METHODS FOR COMPENSATING THERMAL DEFORMATIONS OF MIRRORS IN UNSTABLE RESONATORS WITH THE HELP OF CONTROLLABLE OPTICAL ELEMENTS

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Received October 3, 1988

The problem of optimizing the optical design of a telescopic resonator when the mirrors are subject to thermal deformation is studied. It is proposed that a mirror with a precomputed aspherical surface be inserted into the resonator in order to correct for the divergence of the output radiation. It is shown that relatively simple methods based on the geometric optics approximation can be effectively employed to calculate the correcting profile.

Thermal deformations of resonators mirrors are one of the principal causes for the difficulty of obtaining laser radiation with high output power and small angular divergence[1,2]. They can be reduced by choosing appropriate materials for the mirrors, improving the technology employed for working the mirror surfaces, and using different cooling systems. Such "passive" methods cannot, however, remove thermal deformations completely.

It is thus of interest to study active methods for interactivity correction. In one such method aspherical mirrors, which are designed taking into account the thermal deformations produced by a given radiant load, are inserted into the resonator. In particular, a mirror with a controllable surface profile can be employed to correct for thermal distortions[3]. The aspherical surface profile of such a mirror is formed by applying precomputed controlling voltages to its drives.

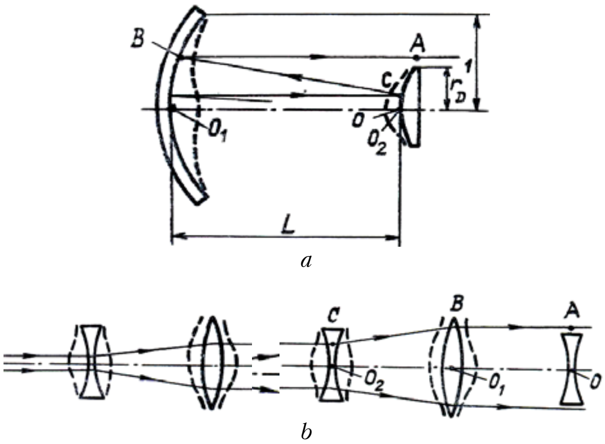

FIG. 1.

In this paper we shall discuss methods for calculating aspherical optical elements to be used to correction for the thermal deformations of resonator mirrors.

1. Consider an unstable telescopic resonator whose concave mirror is a correcting mirror (Fig. 1a). We shall analyze the axisymetric case. We shall represent in the following form the phase shift acquired by the wave on reflection from the concave mirror:

$$\varphi(r)=\varphi_0(r)+\varphi_c(r)+u(r) \tag{1}$$

where $\varphi_0(r)$ is the regular phase corresponding to the undisturbed spherical profile of the mirror, the phase profile $\varphi_c(r)$ takes into account the distortions of the reflecting surface owing to heating, and $u(r)$ is the correcting phase component which we shall represent in the form of a power series:

$$u(r)=\sum_{n=0}^{\infty} c_n r^{2n}. \tag{2}$$

The problem of compensating for the thermal distortions consists of determining the aberrational coefficients c_n that give the minimum divergence of radiation.

In Ref. 2 the aberrational coefficients were computed by gradient methods, which have been successfully employed for solving linear optimization problems[3]. In application to the nonlinear case under study here gradient methods require repeatedly solving the self-consistent problem of the propagation of radiation in the resonator taking into account the thermal deformations of the resonator mirrors, which presents considerable computational difficulties.

In this connection it is important to develop simpler, semi analytical methods for calculating the aspherical profile of the correcting mirror of a resonator.

2. We shall find the distribution of the phase of the radiation in the output plane of a telescopic

resonator (Fig. 1a) which has a magnification M and whose mirrors are deformed owing to heating. For this we shall employ the well-known approach of Ref. 1, based on the geometric-optics approximation. The main assumption is as follows. The ray paths in a resonator with deformed mirrors is identical to that in an ideal resonator. Distortions of the mirror surfaces merely change the optical path length of the corresponding rays (this is true for small deformations).

Because of the presence of thermal deformations of the mirrors the optical path length will be different for different rays in the resonator. As a result the phase front $\Phi(r)$ in the output plane will not be planar, and this difference will equal the difference of the optical path lengths for rays with different coordinates r in the output plane.

Figure 1b shows an optical line equivalent to the telescopic resonator. The resonator mirrors correspond to converging and diverging lenses, and the thermal deformations of the mirrors deformations of the mirrors are modeled by thin phase screens placed near their surfaces. Each cell of the equivalent scheme corresponds to one complete pass of the radiation in thro resonator. We denote by r_D and r_C the radii of the diverging and converging mirrors and by $w_D(r)$ and $w_C(r)$ the displacements of the surfaces of the mirrors owing to heating. We shall represent the thermal deformations in the form of power series:

$$w_D(r)=\sum_{n=0}^{\infty} a_n r^{2n}.$$
$$w_C(r)=\sum_{n=0}^{\infty} b_n r^{2n}. \quad (3)$$

We shall find the difference of the path length between the arbitrary ray OO_1O_2 ..., passing along the axis of the resonator. If the coordinate r of the ray ABC... falls within the diverging mirror [O, r_2], then the difference of the optical path lengths of these rays in one period of the equivalent scheme can be written as follows:

$$\Delta L_I(r)=2(w_D(0)+w_C(0)+u(0))-w_D(r)-2w_C(r)-w_D\left(\frac{r}{M}\right). \quad (4)$$

In the case when the coordinate r lies in the output aperture $]r_D$, 1], the corresponding difference of the path, lengths can be expressed by the relation

$$\Delta L_{II}(r)=2(w_D(0)+w_C(0)+u(0))-2w_C(r)-w_D\left(\frac{r}{M}\right). \quad (5)$$

The expressions (4) and (5) for the difference of the optical path lengths accumulating over one complete passage through the resonator can be represented, substituting (2) and (3), in the form of the following power series:

$$\Delta L_I(r)=\sum_{n=0}^{\infty} \beta_n^I r^{2n}. \quad (6)$$

where $\beta_n^I = -a_n(1+1/M^{2n})-2(b_n+c_n)$, and, correspondingly,

$$\Delta L_{II}(r)=\sum_{n=0}^{\infty} \beta_n^{II} r^{2n}. \quad (7)$$

where $\beta_0^{II}=a_0$; $\beta_n^{II}=-a_n/M^{2n}-2(b_n+c_n)$.

After one complete pass through the resonator the coordinate of the ray ABC... will be r/M. To calculate the path difference in the next sections of the equivalent scheme we can employ the formulas (6) and (7), replacing in them the coordinate r by r/M, r/M^2, r/M^3, etc. Since no ray passes within the output aperture $]r_D$, 1] twice the total difference of the optical paths $\Delta(r)$ of the rays with different coordinates in the output plane can be calculated as follows:

$$\Delta(r)=\begin{cases}\Delta L_I(r)+\Delta L_I(r/M)+\Delta L_I(r/M^2)+..., & r\in[0,\ r_D] \\ \Delta L_{II}(r)+\Delta L_I(r/M)+\Delta L_I(r/M^2)+..., & r\in]r_D,\ 1].\end{cases} \quad (8)$$

Substituting the series (6) and (7) into this expression, and summing the corresponding series gives

$$\Delta(r)=\begin{cases}\sum_{n=1}^{\infty}\frac{M^{2n}}{M^{2n}-1}\cdot\beta_n^I\cdot r^{2n}, & r\in[0,\ r_D] \quad (9a) \\ -\sum_{n=1}^{\infty}\frac{2M^{2n}}{M^{2n}-1}\cdot[a_n/M^{2n}+b_n+c_n]\,r^{2n}, & r\in]r_D,\ 1] \quad (9б)\end{cases}$$

The expression (9) describes the spatial profile of the wavefront of the output radiation of a thermally deformed resonator.

In an ideal telescopic resonator $\Delta(r)$ = const. We shall determine the values of the coefficients c_n for which this equality holds the output aperture $]r_D$, 1] of the resonator with deformed mirrors.

One solution can be obtained by equating to zero the coefficients in the power series (9b). The aberration coefficients c_n sought are then related with the coefficients in the power series for thermal deformations by the simple relation

$$c_n=-a_n/M^{2n}-b_n \quad (10)$$

The rest of the problem is solved by numerical methods.

3. The numerical experiment consists of the following.

As the first step the self-consistent problem of the propagation of radiation in the resonator is solved taking into account the thermal distortions of the resonator mirrors. The method employed to solve the self-consistent problem is studied in detail in Ref. 2 and consists of calculating the distributions of the complex amplitude of the field and the corresponding profiles of the thermal deformations of the resonator mirrors. The calculation of the field at each stage is performed by the method of establishment using the Fox-Lee iteration procedure[1].

The thermal deformations are determined using the semianalytical computational method proposed in Ref. 5. The mirrors are represented in the form of flat circular plates. It is assumed that the temperature of the cylindrical surfaces of the mirrors is maintained constant. The method is based on the combined use of the stationary, uncoupled theory of thermoelasticity of solids and the theory of bending of thin plates. The result of solving the self-consistent problem is the distribution of the stationary field in the resonator with thermal deformations and the profiles of the surfaces of the deformed mirrors.

The second step of the numerical experiment consists of approximation the thermal deformations found $w_D(r)$ and $w_C(r)$ by power-law functions. This is done by the method of least squares. We shall assume that the form of the correcting aspherical mirror can differ from the initial spherical form by the presence of two types of aberrations, corresponding to defocusing $(S_1(r) = \sqrt{3}(2r^2 - 1)$ and spherical aberration $(S_2(r) = \sqrt{5}(6r^4 - 6r^2 + 1)$. Then only three terms need be retained in the representations (2) and (3) for the correcting component and the thermal distortions: $n = 0$, 1, and 2.

At the third stage the coefficients of the power series (2), guaranteeing that within the output aperture the phase of output radiation remains constant ($\Delta(r)$ = const), are determined from the formula (10). The corresponding profile of the correcting mirror is calculated.

The final stage of the numerical experiment consists of solving the self-consistent problem of the propagation of radiation in a resonator whose concave mirror has the form calculated in order to compensate for wavefront distortions introduced by the thermal deformations of the mirrors.

4. The problem of optimizing the divergence of the output radiation was solved for a telescopic resonator with magnification $M = 2.5$ and equivalent Fresnel number $N_e = 5.5$. The radius and thickness of the output mirror $r_0^D = 1.25$ cm and $z_0^D = 0.5$ cm. The dimensions of the concave aspherical mirror $r_0^C = 5$ cm and $z_0^C = 0.5$ cm. The mirrors were made of copper and had reflection coefficients of 98.5%.

The investigations performed revealed that the use of an aspherical mirror, whose form is calculated using the coefficients in the approximation of the exact distributions $w_D(r)$ and $w_c(r)$, obtained from the solution of the self-consistent problem, in the resonator permits virtually complete elimination of the effect of thermal distortions on the spatial structure of the output radiation.

Figure 2 shows the distribution of the phase of the radiation in the output plane of the resonator. The existence of thermal deformations of the resonator mirrors leads to significant distortions of the wavefront of the output radiation. This is confirmed by numerical analysis curve 2). The phase distribution for a resonator with a correcting mirror is virtually identical to the corresponding distribution for an undeformed resonator (curve 1).

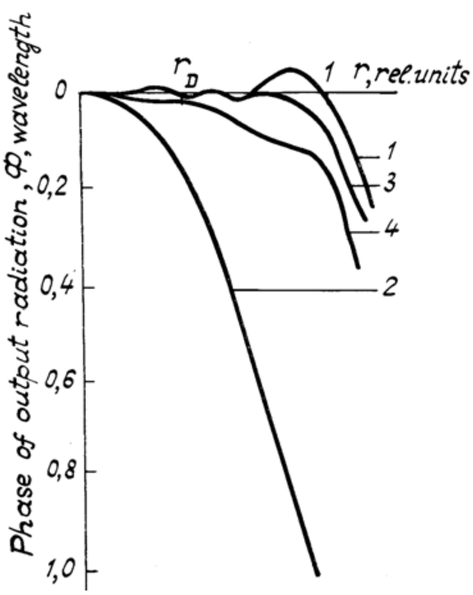

FIG. 2.

We shall employ the Strel number St (the ratio of the intensity at the focal point of the lens positioned at the output of the deformed resonator to the corresponding value for the undisturbed resonator) to evaluate the divergence of the output radiation. In our case St equals 0.065 in the case of a resonator with thermal distortions without compensation and 1.02 with compensation.

The shape of the aspherical mirror was determined using the distribution of the thermal deformations of the mirrors determined by solving the self-consistent problem. In practice it is quite difficult to calculate or measure the thermal deformations. In this connection the approach to the calculation of the form of the corrector can be modified somewhat by concentrating on data accessible in practice. We shall study two variants.

In the first case only one distribution of the intensity in the plane in front of the output mirror is employed. Then, taking into account the geometry of the resonator, the intensity distribution in the plane in front of the converging mirror of the resonator is determined and the thermal deformations of the mirrors, which, generally speaking, differ from $w_D(r)$ and $w_C(r)$, obtained by solving the self-consistent problem, are calculated. All further calculations are performed by the scheme described above. Curve 3 in Fig. 2 characterizes the distribution of the phase corresponding to a resonator the shape of whose aspherical mirror was chosen by the described method. In this case St = 0.98.

In the second case only the average intensity within the output aperture is calculated. It is assumed that the diverging mirror is illuminated uniformly with this intensity. The corresponding intensity in the plane of the correcting mirror is found, and the distributions of the thermal deformations of the mirrors are calculated. In spite of the significant simplifications in calculating the profile of the aspherical mirror St for the output radiation can be increased up to 0.93. The phase distribution in the output plane is shown in Fig. 2 (curve 4).

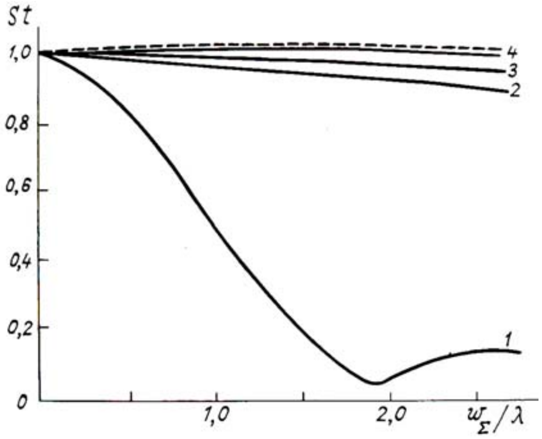

FIG. 3.

Figure 3 shows the dependence of St on the total maximum deflection of the mirrors $w_\Sigma(0) = w_D(0) + w_C(0)$, caused by deformation. The quantity w_Σ characterizes the radiant load, which can be calculated from the given value of the output radiation power. Curve 1 in Fig. 3 corresponds to the standard resonator with thermally deformed mirrors. Curves 2, 3, and 4 illustrate the respective results of optimization of the shape of the concave mirror for the three methods of calculating the correcting profile studied above. The broken line shows the dependence $\mathrm{St}(w_\Sigma)$ determined using gradient methods of optimization.

Table 1 gives some results in order to compare the effectiveness of different methods for calculating correcting mirrors.

The dependences presented were obtained neglecting the saturation of the gain of the active medium. As was checked with the help of a numerical experiment, taking these effects into account does not significantly change the results.

Thus our investigation showed that inserting an aspherical mirror into the resonator enables eliminating practically completely the effect of thermal distortions on the spatial structure of the output radiation and reducing its divergence virtually to zero. It was shown that the correcting profile can be calculated efficiently using relatively simple methods based on the geometric-optics approximation.

TABLE 1.

MAXIMUM DEFLECTIONS ON THE MIRRORS W_Σ	0.4λ	1.82λ	2.4λ
METHOD FOR CALCULATION THE PROFILE OF THE CORRECTION MIRROR	STREL'S NUMBER *St*		
TELESCOPIC REZONATOR WITH SPHERICAL MIRRORS	0.875	0.065	0.118
GRADIENT METHODS	1.01	1.03	1.03
APPROXIMATION OF DEFORMATIONS FOUND FROM THE SOLUTION OF THE SELF-CONSISTENT PROBLEM	1.004	1.02	1.01
CALCULATION OF THERMAL DEFORMATION FROM THE INTENSITY DISTRIBUTION IN FRONT OF THE OUTPUT MIRROR	0.996	0.97	0.96
UNIFORM ILLUMINATION OF THE OUTPUT MIRROR WITH THE AVERAGE (OVER THE OUTPUT APERTURE) INTENSITY	0.994	0.93	0.91

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