ON THE ERRORS IN TESTING OPTICAL COMPONENTS BY RONCHI'S METHOD

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The sensitivity of Ronchi's method to wavefront errors is analyzed. Relations demonstrating that Ronchi's method with visual reading of the shadow pattern does not allow testing of optical components of the highest quality are presented. The sensitivity of Ronchi's null-test method to the position of the screen and the plane of observation of the shadow pattern is evaluated.

To evaluate the quality of an optical system it is very often necessary to have an efficient testing method that does not require cumbersome equipment and gives results that are simple to interpret. Any experimenter attempting to assemble appropriate components for a prototype of an optical system as well as opticians also encounter a similar situation.

Ronchi's method[1] completely meets these requirements. In addition, there exists a series of modifications that make it possible to use this method for testing both spherical (Ronchi grating method) and aspherical (Ronchi null-test or curvilinear screen method) wavefronts. In spite of the fact that Ronchi's method is widely employed complete information about its sensitivity to the parameters and its stability is not available in literature. In this paper the possibilities of Ronchi's method are evaluated.

The essential point of Ronchi's method are illustrated in Fig. 1.

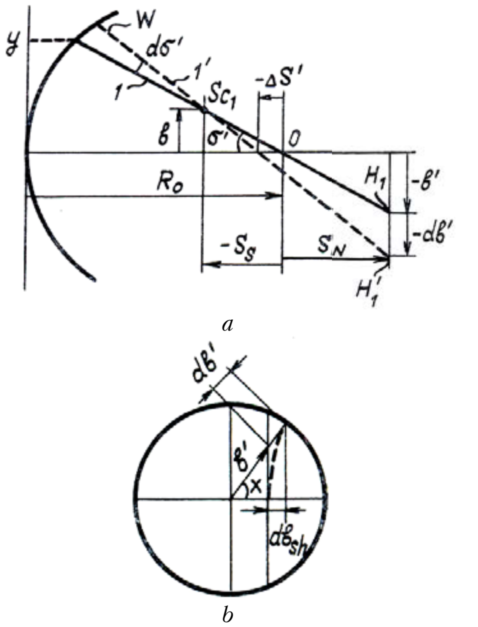

FIG. 1. Essential points of Ronchi method: a) testing scheme employed in Ronchi's method; b) curvature of the fringes of the shadow pattern.

Let the point O be the focal point of the wavefront W. .A Ronchi screen is placed in the path of the pencil of rays at a distance S_s from the focal point O. A shadow pattern representing the projection image of the screen is observed in the plane lying at a distance S_{ob} from the point O. If the tested wavefront is spherical, then Ronchi's screen consists of a grating of straight parallel lines, and the projection image of this grating will also consist of straight parallel lines. If the lines of the shadow are curved in the projection image, then the wavefront W is not spherical – it exhibits wave aberration N_y, as a result of which the zones y of the wavefront have different focal points O_y, i.e., longitudinal spherical aberration $\Delta S'(y)$ exists. The magnitude of the curvature is related with $\Delta S'(y)$ and therefore with $N(y)$.

In testing aspherical wavefronts the lines of Ronchi screen (curvilinear screen) are curved in a manner such that the projection image in the plane of observation of the shadow pattern related with $\Delta S'(y)$ and in this sense the curvilinear screen is called in the foreign literature a Ronchi null test. If the lines of the shadow pattern in the projection image of the null test are curved, then the wavefront does not have the normal form, i.e., the aspherical front exhibits a wave aberration.

It is obvious from the foregoing discussion that both modifications of Ronchi's method must be equally sensitive to wave aberration $N(y)$ when the shadow pattern is observed under identical conditions. But the stability to the conditions of observations (i.e., to errors in positioning S_s and S_{ob}) will be different, since for a grating the error in positioning the screen and the plane of observation of the shadow pattern is of no significance, while for a curvilinear screen this will nullify the compensation of the curvature and produce curvature of the shadow pattern, which can be attributed to aberrations $N(y)$ and will affect the results of the tests, image, which, in turn, can be assigned to be aberration $N(y)$ worsening, as a result, the control quality.

Let the ray 1 project the point S_1 of the screen, located at a height b into the point H_1, located at a height b' in the plane of the shadow pattern.

If the wavefront W has wave aberration N, then the ray 1 will no longer pass through the point S_1;

instead some ray 1′ lying quite close to ray 1 (for sufficiently small wave aberrations) and crossing the optical axis not at the point 0 but at some other point displaced from it by the magnitude of the longitudinal spherical aberration $\Delta S'$ will pass through S_1. Based on Fig. 1, we can write

$$-S_s = b/\tan\sigma'$$

$$-b' = (-S_s + S_{ob})\tan\sigma' - b, \qquad (1)$$

where σ′ is the aperture angle of the zone through which ray 1 passes. Differentiating these relations gives

$$\Delta S' = -dS_s = -\frac{b d\sigma'}{\sin^2\sigma'}; \qquad (2)$$

$$db' = \frac{S_{ob} - S_s}{\cos^2\sigma'} d\sigma' \qquad (3)$$

Equations (1), (2), and (3) give

$$db' = \Delta S' \tan\sigma' \left(\frac{S_{ob}}{S_s} - 1\right) \qquad (4)$$

The relation (4) makes it possible to determine db' — the deviation of the shadow from a straight line.

Let us assume that the wave aberration N is small and therefore[2] the relation

$$\Delta S' = \frac{\partial(N)}{\partial y} \frac{R}{\tan\sigma'}, \qquad (5)$$

where R is the radius of curvature of the wavefront, can be employed.

From (4) and (5) we obtain

$$db' = \frac{\partial(N)}{\partial y} R \left(\frac{S_{ob}}{S_s} - 1\right). \qquad (6)$$

If

$$N(y) = \frac{A_0}{2}\left[1 + \cos 2\pi \frac{y - y_0}{T}\right],$$

where A_0 is the amplitude, y_0 is the coordinate of the top, and T is the period of the detect in the wavefront, then

$$\left(\frac{\partial N}{\partial y}\right)_{max} = \frac{\pi A_0}{T}$$

From here the maximum deviation of the shadow from a straight line is given by

$$db' = \left[\frac{\pi A_0}{T} R_0 \left(\frac{S_{ob}}{S_s} - 1\right)\right] \qquad (7)$$

The projection of the curvature of the line $db'T$ on the horizontal axis (see Fig. 1b) can be determined from the relation

$$db'_T = db' \cos\chi,$$

Since maximum curvature is observed for short lines, close to the edge of the component, $\cos\chi \approx 1$, whence

$$db' \approx db'_T.$$

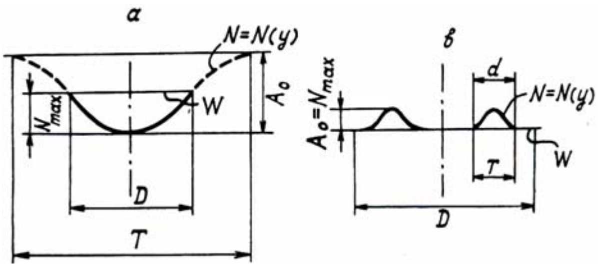

FIG. 2. Diagrams elucidating the relations (8)(a) and (9)(b)

It should be noted that when the relation (7) is used to evaluate the wavefront errors (see Fig. 2) the period should be taken as follows:

$$T = \begin{cases} 2D & \text{for the resultant error} \\ d & \text{for the zonal error} \end{cases} \qquad (8)$$

(The total error is the low-frequency component of the wave aberration, comparable to the wave diameter, and the zonal error is the high-frequency component).

$$A_0 = \begin{cases} 2N_{max} & \text{for the resultant error} \\ N_{max} & \text{for the zonal error} \end{cases} \qquad (9)$$

where D is the light diameter of the tested wavefront, d is the size of the zone in which the curvature of the line is observed, and N_{max} is the maximum deviation of the wavefront from the nominal front.

To check the relation (7) a series of model calculations was performed using the "EKRAN-U" program[3].

The total error was modeled on a spherical mirror with the parameters D/R_0 = 250 mm/1000 mm. The coordinates of the lines of the shadow pattern were calculated and the absolute magnitude of the projection db'_T of the shadow line from a straight line was calculated by varying the eccentricity. The following formula was employed to calculate N_{max}:

$$N_{max} = 2\varepsilon^2 \left(\frac{H^2}{2R_0}\right) / 2R_0 \quad \text{(Ref. 4)}$$

The quantity db' was calculated using the formula (7) under the assumption that the error is cosinusoidal.

The results are given in Table 1.

TABLE 1.

Results of the calculation of the absolute curvature of the shadow pattern for a total error N_{max} using the EKRAN-U – db'_T and the relation (7) for a spherical mirror db'_{max}.

ε	0.25	0.5	1.0	1.5
N_{max} (mkm)	4	17	68	85
$db'_{T max}$ (mm)	0.14	0.51	2.1	4.2
db'_{max} (mm)	0.093	0.36	1.44	4.2

Comparing the last two rows shows that the agreement is good. The discrepancies are not very significant and can be neglected ((7) gives an error of not more than 30%). The differences arise for the following reasons:

– the change in the profile is governed by different laws, in one case by a quadratic polynomial and in the other case by a cosinusoidal law;

– db' is the deviation of the shadow in the radial direction, while db'_T is the projection of the deviation on an axis perpendicular to the lines of the grating.

For visual monitoring of the shadow pattern the sensitivity of the method is limited by the capability of the human eye to distinguish the curvature of a line. By analogy to the distortion we shall take for the limit of sensitivity of the method[5] the error that give to the relative curvature

$$\frac{db'_{max}}{b'} = 3\% \qquad (10)$$

Since

$$b' = -S_{ob} \times \tan\delta'$$

using the relation (7) we can write

$$\frac{db'_{max}}{b} = \left| \frac{\pi A_0 R_0}{T \operatorname{tg} \sigma'} \left(\frac{1}{S_s} - \frac{1}{S_{ob}} \right) \right|. \qquad (11)$$

Therefore according to (8), (9), and (11) the maximum distinguishable total error can be determined from the relation

$$A_0^0 = \left| \frac{0.03}{2\pi} \left(\frac{D}{R_0} \right) \frac{S_{ob} S_s}{S_{ob} - S_s} \right|. \qquad (12)$$

For Ronchi tests the shadow pattern can be formed on the retina of the eye. In the process the crystalline lens is positioned at the paraxial focal point and is accommodated to the component being tested. Therefore $S_{ob} = -R_0$.

For this case the formula (12) is transformed into the relation

$$A_0^0 = \left| \frac{0.03}{2\pi} \left(\frac{D}{R_0} \right) S_s \right|. \qquad (13)$$

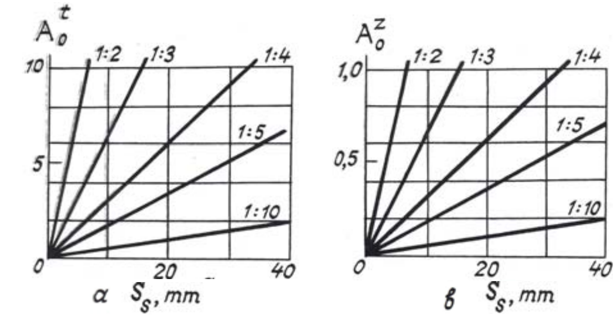

FIG. 3. The maximum distinguishable wavefront error (the total error A_0^0 (a) and the zonal error A_0^z (b)) versus the relative opening of the component and the position of the screen S_s for visual checking with the eye accommodated to the component.

Figure 3a presents graphs of the dependence of the maximum distinguishable total error. $A_0^0 = A_0(S_s)$ for different relative openings.

The maximum distinguishable total error for the case presented in Table 1 ($D/R_0 = 250/1000$; $S_s = -40$, and $S_{ob} = 70$) is

$$A_0 \approx 8 \text{ mkm}$$

and therefore the maximum distinguishable eccentricity $\varepsilon \approx 0.35$.

We shall evaluate the possibilities of Ronchi's method for checking the zonal and local errors. According to Ref. 6 the limit of the zonal errors can be set at $T = D/10$. For this error, which is admissible at the edge of the wavefront, i.e., $\tan \sigma' = \delta/2R_0$, we can write from (10) and (11)

$$A_0^0 = \left| \frac{0.03}{2\pi} \left(\frac{D}{R_0} \right)^2 10^{-1} \frac{S_{ob} S_s}{S_{ob} - S_s} \right|. \qquad (14)$$

The relation (14) gives the distinguishable zonal error with period $T = D/10$.

For visual monitoring with the eye adjusted on the component the maximum distinguishable zonal error is

$$A_0^0 = \left| \frac{0.03}{2\pi} \left(\frac{D}{R_0} \right)^2 10^{-1} S_s \right|. \qquad (15)$$

Figure 3b shows a graph of the dependence of the maximum distinguishable zonal error $A_0^z = A_0(S_s)$ for different relative openings.

Thus, unlike Ref. 7, based on what was said above it can be concluded that Ronchi's method with visual reading can hardly be regarded as a high-quality method in the sense of Rayleigh's criterion, since it is substantially limited by the capabilities of the human eye.

The sensitivity of Ronchi's method can, however, is considerably increased by increasing the accuracy with which the curvature of a fringe is measured. Of course, in the process the shadow pattern must be analyzed photoelectrically. Such a device must be equipped with a distortion-free projection objective, forming an image of the shadow pattern in the plane of the photodetecting element. The distortion of the objective must be corrected (or at least measured) with an accuracy determined by the sensitivity of the apparatus to the curvature of the fringes of the shadow pattern.

Let us assume that we have a device that permits determining the curvature of a fringe with a total error not exceeding

$$\frac{db'_{max}}{b'} = 0.1\%$$

Therefore for an image of the shadow pattern with radius $b' = 8$ mkm the resolution of the photodetector must not be worse than $db' = 8$ μm.

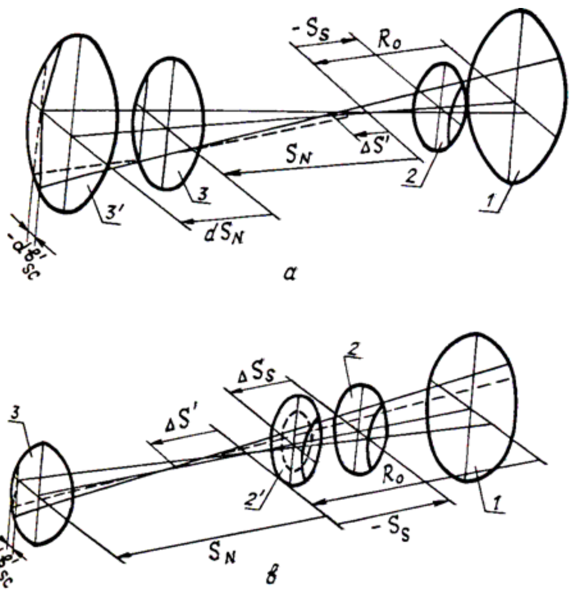

FIG. 4. The sensitivity of the testing scheme to displacements of the observation plane (a) and curvilinear screen (b). 1) tested component; 2) curvilinear screen; 2′) displaced curvilinear screen; 3) plane of compensation of the curvature of the screen; 3′) observation plane.

If $S_s = 4$ mm, and $S_T = -R_0$, for the component D/R_0, then the maximum distinguishable total error in this case, according to (13), is $A_0^0 = 0.08$ μm, which is better than Rayleigh's criterion. To determine the stability of the Ronchi null-test method with respect to the placement of the observation plane we turn to Fig. 4a.

It is not difficult to see from Fig. 4a that when the observation plane is displaced by an amount dS_{ob} the curvature of the shadow (breakdown of the compensation of the curvature) will be given by

$$-db'_T = \Delta S' \frac{D}{2R_0} \frac{dS_{ob}}{S_{ob}}$$

where $\Delta S'$ is the longitudinal aberration of the rays passing through the central line of the curvilinear screen and through its edge. When the curvature of the screen is large it can be assumed that $\Delta S'$ is the magnitude of the longitudinal aberration of the wavefront formed by the component being checked.

Therefore; when the observation plane is displaced by an amount dS_{ob} the relative curvature of the line, can be: determined from the relation

$$\frac{db''}{b''} \approx \frac{db''_T}{b''} = -\frac{\Delta S'' \frac{D}{2R_0} \frac{dS_{ob}}{S_{ob}}}{D/2R_0 \; S_{ob}} = \left| \frac{\Delta S' dS_{ob}}{S_{ob}^2} \right| \qquad (16)$$

In testing' a second-order aspherical mirror[7] the formula

$$\Delta S'' = \varepsilon^2 \frac{(D/2)^2}{2R_0},$$

holds, whence

$$\frac{db'}{b'} = \frac{dS_{ob}}{S_{ob}^2} \varepsilon^2 \frac{(D/2)^2}{2R_0}.$$

Starting from, the relation (10) we obtain, the admissible displacement of the observation plane when testing aspherical mirrors

$$\delta S_{ob} = 3\times10^{-2} \frac{2R_0 S_{ob}^2}{(D/2)^2 \varepsilon^2}.$$

As an example we shall determine dS_{ob} for a parabola $D/R_0 = 250/1000$ with $S_{ob} = -R_0$,

$$\delta S_{ob} \approx 4000 \text{ mm}$$

The testing scheme is more sensitive to small values of S_{ob}. Thus, for example, for the same parabola, with $S_{ob} = 70$ mm

$\delta S_{ob} = 24$ mm

It is obvious from Fig. 4b, that the compensation of the curvature of the lines of the shadow pattern in the observation plane of the pattern indeed breaks down when the curvilinear screen is displaced.

We can write the quite obvious expression

$$db' \approx \delta S_s D/2R_0 \frac{S_{ob} - \Delta S'}{-S_s + \Delta S'},$$

whence

$$\frac{db'}{b'} = \left| \delta S_s \left(1 - \frac{\Delta S'}{S_{ob}}\right) \frac{1}{-S_{ob} + \Delta S'} \right| .$$

For $S_{ob} \gg \Delta S'$, which, as a rule, holds, and from the relation (10) the admissible displacement of the screen is given by

$$\delta S_s = 3 \times 10^{-2} (-S_s + \Delta S') . \quad (17)$$

For a parabola $D/R_0 = 250/1000$, $S_s = -40$ mm, and $\Delta S' = 15.6$ mm

$\delta S_s = 1.6$ mm

The error in matching the light diameter of a curvilinear screen with the boundary of the light cone of the tested wavefront has the form

$$db' = \delta S_s \times \frac{D}{2R_0} = 0.2 \text{ mm}$$

This can be achieved even by visually checking the match.

For an apparatus that permits quantitative Ronchi testing the errors in placement of the screen and the observation plane δS_s and δS_{ob}, must not exceed the following values:

$$\delta S_s = db' \left(\frac{D}{2R_0}\right)^{-1}$$

$$\delta S_s = db' \frac{S_s}{S_{ob}} \left(\frac{D}{2R_0}\right)^{-1} . \quad (18)$$

The relations (18) can be easily obtained from Fig. 4 and they give the accuracy with which the screen and observation plane must be positioned if the maximum curvature of the fringes distinguishable by the apparatus is db'/b', which gives a testing error whose amplitude does not exceed A_0.

Thus, for example, for $db'/b' = 0.1\%$ and the parabola $D/R_0 = 250$ mm/1000 mm, $S_s = 4$ mm, and $S_T = -R_0$ the total testing error $A_0 < 0.08$ μm.

For this the position of the curvilinear screen and observation plane of the shadow pattern must be determined with an error not exceeding

$\delta S_{ob} = 1$ mm; $\delta S_s = 1$ μm

This is already a quite stringent restriction, which is difficult to satisfy.

Thus the following conclusions can be drawn from the foregoing discussion:

– the sensitivity of Ronchi's method depends strongly on the testing scheme, the parameters of the tested component, and the quality of the photodetector apparatus and can be evaluated using the relations (12), (16), and (17);

– qualitative Ronchi testing is quite rough and can be employed only at the initial stages of components fabrication.

– quantitative Ronchi testing can be performed and can satisfy Rayleigh's criterion only if the following are employed: a high-quality photodetector, which makes it possible to transfer the curvature of the fringes of the shadow image not exceeding ~ 0.1%; processing devices with a high resolution of ~ 8 mkm; and, equipment enabling accurate of the Ronchi screen.

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