameter u , in addition to the parameter q , had the anomalous value and the parameter v vanished. Figure 3 shows the case in which in one cloud we observed the layers of the fourth and fifth scattering patterns.

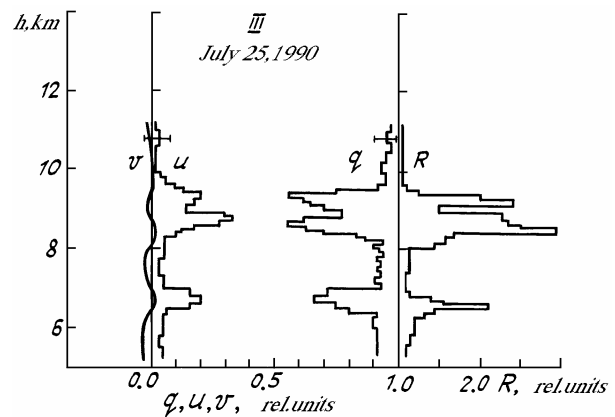


FIG. 2. The Stokes parameters q , u , and v and the scattering ratio R of the aerosol layer characterized by the third scattering pattern.

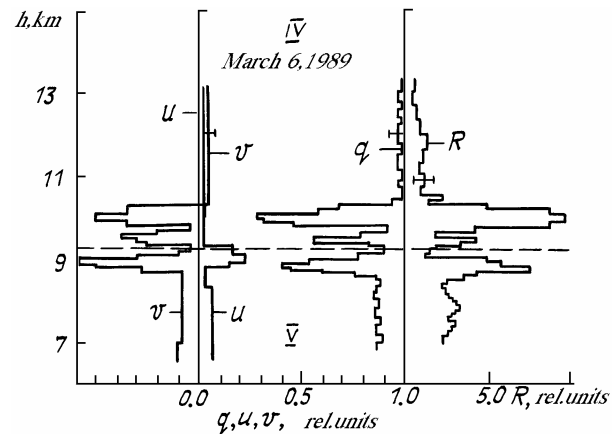


FIG. 3. The Stokes parameters of the fourth (shown above the dashed line) and the fifth (shown under the dashed line) scattering patterns.

Discussion. The obtained results confirm the possibility of existence of a certain preferred orientation of the crystals in the ice clouds. This is indisputable fact, since in the particular cases the orientation is manifested in the anomalous optical phenomena, namely, the mock sun and the light columns. The laser sounding seems to be the perfect instrument to study the phenomena associated with the shape and orientation of the particles. The investigation of even weak manifestations of the crystal clouds with high spatial resolution can be carried out with the help of laser sounding. In addition, there are no principle limitations on the measurements in a day time or at night. If we vary polarization of laser radiation, we can determine the scattering matrix. The latter, as is well known, gives the comprehensive knowledge of the scattering properties of the

ensemble of particles and, hence, of the microphysical characteristics that can be obtained in the experiment on light scattering. However, already the measurements carried out for one state of polarization of laser radiation, whose results are given above, provide grounds for some conclusions about the shape and orientation of particles. For this purpose let us consider the well-known properties of symmetry of the scattering matrix resulting from some properties of symmetry of the ensemble of the scattering particles.¹¹ On the standard assumption that the atmospheric aerosols are the ensembles of the particles which are scattered independently, let us set up several hypotheses and compare them with the result of measurements of the Stokes parameters.

The first scattering pattern. An aerosol is represented by the spherical particles. In this case the result is well known. Only diagonal coefficients of the scattering matrix are nonzero, in addition $\beta_{11} = \beta_{22} = -\beta_{33} = -\beta_{44}$. The polarization state of the scattered radiation remains unchanged if we ignore the inversion of a sign of the third and the fourth Stokes parameters. This is apparently the single case when the parameter q remains unchanged. Therefore, the first scattering pattern in the table can be associated with the ensemble of the spherical particles. However, it should also be kept in mind that the multiple scattering can affect the value of the parameter q . In this case the cloud of the spherical particles will be represented by the second scattering pattern. But, in principle, this case is differed from the case, when the second scattering pattern is caused by the ensemble of the nonspherical particles. For example, the identification can be performed by comparing the rate of growth of the first and the second nonnormalized Stokes parameters.¹² The change of laser polarization will provide the additional possibilities.

The second scattering pattern. An aerosol is represented by the nonspherical particles. Distribution of orientations is such that the entire ensemble is invariant under the arbitrary rotation about the sounding direction. In addition, the principle of specular symmetry about the plane perpendicular to this direction is satisfied. The ensemble, in which the orientations of particles are chaotic or there is the partial orientation of particles in the sounding direction, can meet the enumerated requirements. For example, those are the symmetric crystal flat rosettes whose normals have the preferred orientation along the sensing direction and the random deflections from this direction. The scattering matrix in this case is diagonal with the coefficients $\beta_{11}, \beta_{22} = -\beta_{33}, \beta_{44}$. The second scattering pattern given in the table can be associated most probably with the matrix of similar form. There were experimental evidences of this fact in our first measurements of the total scattering matrix.

The third scattering pattern is characterized by the rotation of the plane of oscillations of the light field vector when the scattered radiation is not elliptic. This can imply that the polarizability tensor of the ensemble of the particles is characterized by the well-pronounced anisotropy, when the polarizability along one direction is much higher than along the others. From the results of simultaneous analysis of the well-known postulates of the light-scattering theory and from the knowledge on the possible shapes of the ice particles we may expect that the scattering volume contains with high probability the partially oriented ice needles. On the basis of this assumption we can indicate the direction of preferred orientation assuming that it coincide with the direction of preferred polarization ϕ . The latter can be found from the well-known relation

$$\psi = \frac{1}{2} \arctan(u/q).$$

The angle ϕ is counted off from the positive direction of the x axis counterclockwise.

The fourth scattering pattern can be associated with the matrix having quite simple and symmetrical form. The nonzero values of this matrix are following: $\beta_{11}, \beta_{22} = -\beta_{33}, \beta_{44}$, and $\beta_{41} = \beta_{14}$. The ensemble of the asymmetrical particles of one type with rotational symmetry about the direction of light propagation is characterized by this matrix. From the well-known shapes of the ice particles parallelepipeds are most suitable for this ensemble.

The fifth scattering pattern represents apparently the case when no assumptions can be made about the symmetry of the ensemble of the particles. For example, such an ensemble could be obtained from the ensemble described in the fourth scattering pattern with the preferred orientation of the plates. Of course, the other variants are also possible, in particular, the ensembles containing the particles of extremely different shapes, in addition it least one type of particles must have the preferred orientation in the plane perpendicular to the direction of sounding. The identification of these nuances will be possible when analyzing the total scattering matrix. This matrix can include up to ten independent parameters and all matrix elements will be nonzero.

In conclusion we note that our assumptions on the form of the scattering matrix discussed above are set up as the probable hypotheses that can be refined after the background of information on the Stokes parameters will be accumulated for four states of polarization of laser beam. Now we have such a confirmation for matrices of the first and second scattering patterns but the possibility exists that the second scattering pattern can be realized for nonzero values of the coefficients β_{12} and β_{21} . As to the possibility of comparing some combinations of the Stokes parameters with one or other types of particles, the exact solution of the problem can be obtained either experimentally or by means of comparing with the collection of the calculated matrix for the modeled ensemble of the particles. Both ways are rather complicated but they can be feasible.

Irrespective of the solution of the foregoing problem, the measurements of the backscattering matrix have the applied significance, since the forward scattering matrix can be predicted on the basis of the same considerations of symmetry and knowledge of the backscattering matrix.

REFERENCES

1. L.V. Kravets, V.N. Marinushkin, and N.D. Smirnov, in: *Radiative Properties of Cirrus*, E.M. Feigel'son, ed. (Nauka, Moscow, 1989) pp. 87–89.
2. C.M.R. Platt, J.C. Scott, and A.C. Diley, *J. Atmos. Sci.* **44**, No. 4, 729–747 (1987).
3. S.R. Pal and A.L. Carswell, *Appl. Opt.* **15**, 1990–1995 (1976).
4. J.P. Houston and A.L. Carswell, *Appl. Opt.* **17**, No. 4, 614–620 (1978).
5. V.E. Zuev, B.V. Kaul', N.V. Kozlov, and I.V. Samokhvalov, in: *Abstracts of Papers at the Ninth International Laser Radar Conference*, Munich, July 1979.
6. *Second International Lidar Researchers Directory*, M.P. McCormick, ed. (Hampton, 1989).
7. B.V. Kaul', O. A. Krasnov, and A.L. Kuznetsov, in: *Data of Complex Experiments "Vertikal'-86"*, ed., G. M. Krekov, Tomsk Research Center, Siberian Branch of the Academy of Science of the USSR, Tomsk, 1989, pp. 49–69.

8. V.E. Zuev, B.V. Kaul', I.V. Samokhvalov, et al., *Laser Sounding of Industrial Aerosols* (Nauka, Novosibirsk, 1986), 185 pp.
9. A. Gerrard and J.M. Burch, *Introduction to Matrix Methods in Optics* (Wiley, New York, 1975). 1978), 341 pp.

10. B.V. Kaul', O.A. Krasnov, A.L. Kuznetsov, and I.Sh. Bikmukhamedov, *Opt. Atm.* **1**, No. 3, 90–97 (1988).
11. H.C. van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957). 536 pp.
12. I.V. Samokhvalov and V.S. Shamanaev, *Izv. Akad. Nauk SSSR, Ser. FAO* **18**, No. 10, 1050–1056 (1982).